Field Aided Semiconductor Superlattices, the Einstein Relation and All That

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ABSTRACT

In this paper we study the Einstein relation for the diffusivity mobility ratio (DMR) under magnetic quantization in III-V, II-VI, IV-VI and HgTe/CdTe SLs with graded interfaces by formulating the appropriate electron statistics. We have also investigated the DMR in III-V, II-VI, IV-VI and HgTe/CdTe effective mass SLs in the presence of quantizing magnetic field respectively. The DMRs in quantum wire $GaAs/Ga_{1-x}Al_xAs$, CdS/CdTe, PbTe/PbSnTe and HgTe/CdTe SLs and the corresponding effective mass SLs have further been studied. It appears that the DMR oscillates both with inverse quantizing magnetic field and electron concentration for $GaAs/Ga_{1-x}Al_xAs$, CdS/CdTe, PbTe/PbSnTe and HgTe/CdTe superlattices with graded interfaces. The DMR decreases with increasing film thickness and decreasing electron concentration for the said superlattices under 2D quantization of wave vector space.

Keywords: *Einstein Relation, Semiconductor Superlattices, Magnetic Quantization, Quantum Wire Superlattices*

Introduction

It is well known that Keldysh [1] first suggested the fundamental concept of a superlattice (SL), although it was successfully experimental realized by Esaki and Tsu [2]. The importance of SLs in the field of nanoelectronics have already been described in [3-5]. The most extensively studied III-V SL is the one consisting of alternate layers of GaAs and $Ga_{1-x}Al_xAs$ owing to the relative ease of fabrication. The GaAs layers forms quantum wells and $Ga_{1-x}Al_xAs$ form potential barriers. The III-V SL's are attractive for the realization of high speed electronic and optoelectronic devices [6]. In addition to SLs with usual structure, SLs with more complex structures such as II-VI [7], IV-VI [8] and HgTe/CdTe [9] SL's have also been proposed.

The IV-VI SLs exhibit quite different properties as compared to the III-V SL due to the peculiar band structure of the constituent materials [10]. The epitaxial growth of II-VI SL is a relatively recent development and the primary motivation for studying the mentioned SLs made of materials with the large band gap is in their potential for optoelectronic operation in the blue [10]. HgTe/CdTe SL's have raised a great deal of attention since 1979, when as a promising new materials for long wavelength infrared detectors and other electro-optical applications [11]. Interest in Hg-based SL's has been further increased as new properties with potential device applications were revealed [11, 12]. These features arise from the unique zero band gap material HgTe [13] and the direct band gap semiconductor CdTe which can be described by the three band mode of Kane [14]. The combination of the aforementioned materials with specified dispersion relation makes HgTe/CdTe SL very attractive, especially because of the possibility to tailor the material properties for various applications by varying the energy band constants of the SLs. In addition to it, for effective mass SLs, the electronic subbands appear continually in real space [15].

We note that all the aforementioned SLs have been proposed with the assumption that the interfaces between the layers are sharply defined, of zero thickness, i.e., devoid of any interface effects. The SL potential distribution may be then considered as a one dimensional array of rectangular potential wells. The aforementioned advanced experimental techniques may produce SLs with physical interfaces between the two materials crystallo graphically abrupt; adjoining their interface will change at least on an atomic scale. As the potential form changes from a well (barrier) to a barrier (well), an intermediate potential region exists for the electrons. The influence of finite thickness of the interfaces on the electron dispersion law is very important, since; the electron energy spectrum governs the electron transport in SLs.

In recent years there has been considerable work in studying the Einstein relation (a very important transport quantity for modern nano-devices) under different physical conditions [19-68]. In this paper, we shall study the Einstein relation for the diffusivity –mobility-ratio (DMR)under magnetic quantization in III-V, II-VI, IV-VI and HgTe/CdTe SLs with graded interfaces in sections 2.1 to 2.4. From sections 2.5 to 2.8, we shall investigate the DMR in III-V, II-VI, IV-VI and HgTe/CdTe effective mass SLs in the presence of quantizing magnetic field respectively. In sections 2.9 to 2.16, we shall investigate the DMR in the aforementioned SLs in the presence of two dimensional size quantizations. In section 3, the doping and magnetic field dependences of the DMRs have been studied by taking $GaAs/Ga_{1-x}Al_xAs$, CdS/CdTe, PbTe/PbSnTe and HgTe/CdTe SLs and the corresponding effective mass SLs as examples. In the same section, we have also studied the doping and thickness dependences of the DMR for the said quantum wire SLs. The section 3 contains the result and discussions as appropriate for this paper.

2 Theoretical Background

2.1 Einstein relation under magnetic quantization in III-V superlattices with graded interfaces

The energy spectrum of the conduction electrons in bulk specimens of the constituent materials of III-V SLs whose energy band structures are defined by three band model of Kane can be written as

$$(\eta^2 k^2)/(2m_i^*) = EG(E, E_{gi}, \Delta_i) \quad (1)$$

$$i = 1,2,\ldots,\; G(E, E_{gi}, \Delta_i) \equiv \frac{\left(E_{gi} + \frac{2}{3}\Delta_i\right)(E + E_{gi} + \Delta_i)(E + E_{gi})}{\left[E_{gi}(E_{gi} + \Delta_i)\left(E + E_{gi} + \frac{2}{3}\Delta_i\right)\right]} .$$

h is Planck constant, k is electron wave vector, m_i^* is the effective electron mass at the edge of the conduction band, E is the electron energy, E_{gi} is the band gap and Δ_i is the spin orbit splitting constant.

Therefore, the dispersion law of the electrons of III-V SLs with graded interfaces can be expressed, following Jiang and Lin [16], as

$$\cos(L_o k) = \frac{1}{2}\Phi(E, k_s) \quad (2)$$

where $L_0 (\equiv a_0 + b_0)$ is the period length, a_0 and b_0 are the widths of the barrier and the well respectively,

$$\Phi(E,k_s) \equiv \Big[2\cosh\{\beta(E,k_s)\}\cos\{\gamma(E,k_s)\} + \varepsilon(E,k_s)\sinh\{\beta(E,k_s)\}\sin\{\gamma(E,k_s)\}$$
$$+\Delta_0\left[\left(\frac{K_1^2(E,k_s)}{K_2(E,k_s)} - 3K_2(E,k_s)\right)\cosh\{\beta(E,k_s)\}\sin\{\gamma(E,k_s)\} + \left(3K_1(E,k_s) - \frac{\{K_2(E,k_s)\}^2}{K_1(E,k_s)}\right)\sinh\{\beta(E,k_s)\}\cos\{\gamma(E,k_s)\}\right]$$
$$+\Delta_0\Big[2\left(\{K_1(E,k_s)\}^2 - \{K_2(E,k_s)\}^2\right)\cosh\{\beta(E,k_s)\}\cos\{\gamma(E,k_s)\}$$
$$+\frac{1}{12}\left[\frac{5\{K_2(E,k_s)\}^3}{K_1(E,k_s)} + \frac{5\{K_1(E,k_s)\}^3}{K_2(E,k_s)} - 34K_2(E,k_s)K_1(E,k_s)\right]\sinh\{\beta(E,k_s)\}\sin\{\gamma(E,k_s)\}\Big]\Big]$$

, $\varepsilon(E,k_s) \equiv \left[\frac{K_1(E,k_s)}{K_2(E,k_s)} - \frac{K_2(E,k_s)}{K_1(E,k_s)}\right]$, $\beta(E,k_s) \equiv K_1(E,k_s)[a_0 - \Delta_0]$, Δ_o is the interface width, $\gamma(E,k_s)=K_2(E,k_s)[b_0-\Delta_0]$, $K_1(E,k_s)\equiv\left[\frac{2m_2^*E'}{\hbar^2}G(E-V_o,\alpha_2,\Delta_2)+k_s^2\right]^{1/2}$, $E'\equiv V_0-E$, V_0 is the potential barrier encountered by the electron $\left(V_0\equiv\left|E_{g_2}-E_{g_1}\right|\right)$, $\alpha_i\equiv 1/E_{gi}$,

$K_2(E,k_s)\equiv\left[\frac{2m_1^*E}{\hbar^2}G(E,\alpha_1,\Delta_1)-k_s^2\right]^{1/2}$ and $k_s^2=k_x^2+k_y^2$.

In the presence of a quantizing magnetic field B along z-direction, the simplified magneto-dispersion relation can be, written as=

$$k_z=\frac{1}{L_o}\left[\rho(n,E)-\left\{\frac{2|e|B}{\hbar}L_0^2\left(n+\frac{1}{2}\right)\right\}\right]^{1/2} \tag{3}$$

where $\rho(n,E)=[\cos^{-1}\{\frac{1}{2}\psi(n,E)\}]^2$, n is the Landau quantum number,

$$\psi(n,E)=\Big[2\cosh\{\beta(n,E)\}\cos\{\gamma(n,E)\}+\varepsilon(n,E)\sinh\{\beta(n,E)\}$$

$$\sin\{\gamma(n,E)\}+\Delta_0\left[\left(\frac{\{K_1(n,E)\}^2}{K_2(n,E)}-3K_2(n,E)\right)\cosh\{\beta(n,E)\}\right.$$

$$\sin\{\gamma(n,E)\}+\left(3K_1(n,E)-\frac{\{K_2(n,E)\}^2}{K_1(n,E)}\right)\sinh\{\beta(n,E)\}\cos\{\gamma(n,E)\}$$

$$+\Delta_0\left[2\left(\{K_1(n,E)\}^2-\{K_2(n,E)\}^2\right)\cosh\{\beta(n,E)\}\cos\{\gamma(n,E)\}\right.$$

$$\left.\left.+\frac{1}{12}\left(\frac{5\{K_1(n,E)\}^3}{K_2(n,E)}+\frac{5\{K_2(n,E)\}^3}{K_1(n,E)}-\{34K_2(n,E)K_1(n,E)\}\right)\sinh\{\beta(n,E)\}\sin\{\gamma(n,E)\}\right]\right]$$

$$\varepsilon(n,E)\equiv\left[\frac{K_1(n,E)}{K_2(n,E)}-\frac{K_2(n,E)}{K_1(n,E)}\right],\beta(n,E)\equiv K_1(n,E)[a_0-\Delta_0]$$

$$K_1(n,E)\equiv\left[\frac{2m_2^*E'}{\hbar^2}G(E-V_o,\alpha_2,\Delta_2)+\frac{2|e|B}{\hbar}\left(n+\frac{1}{2}\right)\right]^{1/2}$$

$\gamma(n,E)=K_2(n,E_s)[b_0-\Delta_0]$ *and* $K_2(n,E)\equiv\left[\frac{2m_1^*E}{\hbar^2}G(E,\alpha_1,\Delta_1)-\left\{\frac{2|e|B}{\hbar}\left(n+\frac{1}{2}\right)\right\}\right]^{1/2}$ Considering only the lowest miniband, since in an actual SL only the lowest miniband is significantly populated at low temperatures, where the quantum effects become prominent, the electron concentration (n_0

)can be written as, $n_0 = \left(\frac{|e| B g_v}{\pi^2 \hbar L_0}\right) \sum_{n=0}^{n_{\max}} \left[T_{91}\left(n, E_{FSL}\right) + T_{92}\left(n, E_{FSL}\right)\right]$ (4)

where g_v is the valley degeneracy,

$$T_{91}\left(n, E_{FSL}\right) \equiv \left[\rho(E_{FSL}, n) - \left\{\frac{2|e|B}{\hbar}\left(n+\frac{1}{2}\right)L_0^2\right\}\right]^{1/2}, \qquad T_{92}\left(n, E_{FSL}\right) \equiv \sum_{r=1}^{s} L\left(r\right)\left[T_{91}\left(n, E_{FSL}\right)\right],$$

$L(r) \equiv \left[2\left(k_B T\right)^{2r}\left(1-2^{1-2r}\right)\xi\left(2r\right)\right]\left[\frac{\partial^{2r}}{\partial E_{FSL}^{2r}}\right]$, *r is the set of real positive integers whose upper limit is s*, $\zeta(2r)$ *is the Zeta function of order* $2r$, k_B is the Boltzmann constant and T is the temperature.

The DMR can be expressed as

$$\frac{D}{\mu} = \frac{1}{|e|} n_0 \left[\frac{\partial n_0}{\partial E_{FSL}}\right]^{-1} \quad (5)$$

The use of equations (4) and (5) leads to the expression of the DMR in this case as

$$\frac{D}{\mu} = \frac{1}{|e|} \frac{\sum_{n=0}^{n_{\max}} \left[T_{91}\left(n, E_{FSL}\right) + T_{92}\left(n, E_{FSL}\right)\right]}{\sum_{n=0}^{n_{\max}} \left[\left\{T_{91}\left(n, E_{FSL}\right)\right\}' + \left\{T_{92}\left(n, E_{FSL}\right)\right\}'\right]} \quad (6)$$

2.2 Einstein relation under magnetic quantization in II-VI superlattices with graded interfaces

The energy spectrum of the conduction electrons of the constituent materials of II-VI SLs are given by [17]

$$E = \frac{\hbar^2 k_s^2}{2m_{\perp,1}^*} + \frac{\hbar^2 k_z^2}{2m_{\parallel,1}^*} + \bar{\lambda}_0 k_s \quad (7)$$

and

$$\frac{\hbar^2 k^2}{2m_2^*} = EG\left(E, E_{g2}, \Delta_2\right) \quad (8)$$

where $m_{\perp,1}^*$ and $m_{\parallel,1}^*$ are the transverse and longitudinal effective electron masses respectively at the edge of the conduction band for the first material and $\bar{\lambda}_0$ is the splitting of the two spin-states by the spin –orbit coupling and the crystalline field .

