WAVELET BASED NUMERICAL SOLUTION OF BURGERS-FISHER EQUATIONS BY LIFTING SCHEMES

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Abstract

The Burgers-Fisher equation is a non linear partial differential equation that models various phenomena for example fluid dynamics, gas dynamics, heat transfers and etc. In this paper, we presented wavelet based Lifting schemes for the numerical solution of Burgers-Fisher equations using orthogonal and biorthogonal wavelet filter coefficients. The numerical results obtained by this scheme are compared with the exact solution to demonstrate the accuracy and also speeds up convergence in lesser computational time as compared with existing scheme. Some test problems are presented for the applicability and validity of the scheme.

Keywords: *Orthogonal and Biorthogonal wavelets; Lifting schemes; Burgers-Fisher equation.*

1. Introduction

We consider the nonlinear partial differential equation known as generalized Burgers-Fisher equation. This nonlinear partial differential equation appears in many fields of applied physics. Some of widely used applications are in fluid dynamics, turbulence and shock wave formation but also in financial mathematics. Burgers-Fisher equation is nonlinear partial differential equation that describes different physical phenomena as convection effect, diffusion transport or interaction between the reaction mechanisms**.** The equation has properties of convection from Burgers differential equation and properties of diffusion transport from Fisher differential equation. This nonlinear partial differential equation was discovered by J.M. Burgers and R.A. Fisher [1]. Also, this equation is a mix of parabolic-hyperbolic type of nonlinear partial differential equation which consists of diffusion term, reaction term, and convection term. Many methods has been developed to find the solution of this problem.

Recently, some of the iterative methods are used for the numerical and analytical solutions of linear and nonlinear partial differential equations. For example, Adomian decomposition method [2], Adaptive gird Haar wavelet collocation method [3] and Biorthogonal Wavelet Based Multigrid and Full Approximation Scheme [4], etc. Mathematical models of basic flow equations which describe unsteady transport problems are governed by a single or a system of nonlinear PDEs.

Wavelet analysis assumed significance due to successful applications in signal and image processing during the 1980s. The smooth orthonormal basis obtained by the translation and dilation of a single function in a hierarchical fashion proved very useful to develop compression algorithms for signals and images up to a chosen threshold of relevant amplitudes. Some of the major contributors to this theory are: multiresolution signal processing used in computer vision; sub band coding, developed for speech and image compression; and wavelet series expansion, developed in applied mathematics.

Some of the earlier works on wavelet based methods can be found in [5]. A collection of the discrete wavelet transforms (DWT) and the full approximation scheme (FAS) were introduced recently in [6-7]. The wavelet based full approximation scheme (WFAS) has exposed to be a very efficient and favorable method for numerous problems related to computational science and engineering fields [8]. These methods can be either used as an iterative solver or as a preconditioning technique, offering in many cases a better performance than some of the most innovative and existing FAS algorithms. Due to the efficiency and potentiality of WFAS, researches further have been carried out for its enrichment. In order to realize this task, work build that is orthogonal/biorthogonal discrete wavelet transform using lifting scheme [9]. Wavelet based lifting technique is introduced by Sweldens [10], which permits some improvements on the properties of existing wavelet transforms. Wavelet based numerical solution of elasto-hydrodynamic lubrication problems via lifting scheme was introduced by Shiralashetti et al. [11] .The technique has some numerical benefits as a reduced number of operations which are fundamental in the context of the iterative solvers. Evidently all attempts to simplify the wavelet solutions for partial differential equation (PDE) are welcome. In PDE, matrices arising from system are dense with non-smooth diagonal and smooth away from the diagonal. This smoothness of the matrix transforms into smallness using wavelet transform and it leads to design the effective wavelets based lifting scheme.

Lifting scheme is a new approach to construct the so-called second generation wavelets that are not necessarily translations and dilations of one function. The latter we refer to as a first generation wavelets or classical methods. The lifting scheme has some additional advantages in comparison with the classical wavelets. This transform works for signals of an arbitrary size with correct treatment of boundaries. Another feature of the lifting scheme is that all constructions are derived in the spatial domain. This is in contrast to the traditional approach, which relies heavily on the frequency domain. The two major advantageous are:

- **i.** It leads to a more intuitively appealing treatment better suited to those in interested in applications than mathematical foundations.
- **ii.** It makes a computational time optimal and sometimes increasing the speed of calculations.