The electron dispersion law in II-VI SLs with graded interfaces can be expressed as

$$\cos\left(L_o k\right)=\frac{1}{2}\Phi_1\left(E,k_s\right) \tag{9}$$

where,

$$\Phi_1(E,k_s)\equiv\left[2\cosh\{\beta_1(E,k_s)\}\cos\{\gamma_1(E,k_s)\}+\varepsilon_1(E,k_s)\sinh\{\beta_1(E,k_s)\}\sin\{\gamma_1(E,k_s)\}\right.$$
$$+\Delta_0[(\frac{\{K_3(E,k_s)\}^2}{K_4(E,k_s)}-3K_4(E,k_s))\cosh\{\beta_1(E,k_s)\}\sin\{\gamma_1(E,k_s)\}+(3K_3(E,k_s)-\frac{\{K_4(E,k_s)\}^2}{K_3(E,k_s)})$$
$$\sinh\{\beta_1(E,k_s)\}\cos\{\gamma_1(E,k_s)\}]$$
$$+\Delta_0[2(\{K_3(E,k_s)^2-\{K_4(E,k_s)\}^2)\cosh\{\beta_1(E,k_s)\}\cos\{\gamma_1(E,k_s)\}$$
$$+\frac{1}{12}[\frac{5\{K_3(E,k_s)^3}{K_4(E,k_s)}+\frac{5\{K_4(E,k_s)\}^3}{K_3(E,k_s)}-34K_4(E,k_s)K_3(E,k_s)]\sinh\{\beta_1(E,k_s)\}\sin\{\gamma_1(E,k_s)\}]]$$
$$\varepsilon_1(E,k_s)\equiv[\frac{K_3(E,k_s)}{K_4(E,k_s)}-\frac{K_4(E,k_s)}{K_3(E,k_s)}],\beta_1(E,k_s)\equiv K_3(E,k_s)[a_0-\Delta_0],$$
$$\gamma_1(E,k_s)=K_4(E,k_s)[b_0-\Delta_0],K_3(E,k_s)\equiv[\frac{2m_2^*E'}{\hbar^2}G(E-V_o,\alpha_2,\Delta_2)+k_s^2]^{1/2}$$

and

$$K_4\left(E,k_s\right)\equiv[\frac{2m_{\|,1}^*}{\hbar^2}[E-\frac{\hbar^2k_s^2}{2m_{\perp,1}^*}\mp\bar{\lambda}_0k_s]]^{1/2} \tag{10}$$

In the presence of a quantizing magnetic field B along z-direction, the simplified magneto-dispersion relation can be, written as

$$k_z^2=\frac{1}{L_0^2}\left[\rho_1\left(n,E\right)-\left\{\frac{2|e|B}{\hbar}L_o^2\left(n+\frac{1}{2}\right)\right\}\right] \tag{11}$$

where,

$$\rho_1\left(n,E\right)=\left[\cos^{-1}\left\{\frac{1}{2}\psi_1\left(n,E\right)\right\}\right]^2$$

$$\psi_1(n,E)=\left[2\cosh\{\beta_1(n,E)\}\cos\{\gamma_1(n,E)\}+\varepsilon_1(n,E)\sinh\{\beta_1(n,E)\}\right.$$

$$\sin\{\gamma_1(n,E)\}+\Delta_0\left[\left(\frac{\{K_3(n,E)\}^2}{K_4(n,E)}-3K_4(n,E)\right)\cosh\{\beta_1(n,E)\}\right.$$

$$\sin\{\gamma_1(n,E)\}+\left(3K_3(n,E)-\frac{\{K_4(n,E)\}^2}{K_3(n,E)}\right)\sinh\{\beta_1(n,E)\}\cos\{\gamma_1(n,E)\}$$

$$+\Delta_0\left[2\left(K_3(n,E)-\{K_4(n,E)\}^2\right)\cosh\{\beta_1(n,E)\}\cos\{\gamma_1(n,E)\}\right.$$

$$\left.\left.+\frac{1}{12}\left(\frac{5\{K_3(n,E)\}^3}{K_4(n,E)}+\frac{5\{K_4(n,E)\}^3}{K_3(n,E)}-\{34K_4(n,E)K_3(n,E)\}\right)\sinh\{\beta_1(n,E)\}\sin\{\gamma_1(n,E)\}\right]\right]$$

$$\varepsilon_1(n,E)\equiv\left[\frac{K_3(n,E)}{K_4(n,E)}-\frac{K_4(n,E)}{K_3(n,E)}\right],\beta_1(n,E)\equiv K_3(n,E)[a_0-\Delta_0],\gamma_1(n,E)=K_4(n,E)[b_0-\Delta_0],$$

$$K_3(n,E)\equiv\left[\frac{2m_2^*}{\hbar^2}E'G(E-V_0,\alpha_2,\Delta_2)+\frac{2|e|B}{\hbar}\left(n+\frac{1}{2}\right)\right]^{1/2}$$

and

$$K_4(n,E)\equiv\left[\frac{2m_{\mathrm{P},1}^*}{\hbar^2}\left[E-\frac{\hbar|e|B}{m_{\perp,1}^*}\left(n+\frac{1}{2}\right)\mp\bar{\lambda}_0\left\{\frac{2|e|B}{\hbar}\left(n+\frac{1}{2}\right)\right\}^{1/2}\right]\right]^{1/2}$$

The electron concentration in this case can be expressed as

$$n_0=\left(\frac{|e|Bg_v}{2\pi^2\hbar L_0}\right)\sum_{n=0}^{n_{\max}}\left[T_{93}(n,E_{FSL})+T_{94}(n,E_{FSL})\right] \tag{12}$$

where,

$$T_{93}(n,E_{FSL})\equiv\left[\rho_1(n,E_{FSL})-\left\{\frac{2|e|B}{\hbar}\left(n+\frac{1}{2}\right)L_0^2\right\}\right]^{1/2} \text{ and } T_{94}(n,E_{FSL})\equiv\sum_{r=1}^{s}L(r)\left[T_{93}(n,E_{FSL})\right]$$

The use of equations (12) and (5) leads to the expression of the DMR as

$$\frac{D}{\mu}=\frac{1}{|e|}\frac{\sum_{n=0}^{n_{\max}}\left[T_{93}(n,E_{FSL})+T_{94}(n,E_{FSL})\right]}{\sum_{n=0}^{n_{\max}}\left[\{T_{93}(n,E_{FSL})\}'+\{T_{94}(n,E_{FSL})\}'\right]} \tag{13}$$

2.3 Einstein relation under magnetic quantization in IV-VI superlattices with graded interfaces

The **E-k** dispersion relation of the conduction electrons of the constituent materials of the IV-VI SLs can be expressed [18] as

$$E = a_i k_s^2 + b_i k_z^2 + \left[\left[c_i k_s^2 + d_i k_z^2 \right] + \left(e_i k_s^2 + f_i k_y^2 + \frac{E_{g_i}}{2} \right)^2 \right]^{1/2} - \frac{E_{g_i}}{2} \qquad (14)$$

where, $a_i \equiv \left[\frac{\text{h}^2}{2m_{\perp,i}^-} \right]$, $b_i \equiv \left(\frac{\text{h}^2}{2m_{\text{P},i}^-} \right)^2$, $c_i \equiv P_{\perp,i}^2$, $d_i \equiv P_{\perp,i}^2$, $e_i \equiv \left[\frac{\text{h}^2}{2m_{\perp,i}^+} \right]$ and $f_i \equiv \left(\frac{\text{h}^2}{2m_{\text{P},i}^+} \right)^2$.

The electron dispersion law in IV-VI SLs with graded interfaces can be expressed as

$$\cos\left(L_o k\right)=\frac{1}{2}\Phi_2\left(E,k_s\right) \tag{15}$$

where,

$$\Phi_2\left(E,k_s\right)\equiv\Big[2\cosh\left\{\beta_2\left(E,k_s\right)\right\}\cos\left\{\gamma_2\left(E,k_s\right)\right\}+\varepsilon_2\left(E,k_s\right)\sinh\left\{\beta_2\left(E,k_s\right)\right\}\sin\left\{\gamma_2\left(E,k_s\right)\right\}$$

$$+\Delta_0[(\frac{\left\{K_5\left(E,k_s\right)\right\}^2}{K_6\left(E,k_s\right)}-3K_6\left(E,k_s\right))\cosh\left\{\beta_2\left(E,k_s\right)\right\}\sin\left\{\gamma_2\left(E,k_s\right)\right\}+(3K_5\left(E,k_s\right)-\frac{\left\{K_6\left(E,k_s\right)\right\}^2}{K_5\left(E,k_s\right)})$$

$$\sinh\left\{\beta_2\left(E,k_s\right)\right\}\cos\left\{\gamma_2\left(E,k_s\right)\right\}]$$

$$+\Delta_0\left[2\left(\left\{K_5\left(E,k_s\right)\right\}^2-\left\{K_6\left(E,k_s\right)\right\}^2\right)\cosh\left\{\beta_2\left(E,k_s\right)\right\}\cos\left\{\gamma_2\left(E,k_s\right)\right\}\right.$$

$$\left.\left.+\frac{1}{12}\left[\frac{5\left\{K_5\left(E,k_s\right)\right\}^3}{K_6\left(E,k_s\right)}+\frac{5\left\{K_6\left(E,k_s\right)\right\}^3}{K_5\left(E,k_s\right)}-34K_6\left(E,k_s\right)K_5\left(E,k_s\right)\right]\sinh\left\{\beta_2\left(E,k_s\right)\right\}\sin\left\{\gamma_2\left(E,k_s\right)\right\}\right]\right]$$

$$\varepsilon_2\left(E,k_s\right)\equiv\left[\frac{K_5\left(E,k_s\right)}{K_6\left(E,k_s\right)}-\frac{K_6\left(E,k_s\right)}{K_5\left(E,k_s\right)}\right],\beta_2\left(E,k_s\right)\equiv K_5\left(E,k_s\right)\left[a_0-\Delta_0\right],\gamma_2\left(E,k_s\right)=K_6\left(E,k_s\right)\left[b_0-\Delta_0\right]$$

$$K_6\left(E,k_x,k_y\right)\equiv\left[\left[EH_{11}+H_{21}\left(k_x,k_y\right)\right]-\left[H_{31}E^2+EH_{41}\left(k_x,k_y\right)+H_{51}\left(k_x,k_y\right)\right]^{1/2}\right]^{1/2},H_{1i}=\left(b_i^2-f_i^2\right)^{-1}$$

and

$$H_{2i}\left(k_x,k_y\right)=\left[2H_{1i}\right]^{-1}\left[E_{g_i}b_i+d_i+f_iE_{g_i}+2\left(e_if_i-a_ib_i\right)\left(k_x^2+k_y^2\right)\right],H_{3i}=\frac{f_i^2}{\left(b_i^2-f_i^2\right)^2},$$

$$H_{4i}\left(k_x,k_y\right)=\left[4H_{1i}^2\right]^{-1}\left[4b_id_i+4b_if_iE_{g_i}+4f_i^2E_{g_i}+8\left(k_x^2+k_y^2\right)\left[e_if_ib_i-a_if_i^2\right]\right],$$

$$H_{5i}\left(k_x,k_y\right)\equiv\left[4H_{1i}^2\right]\left[\left(k_x^2+k_y^2\right)^2\left[-8a_ib_ie_if_i+4b_i^2e_i^2+4f_i^2a_i^2\right]+\left(k_x^2+k_y^2\right)\left[4e_if_iE_{g_i}b_i\right.\right.$$

$$\left.-4e_if_id_i+4e_if_i^2E_{g_i}-4a_ib_i^2E_{g_i}-4a_ib_id_i-4a_ib_if_iE_{g_i}+4b_i^2e_iE_{g_i}+4b_i^2c_i+4b_i^2E_{g_i}a_i-4f_i^2e_iE_{g_i}-4f_i^2c_i-4f_i^2E_{g_i}a_i\right]$$

$$\left.+\left[E_{g_i}^2b_i^2+d_i^2+f_i^2g_i^2+2E_{g_i}b_id_i+2E_{g_i}^2b_if_i+2d_if_iE_{g_i}\right]\right]$$

and

$$K_5\left(E,k_x,k_y\right)\equiv\left[\left[\left(E-V_0\right)^2H_{32}+\left(E-V_0\right)H_{42}\left(k_x,k_y\right)+H_{52}\left(k_x,k_y\right)\right]^{1/2}-\left[\left(E-V_0\right)H_{12}+H_{22}\left(k_x,k_y\right)\right]\right]^{1/2}$$