The lifting scheme starts with a set of well-known filters, thereafter lifting steps are used an attempt to improve (lift) the properties of a corresponding wavelet decomposition. This procedure has some mathematical benefits as a reduced number of operations which are essential in the context of the iterative solvers. In addition to this, the present paper illustrates that the application of the lifting scheme for the numerical solution of Burger-Fisher equations.

The present paper is organized as follows: Preliminaries of wavelets and lifting scheme are given in section 2. Section 3 describes the method of solution. Numerical simulations of test problems are presented in section 4. Finally, the conclusion of the proposed work is given in section 5.

2. Preliminaries of wavelets and Lifting scheme

The lifting scheme starts with a set of well-known filters; thereafter lifting steps are used in attempt to improve the properties of corresponding wavelet decomposition.

Now, we have discussed about different wavelet filters as follows:

a) Haar wavelet filter coefficients

We know that low pass filter coefficients $\left[a_0,\ a_1\right]^T = \left[\frac{1}{\sqrt{2}},\ \frac{1}{\sqrt{2}}\right]^T$ and high pass filter coefficients $\left[b_0,\ b_1\right]^T = \left[\frac{1}{\sqrt{2}},\ \frac{1}{\sqrt{2}}\right]^T$ play an important role in decomposition. Thus it is natural to wonder that, it is possible to model the decomposition in terms of linear transformations i.e. matrices. Moreover, since digital signals and images are composed of discrete data, we need a discrete analog of the decomposition algorithm so that we can process signal and image data.

b) Daubechies wavelet filter coefficients

Daubechies introduced scaling functions having the shortest possible support. The scaling function ϕ_N has support $[0, N-1]$, while the corresponding wavelet ψ_N has support in the interval $[1-N/2, N/2]$.

We have low pass filter coefficients $[a_0, a_1, a_2, a_3]^T = \left[\frac{1+\sqrt{3}}{4\sqrt{2}}, \frac{3+\sqrt{3}}{4\sqrt{2}}, \frac{3-\sqrt{3}}{4\sqrt{2}}, \frac{1-\sqrt{3}}{4\sqrt{2}}\right]^T$

and high pass filter coefficients $[b_0, b_1, b_2, b_3]^T = \left[\frac{1-\sqrt{3}}{4\sqrt{2}}, -\frac{3-\sqrt{3}}{4\sqrt{2}}, \frac{3+\sqrt{3}}{4\sqrt{2}}, -\frac{1+\sqrt{3}}{4\sqrt{2}}\right]^T$

c) Biorthogonal (CDF (2,2)) wavelets

Let's consider the (5, 3) biorthogonal spline wavelet filter pair, the low pass filter pair are $(\tilde{a}_{-1}, \tilde{a}_0, \tilde{a}_1) = \left(\frac{1}{2\sqrt{2}}, \frac{1}{\sqrt{2}}, \frac{1}{2\sqrt{2}}\right)$ and $(a_{-2}, a_{-1}, a_0, a_1, a_2) = \left(\frac{-1}{4\sqrt{2}}, \frac{1}{2\sqrt{2}}, \frac{3}{2\sqrt{2}}, \frac{1}{2\sqrt{2}}, \frac{-1}{4\sqrt{2}}\right)$.

But, we have $b_k = (-1)^k \tilde{a}_{1-k}$ and $\tilde{b}_k = (-1)^k a_{1-k}$, the high pass filter pair are

$b_0 = \frac{1}{2\sqrt{2}}$, $b_1 = \frac{-1}{\sqrt{2}}$, $b_2 = \frac{1}{2\sqrt{2}}$ & $\tilde{b}_{-1} = \frac{1}{4\sqrt{2}}$, $\tilde{b}_0 = \frac{1}{2\sqrt{2}}$, $\tilde{b}_1 = \frac{-3}{2\sqrt{2}}$, $\tilde{b}_2 = \frac{1}{2\sqrt{2}}$, $\tilde{b}_3 = \frac{1}{4\sqrt{2}}$

Foundations of lifting scheme:

Consider to numbers *a, b* as two neighbouring samples of a sequence and then these have some correlation which we would like to take advantage. The simple linear transform which replaces *a* and *b* by average ***s*** and difference *d* i.e.

$$s = \frac{a+b}{2} \quad \& \quad d = \frac{a-b}{2}$$

The idea is that if *a* and *b* are highly correlated, the expected absolute value of their difference d will be small and can be represented with fever bits. In case that *a* = *b*, the difference is simply zero. We have not lost any information because we can always recover *a* and *b* from the given *s* and *d* as:

$$a = s - \frac{d}{2} \quad \& \quad b = s + \frac{d}{2}$$

Finally, a wavelet transform built through lifting consists of three steps: split. Predict and update as given in the Figure 1 [12].

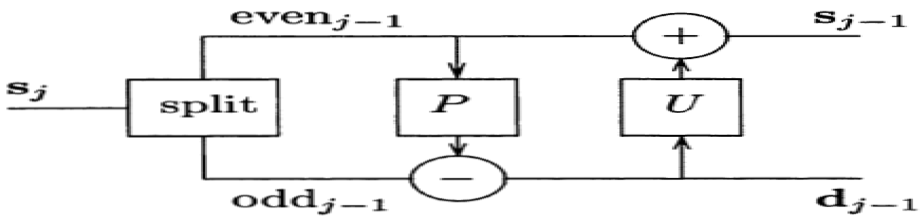


Fig. 1. Steps in lifting scheme

Split: Splitting the signal into two disjoint sets of samples.

Predict: If the signal contains some structure, then we can expect correlation between a sample and its nearest neighbors. i. e. $d_{j-1} = \text{odd}_{j-1} - \text{P}(\text{even}_{j-1})$

Update: Given an even entry, we have predicted that the next odd entry has the same value, and stored the difference. We then update our even entry to reflect our knowledge of the signal. i.e. $s_{j-1} = \text{even}_{j-1} + \text{U}(d_{j-1})$

The detailed algorithm using different wavelets is given in the next section. The general lifting stages for decomposition and reconstruction of a signal are given in Figure 2.

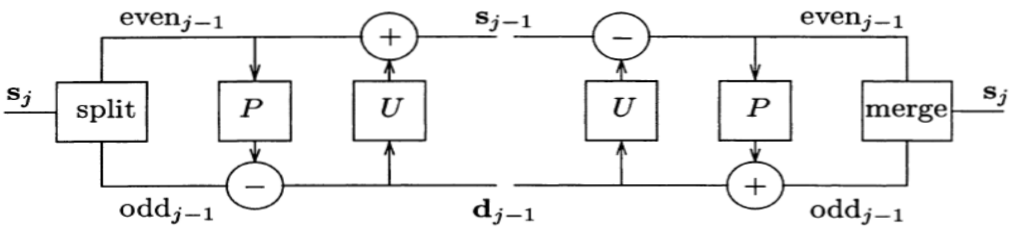


Fig. 2. Lifting wavelet algorithm.

The detailed algorithm using different wavelets is given in the next section.

3. Method of solution

Consider the generalized Burger-Fisher equation

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} - \alpha u^n \frac{\partial u}{\partial x} + \beta u(1 - u^n), \; 0 \le x \le 1 \; \& \; t > 0 \qquad (3.1)$$

Where α is any constant.

After discretizing the equation (3.1) through the finite difference method (FDM), we get system of algebraic equations. Through this system we can write the system as

$$Au = b \qquad (3.2)$$

where A is $N \times N$ coefficient matrix, b is $N \times N$ matrix and u is $N \times N$ matrix to be determined.

where $N = 2^J$, N is the number of grid points and J is the level of resolution.

Solve Eq. (3.2) through the iterative method, we get approximate solution u. Approximate solution containing some error, therefore required solution equals to sum of approximate solution and error. There are many methods to minimize such error to get the accurate solution. Some of them are FAS, WFAS etc. Now we are using the advanced technique based on different wavelets called as lifting scheme. Recently, lifting schemes are useful in the signal analysis and image processing in the area of science and engineering. But currently it extends to approximations in the numerical analysis [8]. Here, we are discussing the algorithm of the lifting schemes as follows.

3.1. Haar wavelet Lifting scheme (HWLS)

In [9], Daubechies and Sweldens have shown that every wavelet filter can be decomposed into lifting steps. More details of the advantages as well as other important structural advantages of the lifting technique can be available in [10]. The representation of Haar wavelet via lifting form presented as;

Decomposition:

Consider approximate solution $S = u$ like as signal and then apply the HWLS decomposition (finer to coarser) procedure as,

$$\left.\begin{aligned} &d^{(1)} = S_{2j} - S_{2j-1}, \\ &s^{(1)} = S_{2j-1} + \frac{1}{2} d^{(1)}, \\ &S_1 = \sqrt{2}\, s^{(1)} \quad \text{and} \\ &D = \frac{1}{\sqrt{2}} d^{(1)} \end{aligned}\right\} \tag{3.3}$$

In this stage finally, we get new approximation as,

$$S = [S_1 \;\; D] \tag{3.4}$$

Reconstruction:

Now apply the HWLS reconstruction (coarser to finer) procedure as,

$$\left.\begin{aligned} &d^{(1)} = \sqrt{2}\, D, \\ &s^{(1)} = \frac{1}{\sqrt{2}} S_1, \\ &S_{2j-1} = s^{(1)} - \frac{1}{2} d^{(1)} \quad \text{and} \\ &S_{2j} = d^{(1)} + S_{2j-1} \end{aligned}\right\} \tag{3.5}$$

which is the required solution of the given equation.