The simplified magneto-dispersion relation in this case can be written as

$$k_z=\frac{1}{L_0}\left[\rho_2\left(n,E\right)-\left\{\frac{2|e|B}{\mathrm{h}}L_o^2\left(n+\frac{1}{2}\right)\right\}\right]^{1/2} \tag{16}$$

where,

$$\rho_2(n,E)=\left[\cos^{-1}\left\{\frac{1}{2}\psi_2(n,E)\right\}\right]^2, \psi_2(n,E)=\left[2\cosh\{\beta_2(n,E)\}\cos\{\gamma_2(n,E)\}+\varepsilon_2(n,E)\sinh\{\beta_2(n,E)\},\right.$$

$$\sin\{\gamma_2(n,E)\}+\Delta_0[(\frac{\{K_5(n,E)\}^2}{K_6(n,E)}-3K_6(n,E))\cosh\{\beta_2(n,E)\}\sin\{\gamma_2(n,E)\}+(3K_5(n,E)-\frac{\{K_6(n,E)\}^2}{K_5(n,E)})$$

$$\sinh\{\beta_2(n,E)\}\cos\{\gamma_2(n,E)\}$$

$$\sin\{\gamma_2(n,E)\}+\left(3K_5(n,E)-\frac{\{K_6(n,E)\}^2}{K_5(n,E)}\right)\sinh\{\beta_2(n,E)\}\cos\{\gamma_2(n,E)\}$$

$$+\Delta_0\left[2\left(K_5(n,E)-\{K_6(n,E)\}^2\right)\cosh\{\beta_2(n,E)\}\cos\{\gamma_2(n,E)\}\right.$$

$$\left.\left.+\frac{1}{12}\left(\frac{5\{K_5(n,E)\}^3}{K_6(n,E)}+\frac{5\{K_6(n,E)\}^3}{K_5(n,E)}-\{34K_6(n,E)K_5(n,E)\}\right)\sinh\{\beta_2(n,E)\}\sin\{\gamma_2(n,E)\}\right]\right]$$

$$\varepsilon_2(n,E)\equiv\left[\frac{K_5(n,E)}{K_6(n,E)}-\frac{K_6(n,E)}{K_5(n,E)}\right], \beta_2(n,E)\equiv K_5(n,E)[a_0-\Delta_0], K_6(n,E)\equiv\left[\left[EH_{11}+H_{21}(n)\right]-\left[E^2H_{31}+EH_{41}(n)+H_{51}(n)\right.\right.$$

$$H_{1i}=\left(b_i^2-f_i^2\right)^{-1}$$

and $H_{2i}(n)=[2H_{1i}]^{-1}\left[E_{g_i}b_i+d_i+f_iE_{g_i}+2(e_if_i-a_ib_i)\left\{\frac{2|e|B}{\text{h}}\left(n+\frac{1}{2}\right)\right\}\right]$, $H_{3i}=\frac{f_i^2}{\left(b_i^2-f_i^2\right)^2}$,

$$H_{4i}(n)=\left[4H_{1i}^2\right]^{-1}\left[4b_id_i+4b_if_iE_{g_i}+4f_i^2E_{g_i}+8\left\{\frac{2|e|B}{\text{h}}\left(n+\frac{1}{2}\right)\right\}\left[e_if_ib_i-a_if_i^2\right]\right]$$

$$H_{5i}(n)\equiv\left[4H_{1i}^2\right]^{-1}\left[\left\{\frac{2|e|B}{\text{h}}\left(n+\frac{1}{2}\right)\right\}^2\left[-8a_ib_ie_if_i+4b_i^2e_i^2+4f_i^2a_i^2\right]+\left\{\frac{2|e|B}{\text{h}}\left(n+\frac{1}{2}\right)\right\}\left[4e_if_iE_{g_i}b_i\right.\right.$$

$$\left.-4e_if_id_i+4e_if_i^2E_{g_i}-4a_ib_i^2E_{g_i}-4a_ib_id_i-4a_ib_if_iE_{g_i}+4b_i^2e_iE_{g_i}+4b_i^2c_i+4b_i^2E_{g_i}a_i-4f_i^2e_iE_{g_i}-4f_i^2c_i-4f_i^2E_{g_i}a_i\right]$$

$$\left.+\left[E_{g_i}^2b_i^2+d_i^2+f_i^2g_i^2+2E_{g_i}b_id_i+2E_{g_i}^2b_if_i+2d_if_iE_{g_i}\right]\right]$$

and

$$K_5(n,E)\equiv\left[\left[(E-V_0)^2H_{32}+(E-V_0)H_{42}(n)+H_{52}(n)\right]^{1/2}-\left[(E-V_0)H_{12}+H_{22}(n)\right]\right]^{1/2}.$$

The electron concentration in this case can be expressed as

$$n_0=\left(\frac{|e|Bg_v}{\pi^2\text{h}L_0}\right)\sum_{n=0}^{n_{\max}}\left[T_{95}(n,E_{FSL})+T_{96}(n,E_{FSL})\right] \quad (17)$$

where, $T_{95}\left(n,E_{FSL}\right)\equiv\left[\rho_2\left(n,E_{FSL}\right)\text{-}\left\{\frac{2|e|B}{\hbar}\left(n+\frac{1}{2}\right)L_0^2\right\}\right]^{1/2}$ and

$T_{96}\left(n,E_{FSL}\right)\equiv\sum_{r=1}^{s}L\left(r\right)\left[T_{95}\left(n,E_{FSL}\right)\right]$.

The use of equations (17) and (5) leads to the expression of the DMR as

$$\frac{D}{\mu}=\frac{1}{|e|}\frac{\sum_{n=0}^{n_{\max}}\left[T_{95}\left(n,E_{FSL}\right)+T_{96}\left(n,E_{FSL}\right)\right]}{\sum_{n=0}^{n_{\max}}\left[\left\{T_{95}\left(n,E_{FSL}\right)\right\}'+\left\{T_{96}\left(n,E_{FSL}\right)\right\}'\right]} \tag{18}$$

2.4 Einstein relation under magnetic quantization in HgTe/CdTe superlattices with graded interfaces

The dispersion relation of the conduction electrons of the constituent materials of HgTe/CdTe SLs can be expressed [13] as

$$E=\frac{\hbar^2k^2}{2m_1^*}+\frac{3|e|^2k}{128\varepsilon_{sc}} \tag{19}$$

$$\frac{\hbar^2k^2}{2m_2^*}=EG\left(E_1E_{g2},\Delta_2\right) \tag{20}$$

The electron energy dispersion law in HgTe/CdTe SL is given by

$$\cos\left(L_ok\right)=\frac{1}{2}\Phi_3\left(E,k_s\right) \tag{21}$$

where, $\Phi_3\left(E,k_s\right)\equiv\Big[2\cosh\left\{\beta_3\left(E,k_s\right)\right\}\cos\left\{\gamma_3\left(E,k_s\right)\right\}+\varepsilon_3\left(E,k_s\right)\sinh\left\{\beta_3\left(E,k_s\right)\right\}\sin\left\{\gamma_3\left(E,k_s\right)\right\}$

$$+\Delta_0\left[\left(\frac{\left\{K_7\left(E,k_s\right)\right\}^7}{K_8\left(E,k_s\right)}\text{-}3K_8\left(E,k_s\right)\right)\cosh\left\{\beta_3\left(E,k_s\right)\right\}\sin\left\{\gamma_3\left(E,k_s\right)\right\}+\left(3K_7\left(E,k_s\right)-\frac{\left\{K_8\left(E,k_s\right)\right\}^2}{K_7\left(E,k_s\right)}\right)\sinh\left\{\beta_3\left(E,k_s\right)\right\}\cos\left\{\gamma_3\left(E,k_s\right)\right\}\right]$$

$$+\Delta_0\Big[2\left(\left\{K_7\left(E,k_s\right)\right\}^2\text{-}\left\{K_8\left(E,k_s\right)\right\}^2\right)\cosh\left\{\beta_3\left(E,k_s\right)\right\}\cos\left\{\gamma_3\left(E,k_s\right)\right\}$$

$$+\frac{1}{12}\left[\frac{5\left\{K_8\left(E,k_s\right)\right\}^3}{K_7\left(E,k_s\right)}+\frac{5\left\{K_7\left(E,k_s\right)\right\}^3}{K_8\left(E,k_s\right)}-34K_7\left(E,k_s\right)K_8\left(E,k_s\right)\right]\sinh\left\{\beta_3\left(E,k_s\right)\right\}\sin\left\{\gamma_3\left(E,k_s\right)\right\}\Big]\Big]$$

$$\varepsilon_3(E,k_s) \equiv \left[\frac{K_7(E,k_s)}{K_8(E,k_s)} - \frac{K_8(E,k_s)}{K_7(E,k_s)}\right], \beta_3(E,k_s) \equiv K_7(E,k_s)[a_0 - \Delta_0]$$

$\gamma_3(E,k_s) = K_8(E,k_s)[b_0 - \Delta_0]$, $K_8(E,k_x,k_y) \equiv \left[\frac{B_0^2 + 2AE - B_0\sqrt{B_0^2 + 4AE}}{2A^2} - k_s^2\right]^{1/2}$, $B_0 = \frac{3|e|^2}{128\varepsilon_{sc}}$, $A = \frac{h^2}{2m_1^*}$ and

$$K_7(E,k_s) \equiv \left[\frac{2m_2^* E'}{h^2} G(E - V_0, E_{g_2}, \Delta_2) + k_s^2\right]^{1/2}.$$

The magneto dispersion relation in this case can be expressed as

$$k_z = \frac{1}{L_o}\left[\rho_3(n,E) - \left\{\frac{2|e|B}{h} L_o^2\left(n + \frac{1}{2}\right)\right\}\right]^{1/2} \qquad (22)$$

where, $\rho_3(n,E) = \left[\cos^{-1}\left\{\frac{1}{2}\psi_3(n,E)\right\}\right]^2$,

$$\psi_3(n,E) = \Big[2\cosh\{\beta_3(n,E)\}\cos\{\gamma_3(n,E)\} + \varepsilon_3(n,E)\sinh\{\beta_3(n,E)\}$$

$$\sin\{\gamma_3(n,E)\} + \Delta_0\Bigg[\left(\frac{\{K_7(n,E)\}^2}{K_8(n,E)} - 3K_8(n,E)\right)\cosh\{\beta_3(n,E)\}$$

$$\sin\{\gamma_3(n,E)\} + \left(3K_7(n,E) - \frac{\{K_8(n,E)\}^2}{K_7(n,E)}\right)\sinh\{\beta_3(n,E)\}\cos\{\gamma_3(n,E)\}$$

$$+\Delta_0\Bigg[2\left(\{K_7(n,E)\}^2 - \{K_8(n,E)\}^2\right)\cosh\{\beta_3(n,E)\}\cos\{\gamma_3(n,E)\}$$

$$+\frac{1}{12}\left(\frac{5\{K_7(n,E)\}^3}{K_8(n,E)} + \frac{5\{K_8(n,E)\}^3}{K_7(n,E)} - \{34K_8(n,E)K_7(n,E)\}\right)\sinh\{\beta_3(n,E)\}\sin\{\gamma_3(n,E)\}\Bigg]\Bigg]$$

$\varepsilon_3(n,E) \equiv \left[\frac{K_7(n,E)}{K_8(n,E)} - \frac{K_8(n,E)}{K_7(n,E)}\right]$, $\beta_3(n,E) \equiv K_7(n,E)[a_0 - \Delta_0]$, $\gamma_3(n,E) = K_8(n,E)[b_0 - \Delta_0]$,

$K_8(n,E) \equiv \left[\frac{B_0^2 + 2AE - B_0\sqrt{B_0^2 + 4AE}}{2A^2} - \left\{\frac{2|e|B}{h}\left(n + \frac{1}{2}\right)\right\}\right]^{1/2}$, and

$$K_7(n,E) \equiv \left[\left(\frac{2m_2^* E'}{h^2}\right)G(E - V_0, \alpha_2, \Delta_2) + \left\{\frac{2|e|B}{h}\left(n + \frac{1}{2}\right)\right\}\right]^{1/2}.$$

The electron concentration in this case can be expressed as

$$n_0 = \left(\frac{|e| B g_v}{\pi^2 \hbar L_0} \right) \sum_{n=0}^{n_{\max}} \left[T_{97}\left(n, E_{FSL}\right) + T_{98}\left(n, E_{FSL}\right) \right] \tag{23}$$

where, $T_{97}\left(n, E_{FSL}\right) \equiv \left[\rho_3\left(n, E_{FSL}\right) - \left\{ \frac{2|e|B}{\hbar}\left(n + \frac{1}{2}\right) L_0^2 \right\} \right]^{1/2}$ and

$T_{98}\left(n, E_{FSL}\right) \equiv \sum_{r=1}^{s} L(r)\left[T_{97}\left(n, E_{FSL}\right) \right]$.

The use of equations (23) and (5) leads to the expression of the DMR as

$$\frac{D}{\mu} = \frac{1}{|e|} \frac{\sum_{n=0}^{n_{\max}} \left[T_{97}\left(n, E_{FSL}\right) + T_{98}\left(n, E_{FSL}\right) \right]}{\sum_{n=0}^{n_{\max}} \left[\left\{ T_{97}\left(n, E_{FSL}\right) \right\}' + \left\{ T_{98}\left(n, E_{FSL}\right) \right\}' \right]} \tag{24}$$

2.5 Einstein relation under magnetic quantization in III-V effective mass superlattices

Following Sasaki [15], the electron dispersion law in III-V effective mass superlattices (EMSLs) can be written as

$$k_x^2 = \left[\frac{1}{L_0^2} \left\{ \cos^{-1}\left(f\left(E, k_y, k_z\right) \right) \right\}^2 - k_\perp^2 \right] \tag{25}$$

in which, $f\left(E, k_y, k_z\right) = a_1 \cos\left[a_0 C_1\left(E, k_\perp\right) + b_0 D_1\left(E, k_\perp\right) \right] - a_2 \cos\left[a_0 C_1\left(E, k_\perp\right) - b_0 D_1\left(E, k_\perp\right) \right]$, $k_\perp^2 = k_y^2 + k_z^2$,

$a_1 = \left[\sqrt{\frac{m_2^*}{m_1^*}} + 1 \right]^2 \left[4\left(\frac{m_2^*}{m_1^*} \right)^{1/2} \right]^{-1}$, $a_2 = \left[-1 + \sqrt{\frac{m_2^*}{m_1^*}} \right]^2 \left[4\left(\frac{m_2^*}{m_1^*} \right)^{1/2} \right]^{-1}$,

$C_1\left(E, k_\perp\right) \equiv \left[\left(\frac{2 m_1^* E}{\hbar^2} \right) G\left(E, E_{g_1}, \Delta_1\right) - k_\perp^2 \right]^{1/2}$ and $D_1\left(E, k_\perp\right) \equiv \left[\left(\frac{2 m_2^* E}{\hbar^2} \right) G\left(E, E_{g_2}, \Delta_2\right) - k_\perp^2 \right]^{1/2}$.