3.2. Daubechies wavelet Lifting scheme (DWLS)

As discussed in the previous section 3.1, we follow the same procedure but we used different wavelet i.e., Daubechies 4th order wavelet coefficient. The DWLS procedure is as follows;

Decomposition:

$$\left.\begin{aligned} s^{(1)} &= S_{2j-1} + \sqrt{3}\,S_{2j}, \\ d^{(1)} &= S_{2j} - \frac{\sqrt{3}}{4} s^{(1)} - \left(\frac{\sqrt{3}-2}{4}\right) s_1^{(j-1)}, \\ s^{(2)} &= s^{(1)} - d_1^{(j+1)}, \\ S_1 &= \frac{\sqrt{3}-1}{\sqrt{2}} s^{(2)} \quad \text{and} \\ D &= \frac{\sqrt{3}+1}{\sqrt{2}} d^{(1)} \end{aligned}\right\} \tag{3.6}$$

Here, we get new approximation as,

$$S = [S_1 \ \ D] \tag{3.7}$$

Reconstruction:

Now, we apply the DWLS reconstruction (coarser to finer) procedure as,

$$\left.\begin{aligned} d^{(1)} &= \frac{\sqrt{2}}{\sqrt{3}+1} D, \\ s^{(2)} &= \frac{\sqrt{2}}{\sqrt{3}-1} S_1, \\ s_1^{(j)} &= s^{(2)} + d_1^{(j+1)}, \\ S_{2j} &= d^{(1)} + \frac{\sqrt{3}}{4} s_1^{(j)} + \frac{\sqrt{3}-2}{4} s_1^{(j-1)} \quad \text{and} \\ S_{2j-1} &= s^{(1)} - \sqrt{3}\,S_{2j} \end{aligned}\right\} \tag{3.8}$$

which is the required solution of the given equation.

3.3. Biorthogonal wavelet Lifting scheme (BWLS)

As discussed in the previous sections 3.1 and 3.2, we follow the same procedure here we used another wavelet i.e., biorthogonal wavelet (CDF(2,2)). The BWLS procedure is as follows;

Decomposition:

$$\left.\begin{aligned} d^{(1)} &= S_{2j} - \frac{1}{2}\left[S_{2j-1} + S_{2j+2}\right], \\ s^{(1)} &= S_{2j-1} + \frac{1}{4}\left[d_{j-1}^{(1)} + d^{(1)}\right], \\ D &= \frac{1}{\sqrt{2}} d^{(1)}, \\ S_1 &= \sqrt{2}\, s^{(1)} \end{aligned}\right\} \tag{3.9}$$

In this stage finally, we get new signal as,

$$S = [S_1 \;\; D] \tag{3.10}$$

Reconstruction:

Now, we apply the DWLS reconstruction (coarser to finer) procedure as

$$\left.\begin{aligned} s^{(1)} &= \frac{1}{\sqrt{2}} S_1, \\ d^{(1)} &= \sqrt{2}\, D, \\ S_{2j-1} &= s^{(1)} - \frac{1}{4}\left[d_{j-1}^{(1)} + d^{(1)}\right] \\ S_{2j} &= d^{(1)} + \frac{1}{2}\left[S_{2j-1} + S_{2j+2})\right], \end{aligned}\right\} \tag{3.11}$$

which is the required solution of the given equation.

The coefficients $s_1^{(j)}$ and $d_1^{(j)}$ are the average and detailed coefficients respectively of the approximate solution u_a . The new approaches are tested through some of the numerical problems and the results are shown in next section.