In the presence of an external magnetic field along x-direction, the simplified magneto dispersion law in this case can be written as

$$k_x^2 = \left[\rho_4\left(n, E\right) \right] \tag{26}$$

in which, $\rho_4(n,E)=\frac{1}{L_0^2}\left[\cos^{-1}\left(f(n,E)\right)\right]^2-\left\{\frac{2|e|B}{\hbar}\left(n+\frac{1}{2}\right)\right\}$, $f(n,E)=a_1\cos\left[a_0C_1(n,E)+b_0D_1(n,E)\right]-a_2\cos\left[a_0C_1(n,E)-b_0D_1(n,E)\right]$ $C_1(n,E)\equiv\left[\left(\frac{2m_1^*E}{\hbar^2}\right)G\left(E,E_{g_1},\Delta_1\right)-\left\{\frac{2|e|B}{\hbar}\left(n+\frac{1}{2}\right)\right\}\right]^{1/2}$ and $D_1(n,E)\equiv\left[\left(\frac{2m_2^*E}{\hbar^2}\right)G\left(E,E_{g_2},\Delta_2\right)-\left\{\frac{2|e|B}{\hbar}\left(n+\frac{1}{2}\right)\right\}\right]^{1/2}$.

The electron concentration in this case can be expressed as

$$n_0=\left(\frac{|e|Bg_v}{\pi^2\hbar}\right)\sum_{n=0}^{n_{max}}\left[T_{99}(n,E_{FSL})+T_{910}(n,E_{FSL})\right] \tag{27}$$

where, $T_{99}(n,E_{FSL})\equiv\left[\rho_4(n,E_{FSL})\right]^{1/2}$ and $T_{910}(n,E_{FSL})\equiv\sum_{r=1}^{s}L(r)\left[T_{99}(n,E_{FSL})\right]$.

The use of equations (27) and (5) leads to the expression of the DMR as

$$\frac{D}{\mu}=\frac{1}{|e|}\frac{\sum_{n=0}^{n_{max}}\left[T_{99}(n,E_{FSL})+T_{910}(n,E_{FSL})\right]}{\sum_{n=0}^{n_{max}}\left[\left\{T_{99}(n,E_{FSL})\right\}'+\left\{T_{910}(n,E_{FSL})\right\}'\right]} \tag{28}$$

2.6 Einstein relation under magnetic quantization in II-VI effective mass superlattices

Following Sasaki [15], the electron dispersion law in II-VI EMSLs can be written as

$$k_z^2=\left[\frac{1}{L_0^2}\left\{\cos^{-1}\left(f_1(E,k_x,k_y)\right)\right\}^2-k_s^2\right] \tag{29}$$

in which, $f_1(E,k_x,k_y)=a_3\cos\left[a_0C_2(E,k_s)+b_0D_2(E,k_s)\right]-a_4\cos\left[a_0C_2(E,k_s)-b_0D_2(E,k_s)\right]$, $k_s^2=k_x^2+k_y^2$, $a_3=\left[\sqrt{\frac{m_2^*}{m_{P,1}^*}}+1\right]^2\left[4\left(\frac{m_2^*}{m_{P,1}^*}\right)^{1/2}\right]^{-1}$, $a_4=\left[-1+\sqrt{\frac{m_2^*}{m_{P,1}^*}}\right]^2\left[4\left(\frac{m_2^*}{m_{P,1}^*}\right)^{1/2}\right]^{-1}$, $C_2(E,k_s)\equiv\left(\frac{2m_{P,1}^*}{\hbar^2}\right)^{1/2}\left[E-\frac{\hbar^2k_s^2}{2m_{\perp,1}^*}\,\mathrm{m}\bar{\lambda}_0k_s\right]^{1/2}$ and $D_2(E,k_s)\equiv\left[\left(\frac{2m_2^*}{\hbar^2}\right)EG\left(E,E_{g_2},\Delta_2\right)-k_s^2\right]^{1/2}$.

Under magnetic quantization along z-direction, the simplified magneto dispersion law can be expressed as

$$k_z^2 = \left[\rho_5(n,E)\right] \tag{30}$$

in which, $\rho_5(n,E) = \frac{1}{L_0^2}\left[\cos^{-1}\left(f_1(n,E)\right)\right]^2 - \left\{\frac{2|e|B}{\hbar}\left(n+\frac{1}{2}\right)\right\}$,

$f_1(n,E) = a_3\cos\left[a_0C_2(n,E)+b_0D_2(n,E)\right] - a_4\cos\left[a_0C_2(n,E)-b_0D_2(n,E)\right]$

$C_2(E,k_\perp) \equiv \left(\frac{2m_{\parallel,1}^*}{\hbar^2}\right)^{1/2}\left[E - \frac{\hbar|e|B}{m_{\perp,1}^*}\left(n+\frac{1}{2}\right) \mp \bar{\lambda}_0\left\{\frac{2|e|B}{\hbar}\left(n+\frac{1}{2}\right)\right\}^{1/2}\right]^{1/2}$ and

$D_2(n,E) \equiv \left[\left(\frac{2m_2^*}{\hbar^2}\right)EG\left(E,E_{g_2},\Delta_2\right) - \frac{2|e|B}{\hbar}\left(n+\frac{1}{2}\right)\right]^{1/2}$.

The electron concentration in this case can be expressed as

$$n_0 = \left(\frac{|e|Bg_v}{2\pi^2\hbar}\right)\sum_{n=0}^{n_{\max}}\left[T_{911}(n,E_{FSL}) + T_{912}(n,E_{FSL})\right] \tag{31}$$

where, $T_{911}(n,E_{FSL}) \equiv \left[\rho_5(n,E_{FSL})\right]^{1/2}$ and $T_{912}(n,E_{FSL}) \equiv \sum_{r=1}^{s} L(r)\left[T_{911}(n,E_{FSL})\right]$.

Thus, using equation (41) and (1.11) leads to the expression of the DMR as

$$\frac{D}{\mu} = \frac{1}{|e|}\frac{\sum_{n=0}^{n_{\max}}\left[T_{911}(n,E_{FSL}) + T_{912}(n,E_{FSL})\right]}{\sum_{n=0}^{n_{\max}}\left[\left\{T_{911}(n,E_{FSL})\right\}' + \left\{T_{912}(n,E_{FSL})\right\}'\right]} \tag{32}$$

2.7 Einstein relation under magnetic quantization in IV-VI effective mass superlattices

Following Sasaki [15], the electron dispersion law in IV-VI, EMSLs can be written as

$$k_x^2 = \left[\frac{1}{L_0^2}\left\{\cos^{-1}\left(f_2(E,k_y,k_z)\right)\right\}^2 - k_\perp^2\right] \tag{33}$$

in which, $f_2(E,k_y,k_z) = a_5\cos\left[a_0C_3(E,k_y,k_z) + b_0D_3(E,k_y,k_z)\right] - a_6\cos\left[a_0C_3(E,k_y,k_z) - b_0D_3(E,k_y,k_z)\right]$,

$$a_5 = \left[\sqrt{\frac{m_2^*}{m_1^*}} + 1\right]^2\left[4\left(\frac{m_2^*}{m_1^*}\right)^{1/2}\right]^{-1}$$

$$m_i^* = \left[\frac{2\hbar^2}{\{a_i^2 - b_i^2\}}\right]\left[a_i + \left[a_iC_i + a_ie_iE_{g_i} - e_i^2E_{g_i}\right]\left[E_{g_i}^2a_i^2 + C_i^2 + e_i^2E_{g_i}^2 + 2C_ie_iE_{g_i} - 2E_{g_i}a_iC_i - 2e_ia_iE_{g_i}^2\right]^{1/2}\right]$$

$$C_3\left(E,k_y,k_z\right) \equiv \left[\left[EH_{11} + H_{21}\left(k_y,k_z\right)\right] - \left[E^2H_{31} + EH_{41}\left(k_y,k_z\right) + H_{51}\left(k_y,k_z\right)\right]^{1/2}\right]^{1/2},$$

$$D_3\left(E,k_y,k_z\right) \equiv \left[\left[EH_{12} + H_{22}\left(k_y,k_z\right)\right] - \left[E^2H_{32} + EH_{42}\left(k_y,k_z\right) + H_{52}\left(k_y,k_z\right)\right]^{1/2}\right]^{1/2}$$ an

$$a_6 = \left[-1 + \sqrt{\frac{m_2^*}{m_1^*}}\right]^2\left[4\left(\frac{m_2^*}{m_1^*}\right)^{1/2}\right]^{-1}.$$

Thus, in the presence of a quantizing magnetic field along x-direction, the simplified magneto dispersion law in this case can be written as

$$k_x^2 = \left[\rho_6\left(n,E\right)\right] \tag{34}$$

in which, $\rho_6\left(n,E\right) = \frac{1}{L_0^2}\left[\cos^{-1}\left(f_2\left(n,E\right)\right)\right]^2 - \left\{\frac{2|e|B}{\hbar}\left(n+\frac{1}{2}\right)\right\}$,

$$f_2\left(n,E\right) = a_5\cos\left[a_0C_3\left(n,E\right) + b_0D_3\left(n,E\right)\right] - a_6\cos\left[a_0C_3\left(n,E\right) - b_0D_3\left(n,E\right)\right]$$

$$C_3\left(n,E\right) \equiv \left[\left[EH_{11} + H_{21}\left(n\right)\right] - \left[E^2H_{31} + EH_{41}\left(n\right) + H_{51}\left(n\right)\right]^{1/2}\right]^{1/2}$$

$$D_3\left(n,E\right) \equiv \left[\left[EH_{12} + H_{22}\left(n\right)\right] - \left[E^2H_{32} + EH_{42}\left(n\right) + H_{52}\left(n\right)\right]^{1/2}\right]^{1/2}.$$

The electron concentration in this case can be expressed as

$$n_0 = \left(\frac{|e|Bg_v}{\pi^2\hbar}\right)\sum_{n=0}^{n_{\max}}\left[T_{913}\left(n,E_{FSL}\right) + T_{914}\left(n,E_{FSL}\right)\right] \tag{35}$$

where, $T_{913}\left(n,E_{FSL}\right) \equiv \left[\rho_6\left(n,E_{FSL}\right)\right]^{1/2}$ and $T_{914}\left(n,E_{FSL}\right) \equiv \sum_{r=1}^{s} L(r)\left[T_{913}\left(n,E_{FSL}\right)\right]$.

Thus, using equation (35) and (5) leads to the expression of the DMR as

$$\frac{D}{\mu} = \frac{1}{|e|}\frac{\sum_{n=0}^{n_{\max}}\left[T_{913}\left(n,E_{FSL}\right) + T_{914}\left(n,E_{FSL}\right)\right]}{\sum_{n=0}^{n_{\max}}\left[\left\{T_{913}\left(n,E_{FSL}\right)\right\}' + \left\{T_{914}\left(n,E_{FSL}\right)\right\}'\right]} \tag{36}$$

2.8 Einstein relation under magnetic quantization in HgTe/CdTe effective mass superlattices

Following Sasaki [15], the electron dispersion law in HgTe/CdTe EMSLs can be written as

$$k_x^2 = \left[\frac{1}{L_0^2} \left\{ \cos^{-1}\left(f_3\left(E, k_y, k_z\right)\right)\right\}^2 - k_\perp^2 \right] \tag{37}$$

in which, $f_3\left(E,k_\perp\right) = a_7 \cos\left[a_0 C_4\left(E,k_\perp\right) + b_0 D_4\left(E,k_\perp\right)\right] - a_8 \cos\left[a_0 C_4\left(E,k_\perp\right) - b_0 D_4\left(E,k_\perp\right)\right]$,

$a_1 = \left[\sqrt{\frac{m_2^*}{m_1^*}} + 1\right]^2 \left[4\left(\frac{m_2^*}{m_1^*}\right)^{1/2}\right]^{-1}$, $a_2 = \left[-1 + \sqrt{\frac{m_2^*}{m_1^*}}\right]^2 \left[4\left(\frac{m_2^*}{m_1^*}\right)^{1/2}\right]^{-1}$,

$C_4\left(E,k_\perp\right) \equiv \left[\frac{B_0^2 + 2AE - B_0\sqrt{B_0^2 + 4AE}}{2A^2} - k_\perp^2\right]^{1/2}$ and $D_4\left(E,k_\perp\right) \equiv \left[\left(\frac{2m_2^* E}{\text{h}^2}\right) G\left(E, E_{g_2}, \Delta_2\right) - k_\perp^2\right]^{1/2}$.

In the presence of an external magnetic field along x-direction, the simplified magneto dispersion law in this case can be written as

$$k_x^2 = \left[\rho_6\left(n, E\right)\right] \tag{38}$$

in which, $\rho_6\left(n,E\right) = \left\{\frac{1}{L_0^2}\left[\cos^{-1}\left(f_3\left(n,E\right)\right)\right]^2\right\} - \left\{\frac{2|e|B}{\text{h}}\left(n + \frac{1}{2}\right)\right\}$,

$f_3\left(n,E\right) = a_7 \cos\left[a_0 C_4\left(n,E\right) + b_0 D_4\left(n,E\right)\right] - a_8 \cos\left[a_0 C_4\left(n,E\right) - b_0 D_4\left(n,E\right)\right]$

$C_4\left(n,E\right) \equiv \left[\frac{B_0^2 + 2AE - B_0\sqrt{B_0^2 + 4AE}}{2A^2} - \left\{\frac{2|e|B}{\text{h}}\left(n + \frac{1}{2}\right)\right\}\right]^{1/2}$ and

$D_4\left(n,E\right) \equiv \left[\left(\frac{2m_2^* E}{\text{h}^2}\right) G\left(E, E_{g_2}, \Delta_2\right) - \left\{\frac{2|e|B}{\text{h}}\left(n + \frac{1}{2}\right)\right\}\right]^{1/2}$.

The electron concentration in this case can be expressed as

$$n_0 = \left(\frac{|e| B g_v}{\pi^2 \text{h}}\right) \sum_{n=0}^{n_{\max}} \left[T_{915}\left(n, E_{FSL}\right) + T_{916}\left(n, E_{FSL}\right)\right] \tag{39}$$

where, $T_{915}\left(n, E_{FSL}\right) \equiv \left[\rho_6\left(n, E_{FSL}\right)\right]^{1/2}$ and $T_{916}\left(n, E_{FSL}\right) \equiv \sum_{r=1}^{s} L(r)\left[T_{915}\left(n, E_{FSL}\right)\right]$.