4. Numerical simulation

In this section, we applied Lifting scheme for the numerical solution of Burger-Fisher equations and also show the capability and applicability of HWLS, DWLS and BWLS. The error is computed by

$$E_{\max} = \max\left| u_e - u_a \right|,$$

where u_e and u_a are exact and approximate solution respectively

Problem 4.1: Consider the Burger-Fisher equation, $\left(\text{In Eq. } (3.1)\text{: } \alpha = 1,\ n = 1\ \&\ \beta = 1\right)$

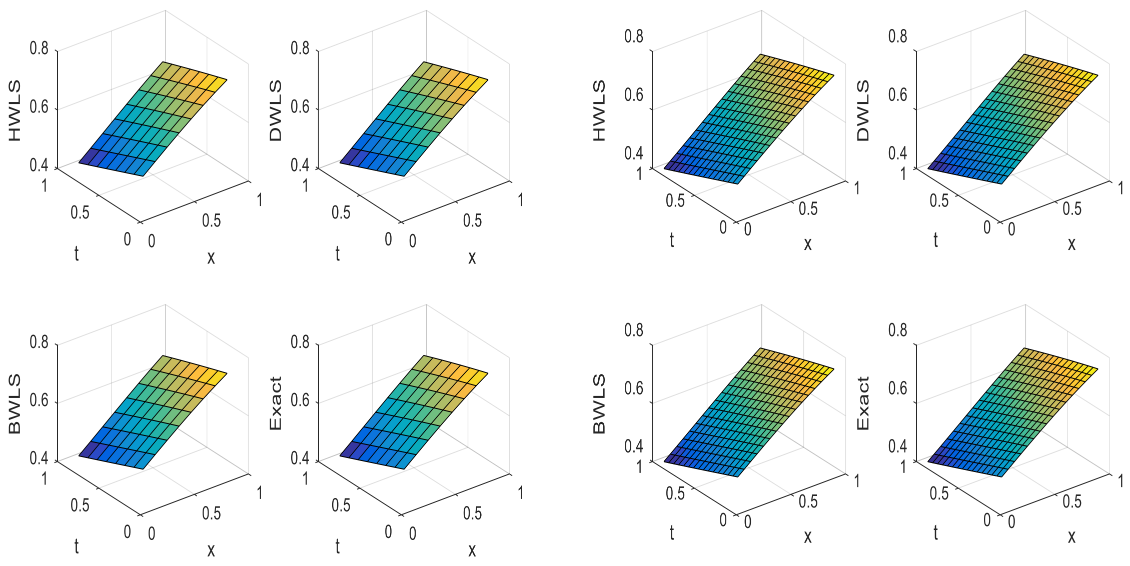
$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} - u\frac{\partial u}{\partial x} + u(1-u),\ 0 \le x \le 1\ \&\ t > 0 \tag{4.1.1}$$

subject to the I.C.:
$$u(x, 0) = \frac{1}{2} - \frac{1}{2}\tanh\left(\frac{x}{4}\right) \tag{4.1.2}$$

and B.C.s:

$$\left.\begin{aligned} u(0,t) &= \frac{1}{2} - \frac{1}{2}\tanh\left\{\frac{1}{4}\left(\frac{5}{2}t\right)\right\} \\ u(1,t) &= \frac{1}{2} - \frac{1}{2}\tanh\left\{\frac{1}{4}\left(1 - \frac{5}{2}t\right)\right\} \end{aligned}\right\}, \tag{4.1.3}$$

Which has the exact solution $u(x,t) = \frac{1}{2} - \frac{1}{2}\tanh\left\{\frac{1}{4}\left(x + \frac{5}{2}t\right)\right\}$ [13]. By applying the methods explained in the section 3, we obtain the numerical solutions and compared with exact solution are presented in figure 3. The maximum absolute errors with CPU time of the methods are presented in table 1.



$N \times N = 8\times 8$ $\qquad$ $N \times N = 16\times 16$

Fig. 3. Comparison of numerical solutions with exact solution of problem 4.1 for $N \times N = 8\times 8$ & 16×16.

Table 1. Maximum error and CPU time (in seconds) of the methods of problem 4.1.

$N \times N$	Method	E_{max}	Setup time	Running time	Total time
4 × 4	FDM	1.2712e-03	3.2169	0.0022	3.2191
	HWLS	1.2712e-03	0.0010	0.0034	0.0040
	DWLS	1.2712e-03	0.0004	0.0110	0.0114
	BWLS	1.2712e-03	0.0003	0.0040	0.0043
8 × 8	FDM	8.6089e-04	5.1159	0.0022	5.1181
	HWLS	8.6089e-04	0.0010	0.0032	0.0042
	DWLS	8.6089e-04	0.0003	0.0096	0.0099
	BWLS	8.6089e-04	0.0004	0.0041	0.0045
16 × 16	FDM	5.0257e-04	4.4860	0.0022	4.4882
	HWLS	5.0257e-04	0.0009	0.0029	0.0038
	DWLS	5.0257e-04	0.0003	0.0096	0.0099

	BWLS	5.0257e-04	0.0003	0.0040	0.0043
32×32	FDM	2.7521e-04	3.6156	0.0028	3.6184
	HWLS	2.7521e-04	0.0009	0.0029	0.0038
	DWLS	2.7521e-04	0.0003	0.0096	0.0099
	BWLS	2.7521e-04	0.0003	0.0040	0.0043
64×64	FDM	1.4855e-04	9.0913	0.0041	9.0954
	HWLS	1.4855e-04	0.0010	0.0030	0.0040
	DWLS	1.4855e-04	0.0003	0.0098	0.0101
	BWLS	1.4855e-04	0.0003	0.0041	0.0044

Problem 4.2 Consider the Burger-Fisher equation, $\left(\text{In Eq. } (3.1)\text{: } \alpha = -1,\ n = 1\ \&\ \beta = 2\right)$

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + u\frac{\partial u}{\partial x} + 2u(1-u),\ 0 \le x \le 1\ \&\ t > 0 \tag{4.2.1}$$

subject to the I.C.:

$$u(x,0) = \frac{1}{2} + \frac{1}{2}\tanh\left(\frac{x}{4}\right) \tag{4.2.2}$$

and B.C.s:

$$\left.\begin{aligned} u(0,t) &= \frac{1}{2} + \frac{1}{2}\tanh\left\{\frac{1}{4}\left(\frac{9}{2}t\right)\right\} \\ u(1,t) &= \frac{1}{2} + \frac{1}{2}\tanh\left\{\frac{1}{4}\left(1 + \frac{9}{2}t\right)\right\} \end{aligned}\right\}, \tag{4.2.3}$$

Which has the exact solution $u(x,t) = \frac{1}{2} + \frac{1}{2}\tanh\left\{\frac{1}{4}\left(x + \frac{9}{2}t\right)\right\}$ [14]. By applying the methods explained in the section 3, we obtain the numerical solutions and compared with exact solution are presented in figure 4. The maximum absolute errors with CPU time of the methods are presented in table 2.

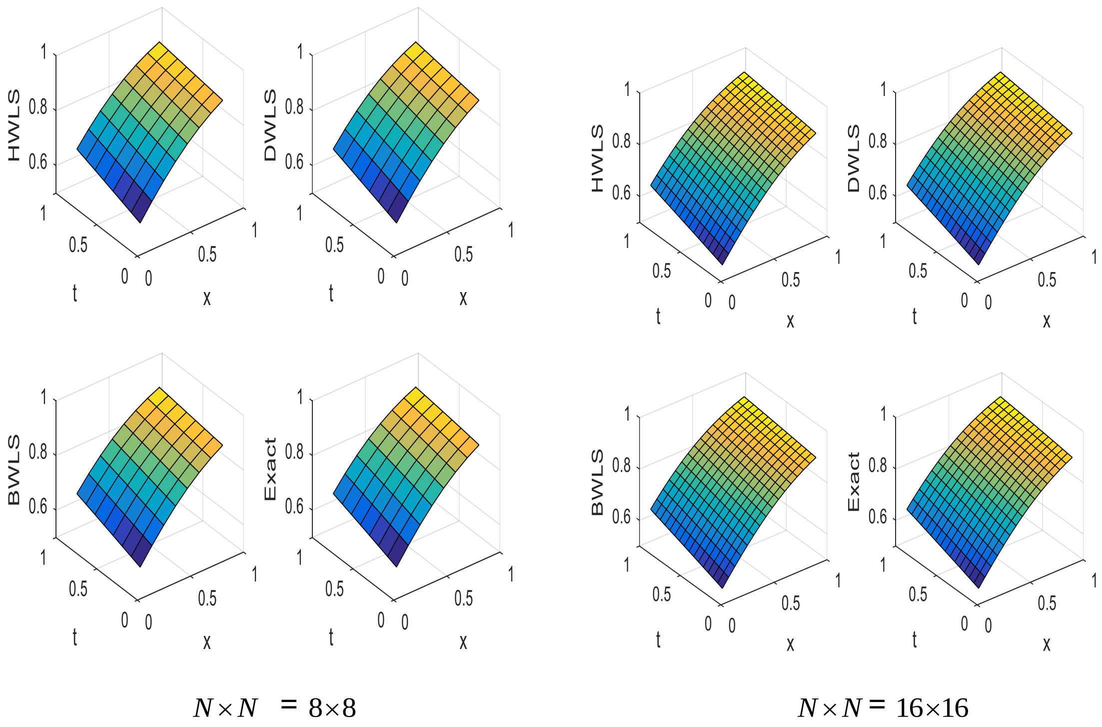


Fig. 4. Comparison of numerical solutions with exact solution of problem 4.2 for $N \times N = 8\times8$ & 16×16.