The use of equations (39) and (5) leads to the expression of the DMR as

$$\frac{D}{\mu}=\frac{1}{|e|}\frac{\sum_{n=0}^{n_{\max}}\left[T_{915}\left(n,E_{FSL}\right)+T_{916}\left(n,E_{FSL}\right)\right]}{\sum_{n=0}^{n_{\max}}\left[\left\{T_{915}\left(n,E_{FSL}\right)\right\}'+\left\{T_{916}\left(n,E_{FSL}\right)\right\}'\right]} \tag{40}$$

2.9 Einstein relation in III-V quantum wire superlattices with graded interfaces

The electron dispersion law in III-V quantum wire superlattices (QWSLs), can be written following equation (2) as

$$k_z^2=\left[\frac{1}{L_0^2}\left\{\rho_8\left(n_x,n_y,E\right)\right\}-\phi\left(n_x,n_y\right)\right] \tag{41}$$

where $\rho_8\left(n_x,n_y,E\right)=\left[\cos^{-1}\left\{\frac{1}{2}\psi_8\left(n_x,n_y,E\right)\right\}\right]^2$, $\phi\left(n_x,n_y\right)=\left(n_x\pi/d_x\right)^2+\left(n_y\pi/d_y\right)^2$,

$n_x(=1,2,3,...)$ and $n_y(=1,2,3,...)$ are the size quantum numbers along the x- and y- direction respectively and d_x and d_y are the nanothickness along the respective directions, $\psi_8\left(n_x,n_y,E\right)=\Big[2\cosh\left\{\beta_8\left(n_x,n_y,E\right)\right\}\cos\left\{\gamma_8\left(n_x,n_y,E\right)\right\}+\varepsilon_8\left(n_x,n_y,E\right)\sinh\left\{\beta_8\left(n_x,n_y,E\right)\right\}$

$$\sin\left\{\gamma_8\left(n_x,n_y,E\right)\right\}+\Delta_0\Bigg[\left(\frac{\left\{K_9\left(n_x,n_y,E\right)\right\}^2}{K_{10}\left(n_x,n_y,E\right)}-3K_{10}\left(n_x,n_y,E\right)\right)\cosh\left\{\beta_8\left(n_x,n_y,E\right)\right\}$$

$$\sin\left\{\gamma_8\left(n_x,n_y,E\right)\right\}+\left(3K_9\left(n_x,n_y,E\right)-\frac{\left\{K_{10}\left(n_x,n_y,E\right)\right\}^2}{K_9\left(n_x,n_y,E\right)}\right)\sinh\left\{\beta_8\left(n_x,n_y,E\right)\right\}\cos\left\{\gamma_8\left(n_x,n_y,E\right)\right\}$$

$$+\Delta_0\left[2\left(K_9\left(n_x,n_y,E\right)-\left\{K_{10}\left(n_x,n_y,E\right)\right\}^2\right)\cosh\left\{\beta_8\left(n_x,n_y,E\right)\right\}\cos\left\{\gamma_8\left(n_x,n_y,E\right)\right\}\right.$$

$$\left.+\frac{1}{12}\left(\frac{5\left\{K_9\left(n_x,n_y,E\right)\right\}^3}{K_{10}\left(n_x,n_y,E\right)}+\frac{5\left\{K_{10}\left(n_x,n_y,E\right)\right\}^3}{K_9\left(n_x,n_y,E\right)}-\left\{34K_{10}\left(n_x,n_y,E\right)K_9\left(n_x,n_y,E\right)\right\}\right)\sinh\left\{\beta_8\left(n_x,n_y,E\right)\right\}\sin\left\{\gamma_8\left(n_x,n_y,E\right)\right\}\right]\Bigg]$$

$\varepsilon_8\left(n_x,n_y,E\right)\equiv\left[\frac{K_9\left(n_x,n_y,E\right)}{K_{10}\left(n_x,n_y,E\right)}-\frac{K_{10}\left(n_x,n_y,E\right)}{K_9\left(n_x,n_y,E\right)}\right]$, $\beta_8\left(n_x,n_y,E\right)\equiv K_9\left(n_x,n_y,E\right)\left[a_0-\Delta_0\right]$,

$K_9\left(n_x,n_y,E\right)\equiv\left[\frac{2m_2^*E'}{\hbar^2}G(E-V_0,\alpha_2,\Delta_2)+\phi\left(n_x,n_y\right)\right]^{1/2}$, $\gamma_8\left(n_x,n_y,E\right)=K_{10}\left(n_x,n_y,E\right)\left[b_0-\Delta_0\right]$ and

$K_{10}\left(n_x,n_y,E\right)\equiv\left[\frac{2m_1^*E}{\hbar^2}G(E,\alpha_1,\Delta_1)-\phi\left(n_x,n_y\right)\right]^{1/2}$.

Considering only the lowest miniband, since in an actual SL only the lowest miniband is significantly populated at low temperatures, where the quantum effects become prominent, the relation between the 1D electron concentration (n_{1D}) and the Fermi energy in the present case can be written as,

$$n_{1D} = \left(\frac{2g_v}{\pi}\right)\sum_{n_x=1}^{n_{x\max}}\sum_{n_y=1}^{n_{y\max}}\left[T_{917}\left(n_x,n_y,E_{FQWSL}\right)+T_{918}\left(n_x,n_y,E_{FQWSL}\right)\right] \quad (42)$$

where, $T_{917}\left(n_x,n_y,E_{FQWSL}\right)\equiv\left[\rho_8\left(n_x,n_y,E_{FQWSL}\right)\text{-}\phi\left(n_x,n_y\right)\right]^{1/2}$ and $T_{918}\left(n_x,n_y,E_{FQWSL}\right)\equiv\sum_{r=1}^{s}L(r)\left[T_{917}\left(n_x,n_y,E_{FQWSL}\right)\right]$.

The use of equations (42) and (5) leads to the expression of the DMR in this case as

$$\frac{D}{\mu}=\frac{1}{|e|}\frac{\sum_{n_x=1}^{n_{x\max}}\sum_{n_y=1}^{n_{y\max}}\left[T_{917}\left(n_x,n_y,E_{FQWSL}\right)+T_{918}\left(n_x,n_y,E_{FQWSL}\right)\right]}{\sum_{n_x=1}^{n_{x\max}}\sum_{n_y=1}^{n_{y\max}}\left[\left\{T_{917}\left(n_x,n_y,E_{FQWSL}\right)\right\}'+\left\{T_{918}\left(n_x,n_y,E_{FQWSL}\right)\right\}'\right]} \quad (43)$$

2.10 Einstein relation in II-VI quantum wire superlattices with graded interfaces

The electron dispersion law in II-VI QWSLs, can be written as

$$k_z^2=\left[\frac{1}{L_0^2}\left\{\rho_9\left(n_x,n_y,E\right)\right\}-\phi\left(n_x,n_y\right)\right] \quad (44)$$

where, $\rho_9\left(n_x,n_y,E\right)=\left[\cos^{-1}\left\{\frac{1}{2}\psi_9\left(n_x,n_y,E\right)\right\}\right]^2$,

$$\psi_9\left(n_x,n_y,E\right)=\Big[2\cosh\left\{\beta_9\left(n_x,n_y,E\right)\right\}\cos\left\{\gamma_9\left(n_x,n_y,E\right)\right\}+\varepsilon_9\left(n_x,n_y,E\right)\sinh\left\{\beta_9\left(n_x,n_y,E\right)\right\}$$

$$\sin\left\{\gamma_9\left(n_x,n_y,E\right)\right\}+\Delta_0\left[\left(\frac{\left\{K_{11}\left(n_x,n_y,E\right)\right\}^2}{K_{12}\left(n_x,n_y,E\right)}-3K_{12}\left(n_x,n_y,E\right)\right)\cosh\left\{\beta_9\left(n_x,n_y,E\right)\right\}\right.$$

$$\sin\left\{\gamma_9\left(n_x,n_y,E\right)\right\}+\left(3K_{11}\left(n_x,n_y,E\right)-\frac{\left\{K_{12}\left(n_x,n_y,E\right)\right\}^2}{K_{11}\left(n_x,n_y,E\right)}\right)\sinh\left\{\beta_9\left(n_x,n_y,E\right)\right\}\cos\left\{\gamma_9\left(n_x,n_y,E\right)\right\}$$

$$+\Delta_0\left[2\left(K_{11}\left(n_x,n_y,E\right)-\left\{K_{12}\left(n_x,n_y,E\right)\right\}^2\right)\cosh\left\{\beta_9\left(n_x,n_y,E\right)\right\}\cos\left\{\gamma_9\left(n_x,n_y,E\right)\right\}\right.$$

$$+\frac{1}{12}\left(\frac{5\left\{K_{11}\left(n_x,n_y,E\right)\right\}^3}{K_{12}\left(n_x,n_y,E\right)}+\frac{5\left\{K_{12}\left(n_x,n_y,E\right)\right\}^3}{K_{11}\left(n_x,n_y,E\right)}-\left\{34K_{12}\left(n_x,n_y,E\right)K_{11}\left(n_x,n_y,E\right)\right\}\right)\sinh\left\{\beta_9\left(n_x,n_y,E\right)\right\}\sin\left\{\gamma_9\left(n_x,n_y,E\right)\right\}\Bigg]\Bigg]$$

$\varepsilon_9\left(n_x,n_y,E\right)\equiv\left[\frac{K_{11}\left(n_x,n_y,E\right)}{K_{12}\left(n_x,n_y,E\right)}-\frac{K_{12}\left(n_x,n_y,E\right)}{K_{11}\left(n_x,n_y,E\right)}\right]$, $\beta_9\left(n_x,n_y,E\right)\equiv K_{11}\left(n_x,n_y,E\right)\left[a_0-\Delta_0\right]$,

$\gamma_9\left(n_x,n_y,E\right)=K_{12}\left(n_x,n_y,E\right)\left[b_0-\Delta_0\right]$,

$$K_{11}\left(n_x,n_y,E\right)\equiv\left[\frac{2m_2^*}{\hbar^2}E'G\left(E-V_0,\alpha_2,\Delta_2\right)+\phi\left(n_x,n_y\right)\right]^{1/2}$$

and $K_{12}\left(n_x,n_y,E\right)\equiv\left[\frac{2m_{\parallel,1}^*}{\hbar^2}\left[E-\frac{\hbar^2}{2m_{\perp,1}^*}\phi\left(n_x,n_y\right)\mp\bar{\lambda}_0\left\{\phi\left(n_x,n_y\right)\right\}^{1/2}\right]\right]^{1/2}$.

The electron concentration in this case can be expressed as

$$n_{1D}=\left(\frac{2g_v}{\pi}\right)\sum_{n_x=1}^{n_{x_{mas}}}\sum_{n_y=1}^{n_{y_{\max}}}\left[T_{919}\left(n_x,n_y,E_{FQWSL}\right)+T_{920}\left(n_x,n_y,E_{FQWSL}\right)\right] \quad (45)$$

where, $T_{919}\left(n_x,n_y,E_{FQWSL}\right)\equiv\left[\rho_9\left(n_x,n_y,E_{FQWSL}\right)-\phi\left(n_x,n_y\right)\right]^{1/2}$ and

$T_{920}\left(n_x,n_y,E_{FQWSL}\right)\equiv\sum_{r=1}^{s}L(r)\left[T_{919}\left(n_x,n_y,E_{FQWSL}\right)\right]$.

The use of equations (45) and (5) leads to the expression of the DMR as

$$\frac{D}{\mu}=\frac{1}{|e|}\frac{\sum_{n_x=1}^{n_{x_{mas}}}\sum_{n_y=1}^{n_{y_{\max}}}\left[T_{919}\left(n_x,n_y,E_{FQWSL}\right)+T_{920}\left(n_x,n_y,E_{FQWSL}\right)\right]}{\sum_{n_x=1}^{n_{x_{mas}}}\sum_{n_y=1}^{n_{y_{\max}}}\left[\left\{T_{919}\left(n_x,n_y,E_{FQWSL}\right)\right\}'+\left\{T_{920}\left(n_x,n_y,E_{FQWSL}\right)\right\}'\right]} \quad (46)$$