Table 2. Maximum error and CPU time (in seconds) of the methods of problem 4.2.

$N \times N$	Method	$E_{\max}$	Setup time	Running time	Total time
4×4	FDM	5.6596e-03	3.2373	0.0020	3.2393
	HWLS	5.6596e-03	0.0009	0.0029	0.0038
	DWLS	5.6596e-03	0.0003	0.0097	0.0100
	BWLS	5.6596e-03	0.0003	0.0040	0.0043
8×8	FDM	2.8678e-03	3.6739	0.0021	3.6960
	HWLS	2.8678e-03	0.0010	0.0031	0.0041
	DWLS	2.8678e-03	0.0003	0.0103	0.0106
	BWLS	2.8678e-03	0.0004	0.0042	0.0046
16×16	FDM	1.4963e-03	3.6426	0.0022	3.6448
	HWLS	1.4963e-03	0.0009	0.0029	0.0038
	DWLS	1.4963e-03	0.0003	0.0099	0.0102
	BWLS	1.4963e-03	0.0003	0.0042	0.0045
32×32	FDM	7.6815e-04	5.3950	0.0027	5.3977
	HWLS	7.6815e-04	0.0009	0.0030	0.0039
	DWLS	7.6815e-04	0.0003	0.0096	0.0099
	BWLS	7.6815e-04	0.0003	0.0047	0.0050
64×64	FDM	3.9016e-04	7.7931	0.0040	7.7971
	HWLS	3.9016e-04	0.0009	0.0030	0.0039
	DWLS	3.9016e-04	0.0003	0.0097	0.0100
	BWLS	3.9016e-04	0.0003	0.0041	0.0044

Problem 4.3 Consider the Burger-Fisher equation, $\left(\text{In Eq. } (3.1):\ \alpha = 1,\ n = 2\ \&\ \beta = 1\right)$

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} - u^2 \frac{\partial u}{\partial x} + u(1 - u^2),\ 0 \le x \le 1\ \&\ t > 0 \tag{4.3.1}$$

subject to the I.C.:

$$u(x,0) = \left[\frac{1}{2}\left\{1 - \tanh\left(\frac{x}{3}\right)\right\}\right]^{1/2} \tag{4.3.2}$$

and B.C.s:

$$\left.\begin{aligned} u(0,t) &= \left[\frac{1}{2}\left\{1 + \tanh\left(\frac{10}{9}t\right)\right\}\right]^{1/2} \\ u(1,t) &= \left[\frac{1}{2}\left\{1 - \tanh\left(\frac{1}{3} - \frac{10}{9}t\right)\right\}\right]^{1/2} \end{aligned}\right\}, \tag{4.3.3}$$

Which has the exact solution $u(x,t) = \left[\frac{1}{2}\left\{1 - \tanh\left(\frac{x}{3} - \frac{10}{9}t\right)\right\}\right]^{1/2}$ [1]. By applying the methods explained in the section 3, we obtain the numerical solutions and compared with exact solution are presented in figure 5. The maximum absolute errors with CPU time of the methods are presented in table 3.