2.11 Einstein relation in IV-VI quantum wire superlattices with graded interfaces

The electron dispersion law in IV-VI QWSLs can be written as

$$k_z^2=\left[\frac{1}{L_0^2}\left\{\rho_{10}\left(n_x,n_y,E\right)\right\}-\phi\left(n_x,n_y\right)\right] \quad (47)$$

where, $\rho_{10}\left(n_x,n_y,E\right)=\left[\cos^{-1}\left\{\frac{1}{2}\psi_{10}\left(n_x,n_y,E\right)\right\}\right]^2$,

$$\psi_{10}\left(n_x,n_y,E\right)=\left[2\cosh\left\{\beta_{10}\left(n_x,n_y,E\right)\right\}\cos\left\{\gamma_{10}\left(n_x,n_y,E\right)\right\}+\varepsilon_{10}\left(n_x,n_y,E\right)\sinh\left\{\beta_{10}\left(n_x,n_y,E\right)\right\}\right.$$

$$\sin\left\{\gamma_{10}\left(n_x,n_y,E\right)\right\}+\Delta_0\left[\left(\frac{\left\{K_{11}\left(n_x,n_y,E\right)\right\}^2}{K_{12}\left(n_x,n_y,E\right)}-3K_{12}\left(n_x,n_y,E\right)\right)\cosh\left\{\beta_{10}\left(n_x,n_y,E\right)\right\}\right.$$

$$\sin\left\{\gamma_{10}\left(n_x,n_y,E\right)\right\}+\left(3K_{11}\left(n_x,n_y,E\right)-\frac{\left\{K_{12}\left(n_x,n_y,E\right)\right\}^2}{K_{11}\left(n_x,n_y,E\right)}\right)\sinh\left\{\beta_{10}\left(n_x,n_y,E\right)\right\}\cos\left\{\gamma_{10}\left(n_x,n_y,E\right)\right\}$$

$$+\Delta_0\left[2\left(K_{11}\left(n_x,n_y,E\right)-\left\{K_{12}\left(n_x,n_y,E\right)\right\}^2\right)\cosh\left\{\beta_{10}\left(n_x,n_y,E\right)\right\}\cos\left\{\gamma_{10}\left(n_x,n_y,E\right)\right\}\right.$$

$$\left.\left.+\frac{1}{12}\left(\frac{5\left\{K_{11}\left(n_x,n_y,E\right)\right\}^3}{K_{12}\left(n_x,n_y,E\right)}+\frac{5\left\{K_{12}\left(n_x,n_y,E\right)\right\}^3}{K_{11}\left(n_x,n_y,E\right)}-\left\{34K_{12}\left(n_x,n_y,E\right)K_{11}\left(n_x,n_y,E\right)\right\}\right)\sinh\left\{\beta_{10}\left(n_x,n_y,E\right)\right\}\sin\left\{\gamma_{10}\left(n_x,n_y,E\right)\right\}\right]\right]$$

$$\varepsilon_{10}\left(n_x,n_y,E\right)\equiv\left[\frac{K_{11}\left(n_x,n_y,E\right)}{K_{12}\left(n_x,n_y,E\right)}-\frac{K_{12}\left(n_x,n_y,E\right)}{K_{11}\left(n_x,n_y,E\right)}\right],\qquad \beta_{10}\left(n_x,n_y,E\right)\equiv K_{11}\left(n_x,n_y,E\right)\left[a_0-\Delta_0\right]$$

$$K_{12}\left(n_x,n_y,E\right)\equiv\left[\left[EH_{11}+H_{21}\left(n_x,n_y\right)\right]-\left[E^2H_{31}+EH_{41}\left(n_x,n_y\right)+H_{51}\left(n_x,n_y\right)\right]^{1/2}\right]^{1/2},$$

$$H_{2i}\left(n_x,n_y\right)=\left[2H_{1i}\right]^{-1}\left[E_{g_i}b_i+d_i+f_iE_{g_i}+2\left(e_if_i-a_ib_i\right)\phi\left(n_x,n_y\right)\right],$$

$$H_{4i}\left(n_x,n_y\right)=\left[4H_{1i}^2\right]^{-1}\left[4b_id_i+4b_if_iE_{g_i}+4f_i^2E_{g_i}+8\phi\left(n_x,n_y\right)\left[e_if_ib_i-a_if_i^2\right]\right],$$

$$H_{5i}\left(n_x,n_y\right)\equiv\left[4H_{1i}^2\right]^{-1}\left[\left\{\phi\left(n_x,n_y\right)\right\}^2\left[-8a_ib_ie_if_i+4b_i^2e_i^2+4f_i^2a_i^2\right]+\left\{\phi\left(n_x,n_y\right)\right\}\left[4e_if_iE_{g_i}b_i\right.\right.$$

$$\left.-4e_if_id_i+4e_if_i^2E_{g_i}-4a_ib_i^2E_{g_i}-4a_ib_id_i-4a_ib_if_iE_{g_i}+4b_i^2e_iE_{g_i}+4b_i^2c_i+4b_i^2E_{g_i}a_i-4f_i^2e_iE_{g_i}-4f_i^2c_i-4f_i^2E_{g_i}a_i\right]$$

$$\left.+\left[E_{g_i}^2b_i^2+d_i^2+f_i^2g_i^2+2E_{g_i}b_id_i+2E_{g_i}^2b_if_i+2d_if_iE_{g_i}\right]\right] \text{ and}$$

$$K_{11}\left(n_x,n_y,E\right)\equiv\left[\left[\left(E-V_0\right)^2H_{32}+\left(E-V_0\right)H_{42}\left(n_x,n_y\right)+H_{52}\left(n_x,n_y\right)\right]^{1/2}-\left[\left(E-V_0\right)H_{12}+H_{22}\left(n_x,n_y\right)\right]\right]^{1/2}.$$

The electron concentration in this case can be expressed as

$$n_{1D}=\left(\frac{2g_v}{\pi}\right)\sum_{n_x=1}^{n_{x\max}}\sum_{n_y=1}^{n_{y\max}}\left[T_{921}\left(n_x,n_y,E_{FQWSL}\right)+T_{922}\left(n_x,n_y,E_{FQWSL}\right)\right] \tag{48}$$

where, $T_{921}\left(n_x,n_y,E_{FQWSL}\right)\equiv\left[\rho_{10}\left(n_x,n_y,E_{FQWSL}\right)-\left\{\phi\left(n_x,n_y,E_{FQWSL}\right)\right\}\right]^{1/2}$ and

$$T_{922}\left(n_x,n_y,E_{FQWSL}\right)\equiv\sum_{r=1}^{s}L(r)\left[T_{921}\left(n_x,n_y,E_{FQWSL}\right)\right].$$

The use of equations (48) and (5) leads to the expression of the DMR as

$$\frac{D}{\mu}=\frac{1}{|e|}\frac{\sum_{n_x=1}^{n_{x_{\max}}}\sum_{n_y=1}^{n_{y_{\max}}}\left[T_{921}\left(n_x,n_y,E_{FQWSL}\right)+T_{922}\left(n_x,n_y,E_{FQWSL}\right)\right]}{\sum_{n_x=1}^{n_{x_{\max}}}\sum_{n_y=1}^{n_{y_{\max}}}\left[\left\{T_{921}\left(n_x,n_y,E_{FQWSL}\right)\right\}'+\left\{T_{922}\left(n_x,n_y,E_{FQWSL}\right)\right\}'\right]} \quad (49)$$

2.12 Einstein relation in HgTe/CdTe quantum wire superlattices with graded interfaces

The electron dispersion law in HgTe/CdTe QWSLs can be written as

$$k_z=\left[\frac{1}{L_0^2}\left\{\rho_{11}\left(n_x,n_y,E\right)\right\}-\left\{\phi\left(n_x,n_y,E\right)\right\}\right]^{1/2} \quad (50)$$

where, $\rho_{11}\left(n_x,n_y,E\right)=\left[\cos^{-1}\left\{\frac{1}{2}\psi_{11}\left(n_x,n_y,E\right)\right\}\right]^2$,

$$\psi_{11}\left(n_x,n_y,E\right)=\Big[2\cosh\left\{\beta_{11}\left(n_x,n_y,E\right)\right\}\cos\left\{\gamma_{11}\left(n_x,n_y,E\right)\right\}+\varepsilon_{11}\left(n_x,n_y,E\right)\sinh\left\{\beta_{11}\left(n_x,n_y,E\right)\right\}$$

$$\sin\left\{\gamma_{11}\left(n_x,n_y,E\right)\right\}+\Delta_0\left[\left(\frac{\left\{K_{13}\left(n_x,n_y,E\right)\right\}^2}{K_{14}\left(n_x,n_y,E\right)}-3K_{14}\left(n_x,n_y,E\right)\right)\cosh\left\{\beta_{11}\left(n_x,n_y,E\right)\right\}\right.$$

$$\sin\left\{\gamma_{11}\left(n_x,n_y,E\right)\right\}+\left(3K_{13}\left(n_x,n_y,E\right)-\frac{\left\{K_{14}\left(n_x,n_y,E\right)\right\}^2}{K_{13}\left(n_x,n_y,E\right)}\right)\sinh\left\{\beta_{11}\left(n_x,n_y,E\right)\right\}\cos\left\{\gamma_{11}\left(n_x,n_y,E\right)\right\}$$

$$+\Delta_0\left[2\left(\left\{K_{13}\left(n_x,n_y,E\right)\right\}^2-\left\{K_{14}\left(n_x,n_y,E\right)\right\}^2\right)\cosh\left\{\beta_{11}\left(n_x,n_y,E\right)\right\}\cos\left\{\gamma_{11}\left(n_x,n_y,E\right)\right\}\right.$$

$$\left.\left.+\frac{1}{12}\left(\frac{5\left\{K_{13}\left(n_x,n_y,E\right)\right\}^3}{K_{14}\left(n_x,n_y,E\right)}+\frac{5\left\{K_{14}\left(n_x,n_y,E\right)\right\}^3}{K_{13}\left(n_x,n_y,E\right)}-\left\{34K_{14}\left(n_x,n_y,E\right)K_{13}\left(n_x,n_y,E\right)\right\}\right)\sinh\left\{\beta_{11}\left(n_x,n_y,E\right)\right\}\sin\left\{\gamma_{11}\left(n_x,n_y,E\right)\right\}\right]\right]\Big]$$

$\varepsilon_{11}\left(n_x,n_y,E\right)\equiv\left[\frac{K_{13}\left(n_x,n_y,E\right)}{K_{14}\left(n_x,n_y,E\right)}-\frac{K_{14}\left(n_x,n_y,E\right)}{K_{13}\left(n_x,n_y,E\right)}\right]$, $\beta_{11}\left(n_x,n_y,E\right)\equiv K_{13}\left(n_x,n_y,E\right)\left[a_0-\Delta_0\right]$,

$\gamma_{11}\left(n_x,n_y,E\right)=K_{14}\left(n_x,n_y,E\right)\left[b_0-\Delta_0\right]$,

$$K_{14}\left(n_x,n_y,E\right)\equiv\left[\frac{B_0^2+2AE-B_0\sqrt{B_0^2+4AE}}{2A^2}-\left\{\phi\left(n_x,n_y\right)\right\}\right]^{1/2},$$ and

$$K_{13}\left(n_x,n_y,E\right)\equiv\left[\left(\frac{2m_2^*E'}{\hbar^2}\right)G\left(E-V_0,\alpha_2,\Delta_2\right)+\left\{\phi\left(n_x,n_y\right)\right\}\right]^{1/2}.$$

The electron concentration in this case can be expressed as

$$n_{1D}=\left(\frac{2g_v}{\pi}\right)\sum_{n_x=1}^{n_{\max}}\sum_{n_y=1}^{n_{y\max}}\left[T_{923}\left(n_x,n_y,E_{FQWSL}\right)+T_{924}\left(n_x,n_y,E_{FQWSL}\right)\right] \tag{51}$$

where, $T_{923}\left(n_x,n_y,E_{FQWSL}\right)\equiv\left[\rho_{11}\left(n_x,n_y,E_{FQWSL}\right)-\left\{\phi\left(n_x,n_y\right)\right\}\right]^{1/2}$ and

$$T_{924}\left(n_x,n_y,E_{FQWSL}\right)\equiv\sum_{r=1}^{s}L(r)\left[T_{923}\left(n_x,n_y,E_{FQWSL}\right)\right].$$

The use of equations (51) and (5) leads to the expression of the DMR as

$$\frac{D}{\mu}=\frac{1}{|e|}\frac{\sum_{n_x=1}^{n_{\max}}\sum_{n_y=1}^{n_{y\max}}\left[T_{923}\left(n_x,n_y,E_{FQWSL}\right)+T_{924}\left(n_x,n_y,E_{FQWSL}\right)\right]}{\sum_{n_x=1}^{n_{\max}}\sum_{n_y=1}^{n_{y\max}}\left[\left\{T_{923}\left(n_x,n_y,E_{FQWSL}\right)\right\}'+\left\{T_{924}\left(n_x,n_y,E_{FQWSL}\right)\right\}'\right]} \tag{52}$$

2.13 Einstein relation in III-V effective mass quantum wire superlattices

The electron dispersion law in III-V, effective mass quantum wire superlattices (EMQWSLs) can be written as

$$k_x^2=\left[\rho_{12}\left(n_y,n_z,E\right)\right] \tag{53}$$

in which, $\rho_{12}\left(n_y,n_z,E\right)=\frac{1}{L_0^2}\left[\cos^{-1}\left(f_{12}\left(n_y,n_z,E\right)\right)\right]^2-\left\{\phi\left(n_y,n_z\right)\right\}$, $\phi\left(n_y,n_z\right)=\left\{\left(\frac{n_y\pi}{d_y}\right)^2+\left(\frac{n_z\pi}{d_z}\right)^2\right\}$

$$f_{12}\left(n_y,n_z,E\right)=a_7\cos\left[a_0C_5\left(n_y,n_z,E\right)+b_0D_5\left(n_y,n_z,E\right)\right]-a_8\cos\left[a_0C_5\left(n_y,n_z,E\right)-b_0D_5\left(n_y,n_z,E\right)\right]$$

$C_5\left(n_y,n_z,E\right)\equiv\left[\left(\frac{2m_1^*E}{\hbar^2}\right)G\left(E,E_{g_1},\Delta_1\right)-\left\{\phi\left(n_y,n_z\right)\right\}\right]^{1/2}$ and

$D_5\left(n_y,n_z,E\right)\equiv\left[\left(\frac{2m_2^*E}{\hbar^2}\right)G\left(E,E_{g_1},\Delta_1\right)-\left\{\phi\left(n_y,n_z\right)\right\}\right]^{1/2}$.

The electron concentration in this case can be expressed as

$$n_{1D}=\left(\frac{2g_v}{\pi}\right)\sum_{n_y=1}^{n_{y\max}}\sum_{n_z=1}^{n_{z\max}}\left[T_{925}\left(n_y,n_z,E_{FQWSL}\right)+T_{926}\left(n_y,n_z,E_{FQWSL}\right)\right] \quad (54)$$

where, $T_{925}\left(n_y,n_z,E_{FQWSL}\right)\equiv\left[\rho_{12}\left(n_y,n_z,E_{FQWSL}\right)\right]^{1/2}$ and

$T_{926}\left(n_y,n_z,E_{FQWSL}\right)\equiv\sum_{r=1}^{s}L\left(r\right)\left[T_{925}\left(n_y,n_z,E_{FQWSL}\right)\right]$.