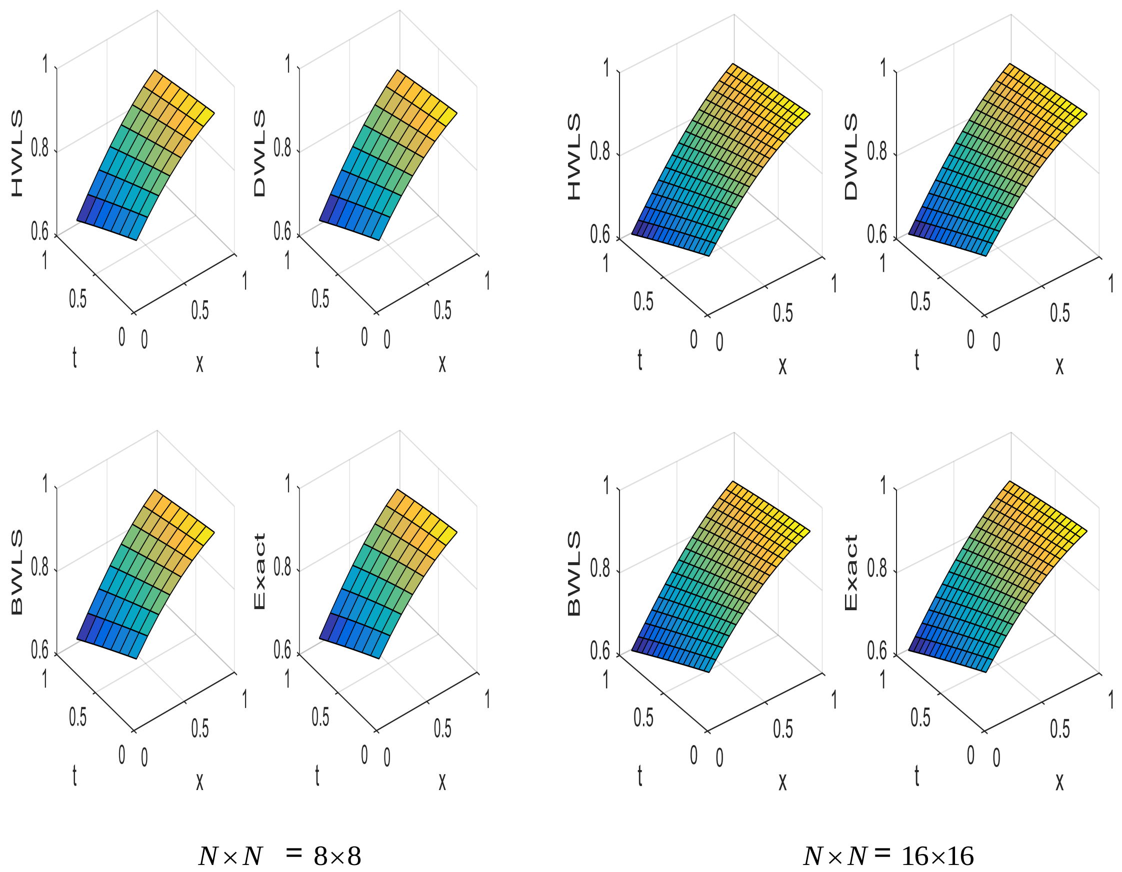


Fig. 5. Comparison of numerical solutions with exact solution of problem 4.3 for $N \times N = 8 \times 8\ \&\ 16 \times 16$.

Table 3. Maximum error and CPU time (in seconds) of the methods of problem 4.3.

$N \times N$	Method	$E_{\max}$	Setup time	Running time	Total time
4×4	FDM	4.4208e-03	3.1919	0.0019	3.1938
	HWLS	4.4208e-03	0.0009	0.0028	0.0037
	DWLS	4.4208e-03	0.0003	0.0096	0.0099
	BWLS	4.4208e-03	0.0003	0.0040	0.0043
8×8	FDM	2.0856e-03	4.6072	0.0019	4.6091
	HWLS	2.0856e-03	0.0008	0.0028	0.0037
	DWLS	2.0856e-03	0.0003	0.0096	0.0099
	BWLS	2.0856e-03	0.0004	0.0039	0.0043
16×16	FDM	1.1249e-03	3.3155	0.0022	3.3177
	HWLS	1.1249e-03	0.0009	0.0035	0.0044
	DWLS	1.1249e-03	0.0003	0.0096	0.0099
	BWLS	1.1249e-03	0.0003	0.0041	0.0044
32×32	FDM	5.8218e-04	4.6153	0.0028	4.6181
	HWLS	5.8218e-04	0.0009	0.0028	0.0037
	DWLS	5.8218e-04	0.0003	0.0100	0.0103
	BWLS	5.8218e-04	0.0004	0.0043	0.0047
64×64	FDM	2.9335e-04	7.5337	0.0041	7.5378
	HWLS	2.9335e-04	0.0009	0.0029	0.0038
	DWLS	2.9335e-04	0.0003	0.0098	0.0101
	BWLS	2.9335e-04	0.0003	0.0041	0.0044

5. Conclusions

In this paper, we obtained the numerical solution of Burger-Fisher equations by Lifting scheme using different wavelet filters. From the above figures and tables, we observed that

i. The numerical solutions obtained by different Lifting schemes are agrees with the exact solution.

ii. Convergence of the presented schemes is observed i.e. the error decreases when the level of resolution N increases.

iii. In addition the calculations involved in Lifting schemes are simple, straight forward and low computation cost compared to classical method i.e. FDM.

Hence the presented Lifting schemes in particular HWLS & BWLS are very effective for solving non-linear partial differential equations.

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