The use of equations (54) and (5) leads to the expression of the DMR as

$$\frac{D}{\mu}=\frac{1}{|e|}\frac{\sum_{n_y=1}^{n_{y\max}}\sum_{n_z=1}^{n_{z\max}}\left[T_{925}\left(n_y,n_z,E_{FQWSL}\right)+T_{926}\left(n_y,n_z,E_{FQWSL}\right)\right]}{\sum_{n_y=1}^{n_{y\max}}\sum_{n_z=1}^{n_{z\max}}\left[\left\{T_{925}\left(n_y,n_z,E_{FQWSL}\right)\right\}'+\left\{T_{926}\left(n_y,n_z,E_{FQWSL}\right)\right\}'\right]} \quad (55)$$

2.14 Einstein relation in II-VI effective mass quantum wire superlattices

The dispersion law in II-VI, EMQWSLs can be written as

$$k_z^2=\left[\rho_{13}\left(n_x,n_y,E\right)\right] \quad (56)$$

in which, $\rho_{13}\left(n_x,n_y,E\right)=\frac{1}{L_0^2}\left[\cos^{-1}\left(f_{13}\left(n_x,n_y,E\right)\right)\right]^2-\left\{\phi\left(n_x,n_y\right)\right\}$,

$f_{13}\left(n_x,n_y,E\right)=a_9\cos\left[a_0C_6\left(n_x,n_y,E\right)+b_0D_6\left(n_x,n_y,E\right)\right]-a_{10}\cos\left[a_0C_6\left(n_x,n_y,E\right)-b_0D_6\left(n_x,n_y,E\right)\right]$

$C_6\left(n_x,n_y,E\right)\equiv\left(\frac{2m_{\parallel,1}^*}{\hbar^2}\right)^{1/2}\left[E-\left\{\frac{\hbar^2}{2m_{\perp,1}^*}\phi\left(n_x,n_y\right)\right\}\mp\bar{\lambda}_0\left\{\phi\left(n_x,n_y\right)\right\}^{1/2}\right]^{1/2}$

and $D_6\left(n_x,n_y,E\right)\equiv\left[\left(\frac{2m_2^*}{\hbar^2}\right)EG\left(E,E_{g_2},\Delta_2\right)-\phi\left(n_x,n_y\right)\right]^{1/2}$.

The electron concentration in this case can be expressed as

$$n_{1D} = \left(\frac{2g_v}{\pi}\right) \sum_{n_x=1}^{n_{x\max}} \sum_{n_y=1}^{n_{y\max}} \left[T_{927}\left(n_x, n_y, E_{FQWSL}\right) + T_{928}\left(n_x, n_y, E_{FQWSL}\right) \right] \quad (57)$$

where, $T_{927}\left(n_x, n_y, E_{FQWSL}\right) \equiv \left[\rho_{13}\left(n_x, n_y, E_{FQWSL}\right)\right]^{1/2}$ and $T_{928}\left(n_x, n_y, E_{FQWSL}\right) \equiv \sum_{r=1}^{s} L(r) \left[T_{927}\left(n_x, n_y, E_{FQWSL}\right)\right]$.

Thus, using equation (57) and (511) leads to the expression of the DMR as

$$\frac{D}{\mu} = \frac{1}{|e|} \frac{\sum_{n_x=1}^{n_{x\max}} \sum_{n_y=1}^{n_{y\max}} \left[T_{927}\left(n_x, n_y, E_{FQWSL}\right) + T_{928}\left(n_x, n_y, E_{FQWSL}\right) \right]}{\sum_{n_x=1}^{n_{x\max}} \sum_{n_y=1}^{n_{y\max}} \left[\left\{ T_{927}\left(n_x, n_y, E_{FQWSL}\right) \right\}' + \left\{ T_{928}\left(n_x, n_y, E_{FQWSL}\right) \right\}' \right]} \quad (58)$$

2.15 Einstein relation in IV-VI effective mass quantum wire superlattices

The dispersion law in IV-VI, EMQWSLs can be written as

$$k_x^2 = \left[\rho_{14}\left(n_y, n_z, E\right) \right] \quad (59)$$

in which, $\rho_{14}\left(n_y, n_z, E\right) = \frac{1}{L_0^2} \left[\cos^{-1}\left(f_{14}\left(n_y, n_z, E\right)\right)\right]^2 - \left\{ \phi\left(n_y, n_z\right) \right\}$

$$f_{14}\left(n_y, n_z, E\right) = a_9 \cos\left[a_0 C_7\left(n_y, n_z, E\right) + b_0 D_7\left(n_y, n_z, E\right)\right] - a_{10} \cos\left[a_0 C_7\left(n_y, n_z, E\right) - b_0 D_7\left(n_y, n_z, E\right)\right]$$

$$C_7\left(n_y, n_z, E\right) \equiv \left[\left[EH_{11} + H_{21}\left(n_y, n_z\right)\right] - \left[E^2 H_{31} + EH_{41}\left(n_y, n_z\right) + H_{51}\left(n_y, n_z\right)\right]^{1/2} \right]^{1/2}$$

$$D_7\left(n_y, n_z, E\right) \equiv \left[\left[EH_{12} + H_{22}\left(n_y, n_z\right)\right] - \left[E^2 H_{32} + EH_{42}\left(n_y, n_z\right) + H_{52}\left(n_y, n_z\right)\right]^{1/2} \right]^{1/2}.$$

The electron concentration in this case can be expressed as

$$n_{1D} = \left(\frac{2g_v}{\pi}\right) \sum_{n_y=1}^{n_{y\max}} \sum_{n_z=1}^{n_{z\max}} \left[T_{929}\left(n_y, n_z, E_{FQWSL}\right) + T_{930}\left(n_y, n_z, E_{FQWSL}\right) \right] \quad (60)$$

where, $T_{929}\left(n_y,n_z,E_{FQWSL}\right)\equiv\left[\rho_{14}\left(n_y,n_z,E_{FQWSL}\right)\right]^{1/2}$ and $T_{930}\left(n_y,n_z,E_{FQWSL}\right)\equiv\sum_{r=1}^{s}L(r)\left[T_{929}\left(n_y,n_z,E_{FQWSL}\right)\right]$.

Thus, using equation (60) and (5) leads to the expression of the DMR as

$$\frac{D}{\mu}=\frac{1}{|e|}\frac{\sum_{n_y=1}^{n_{y\max}}\sum_{n_z=1}^{n_{z\max}}\left[T_{929}\left(n_y,n_z,E_{FQWSL}\right)+T_{930}\left(n_y,n_z,E_{FQWSL}\right)\right]}{\sum_{n_y=1}^{n_{y\max}}\sum_{n_z=1}^{n_{z\max}}\left[\left\{T_{929}\left(n_y,n_z,E_{FQWSL}\right)\right\}'+\left\{T_{930}\left(n_y,n_z,E_{FQWSL}\right)\right\}'\right]} \quad (61)$$

2.16 Einstein relation in HgTe/CdTe effective mass quantum wire superlattices

The dispersion law in HgTe/CdTe, EMQWSLs can be written as

$$k_x^2=\left[\rho_{15}\left(n_y,n_z,E\right)\right] \quad (62)$$

in which, $\rho_{15}\left(n_y,n_z,E\right)=\frac{1}{L_0^2}\left[\cos^{-1}\left(f_{15}\left(n_y,n_z,E\right)\right)\right]^2-\left\{\phi\left(n_y,n_z\right)\right\}$,

$f_{15}\left(n_y,n_z,E\right)=a_{11}\cos\left[a_0C_8\left(n_y,n_z,E\right)+b_0D_8\left(n_y,n_z,E\right)\right]-a_{12}\cos\left[a_0C_8\left(n_y,n_z,E\right)-b_0D_8\left(n_y,n_z,E\right)\right]$

$C_8\left(n_y,n_z,E\right)\equiv\left[\frac{B_0^2+2AE-B_0\sqrt{B_0^2+4AE}}{2A^2}-\left\{\phi\left(n_y,n_z\right)\right\}\right]^{1/2}$ and

$D_8\left(n_y,n_z,E\right)\equiv\left[\left(\frac{2m_2^*E}{\hbar^2}\right)G\left(E,E_{g_2},\Delta_2\right)-\left\{\phi\left(n_y,n_z\right)\right\}\right]^{1/2}$.

The electron concentration in this case can be expressed as

$$n_{1D}=\left(\frac{2g_v}{\pi}\right)\sum_{n_y=1}^{n_{y\max}}\sum_{n_z=1}^{n_{z\max}}\left[T_{931}\left(n_y,n_z,E_{FQWSL}\right)+T_{932}\left(n_y,n_z,E_{FQWSL}\right)\right] \quad (63)$$

where, $T_{931}\left(n_y,n_z,E_{FQWSL}\right)\equiv\left[\rho_{15}\left(n_y,n_z,E_{FQWSL}\right)\right]^{1/2}$ and

$T_{932}\left(n_y,n_z,E_{FQWSL}\right)\equiv\sum_{r=1}^{s}L(r)\left[T_{931}\left(n_y,n_z,E_{FQWSL}\right)\right]$.

The use of equations (63) and (5) leads to the expression of the DMR as

$$\frac{D}{\mu}=\frac{1}{|e|}\frac{\sum_{n_y=1}^{n_{y_{\max}}}\sum_{n_z=1}^{n_{z_{\max}}}\left[T_{931}\left(n_y,n_z,E_{FQWSL}\right)+T_{932}\left(n_y,n_z,E_{FQWSL}\right)\right]}{\sum_{n_y=1}^{n_{y_{\max}}}\sum_{n_z=1}^{n_{z_{\max}}}\left[\left\{T_{931}\left(n_y,n_z,E_{FQWSL}\right)\right\}'+\left\{T_{932}\left(n_y,n_z,E_{FQWSL}\right)\right\}'\right]} \quad (64)$$

Results and discussion

Using the appropriate equations, we have plotted the DMR in figure 1 as a function of inverse quantizing magnetic field for $GaAs/Ga_{1-x}Al_xAs$, CdS/CdTe, PbTe/PbSnTe and HgTe/CdTe superlattices with graded interfaces as shown by curves (a), (b), (c) and (d) respectively. Using the same equations, we have plotted the DMR as function of electron concentration for $GaAs/Ga_{1-x}Al_xAs$, CdS/CdTe superlattices, as shown by curves (a) and (b), where as, in figure 3, the same has been drawn for PbTe/PbSnTe and HgTe/CdTe superlattices respectively as shown by curves (c) and (d) respectively. We have plotted the DMR in figure 4 and as a function of inverse quantizing magnetic field for the aforementioned effective mass superlattices respectively. Using the same equations, we have plotted the DMR as function of electron concentration for $GaAs/Ga_{1-x}Al_xAs$, CdS/CdTe effective mass superlattices, as shown by curves (a) and (b) in figure 5, where as the same has been drawn for PbTe/PbSnTe and HgTe/CdTe effective mass superlattices respectively as shown by curves (c) and (d) in figure 6 respectively. It appears from the said figures that the DMR oscillates both with 1/B and n_0 due to SdH effect, although the rates of variations are totally band structures dependent for all types of superlattices as considered here. We have plotted the normalized 1D DMR as function of film thickness for $GaAs/Ga_{1-x}Al_xAs$, CdS/CdTe, PbTe/PbSnTe and HgTe/CdTe quantum wire superlattices with graded interfaces as shown by curves (a), (b), (c) and (d) of figure 7. In figure 8, we have plotted all cases of figure 7 as function of electron concentration per unit length. It appears from both the figures 7 and 8 that the DMR increases with decreasing film thickness and increasing electron concentration per unit length respectively, although the numerical values are totally band structure dependent. We further plot the normalized 1D DMR in quantum wire effective mass superlattices of the aforementioned materials as functions of film thickness and electron concentration per unit length as shown by curves (a), (b), (c) and (d) in figures 9 and 10 respectively. It appears that the DMR in this case decreases with film thickness and increases with electron concentration for all the cases.

Conclusion

In this paper an attempt is made to study the Einstein relation for the diffusivity mobility ratio (DMR) under magnetic quantization in III-V, II-VI, IV-VI and HgTe/CdTe SLs with graded interfaces by formulating the appropriate electron statistics. We have also investigated the DMR in III-V, II-VI, IV-VI and HgTe/CdTe effective mass SLs in the presence of quantizing magnetic field respectively. The DMRs in quantum wire $GaAs/Ga_{1-x}Al_xAs$, CdS/CdTe, PbTe/PbSnTe and HgTe/CdTe SLs and the corresponding effective mass SLs have further been studied. It appears that the DMR oscillates both with inverse quantizing magnetic field and electron concentration for $GaAs/Ga_{1-x}Al_xAs$, CdS/CdTe, PbTe/PbSnTe and HgTe/CdTe superlattices with graded interfaces. The DMR decreases with increasing film thickness and decreasing electron concentration for the said superlattices under 2D quantization of wave vector space.

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FIGURES

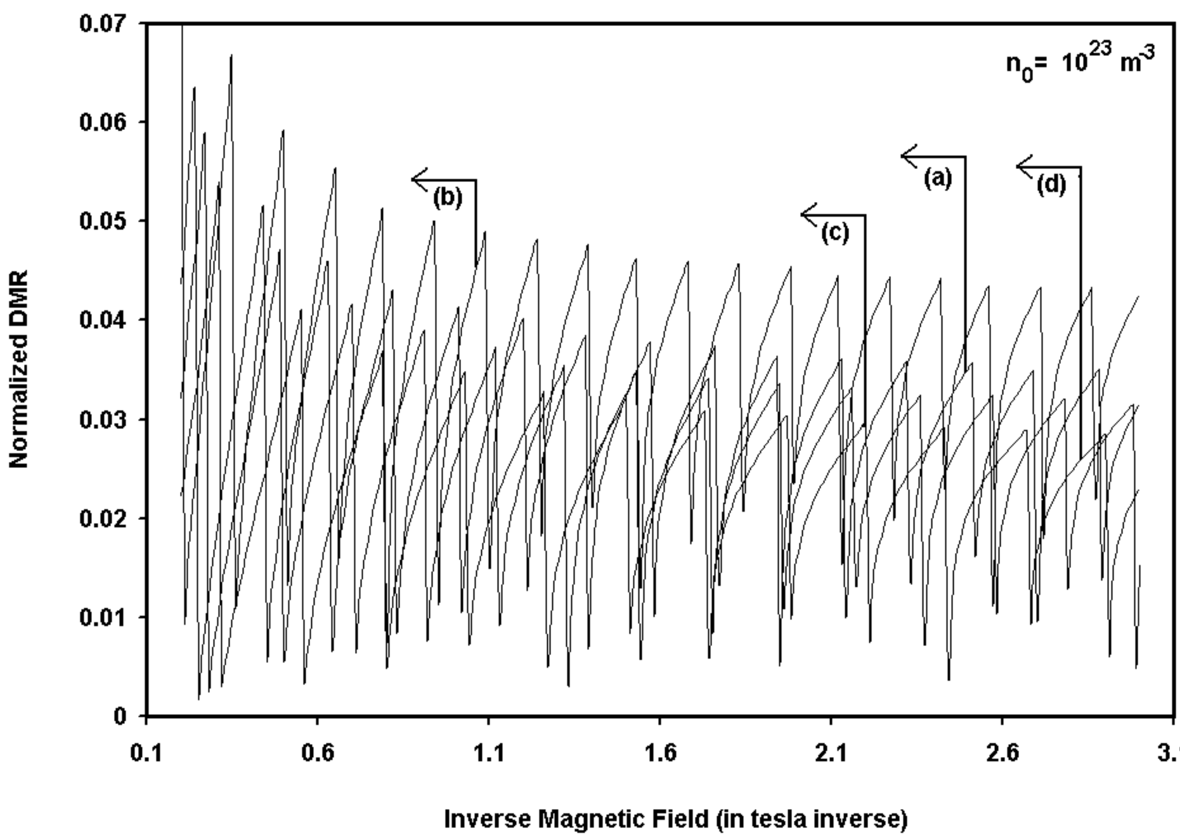

Fig 1 The plot of the DMR as a function of inverse quantizing magnetic field for (a) GaAs/$Ga_{1-x}Al_xAs$ (b) CdS/CdTe (c) PbTe/PbSnTe and (d) HgTe/CdTe superlattices with graded interfaces.

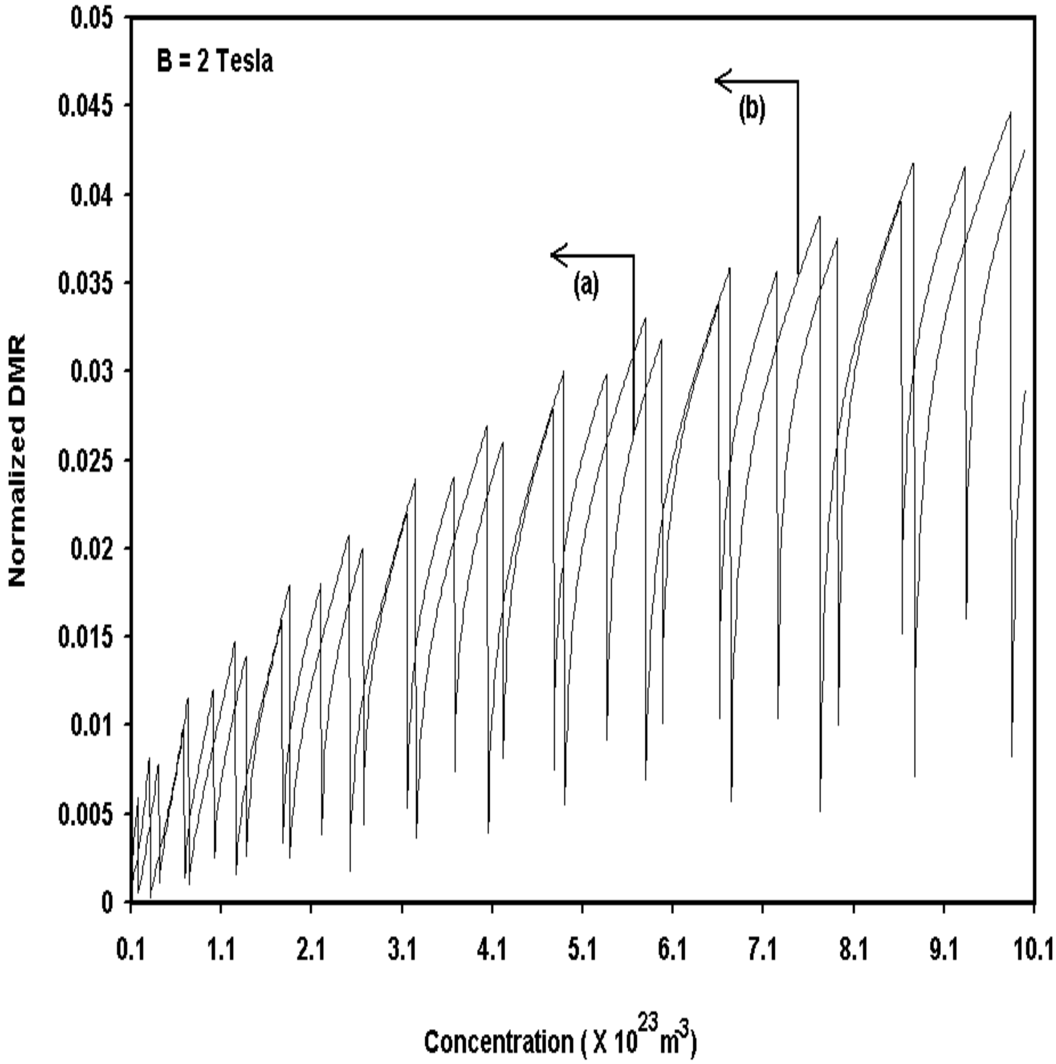

Fig 2 The plot of the DMR as a function of electron concentration for (a) GaAs/$Ga_{1-x}Al_xAs$ and (b) CdS/CdTe superlattices with graded interfaces.

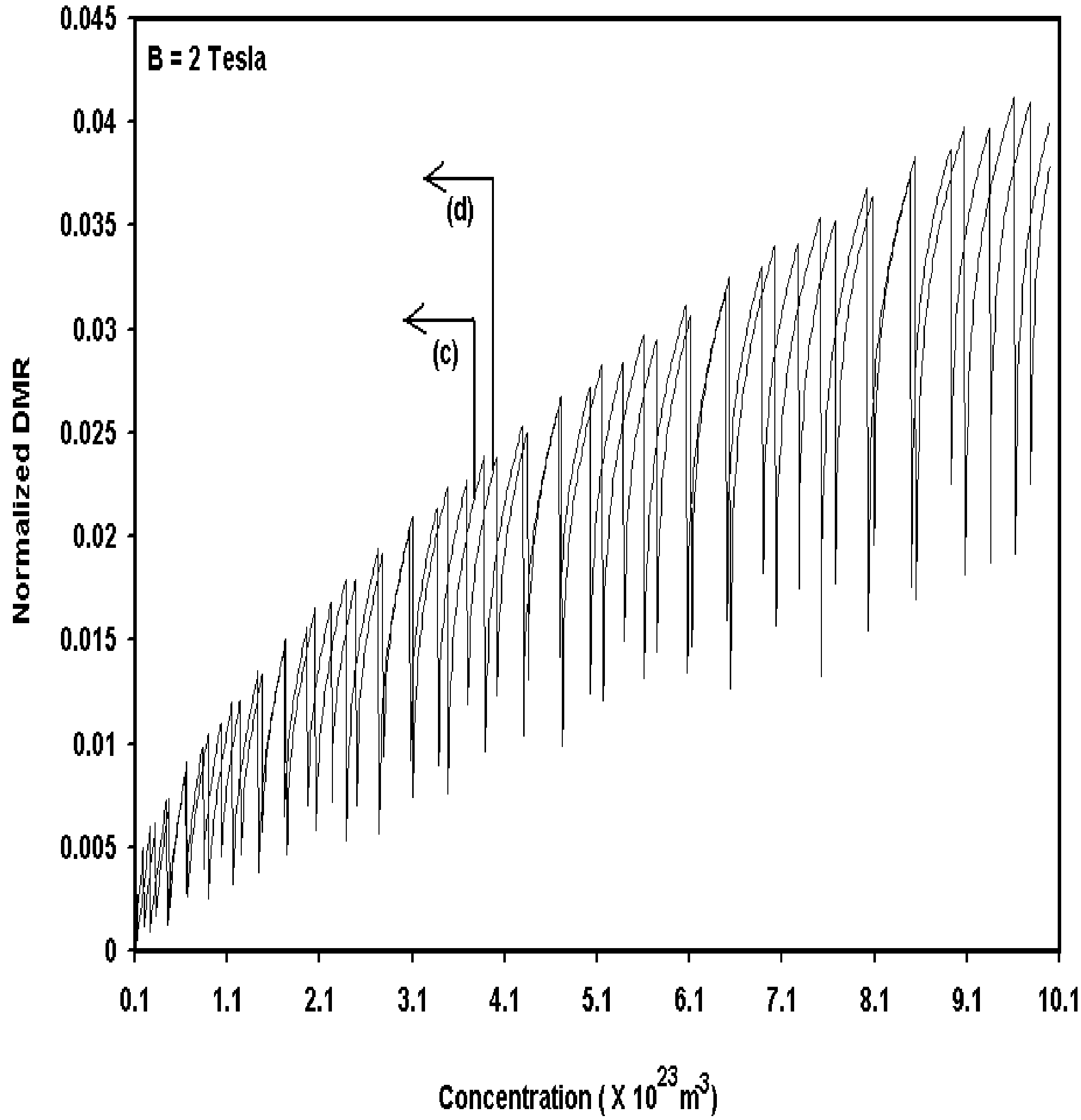

Fig 3 The plot of the DMR as a function of electron concentration for (c) PbTe/PbSnTe and (d) HgTe/CdTe superlattices with graded interfaces.

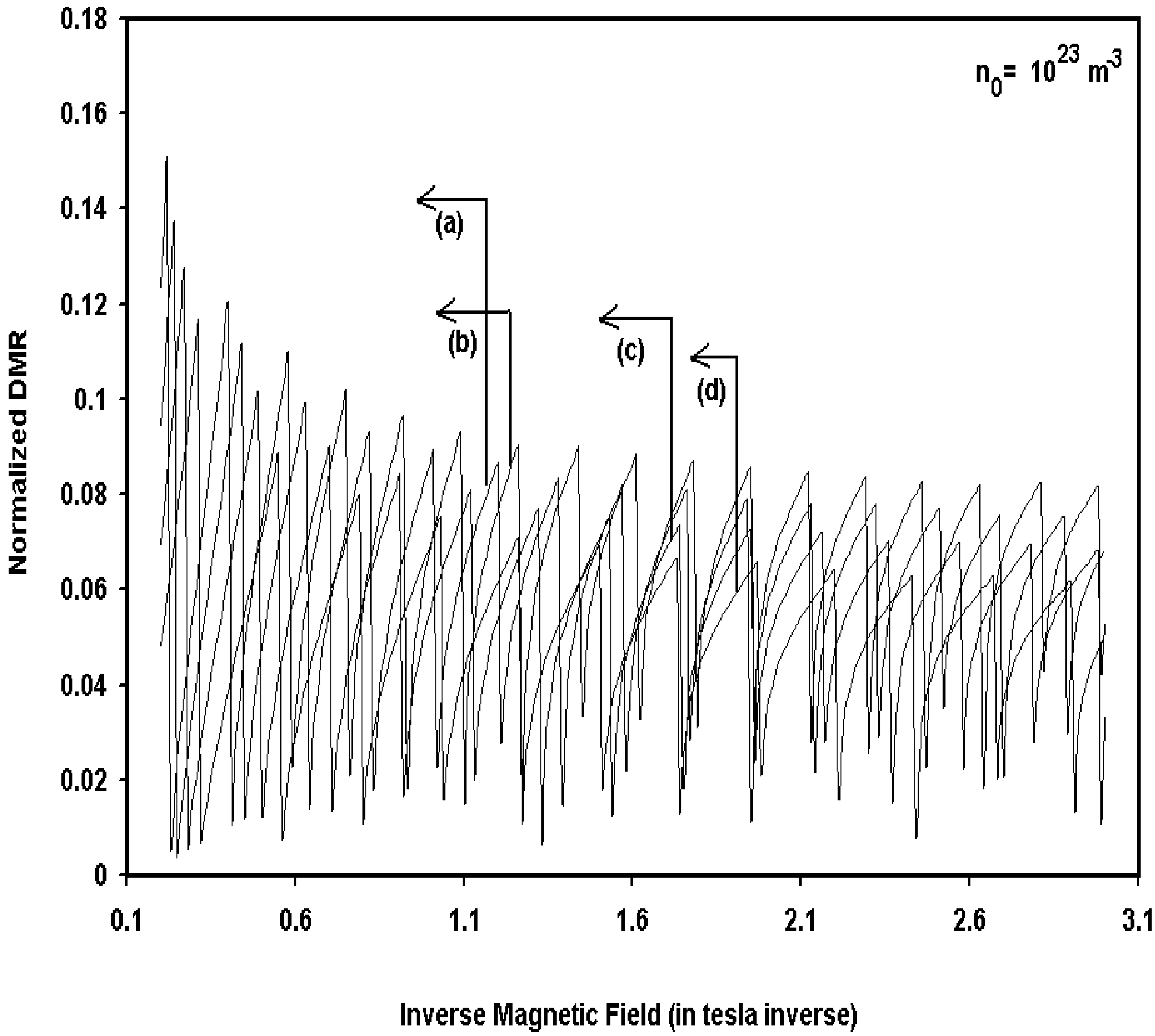

Fig 4 The plot of the DMR as a function of inverse quantizing magnetic field for (a) $GaAs/Ga_{1-x}Al_xAs$ (b) CdS/CdTe (c) PbTe/PbSnTe and (d) HgTe/CdTe effective mass superlattices.

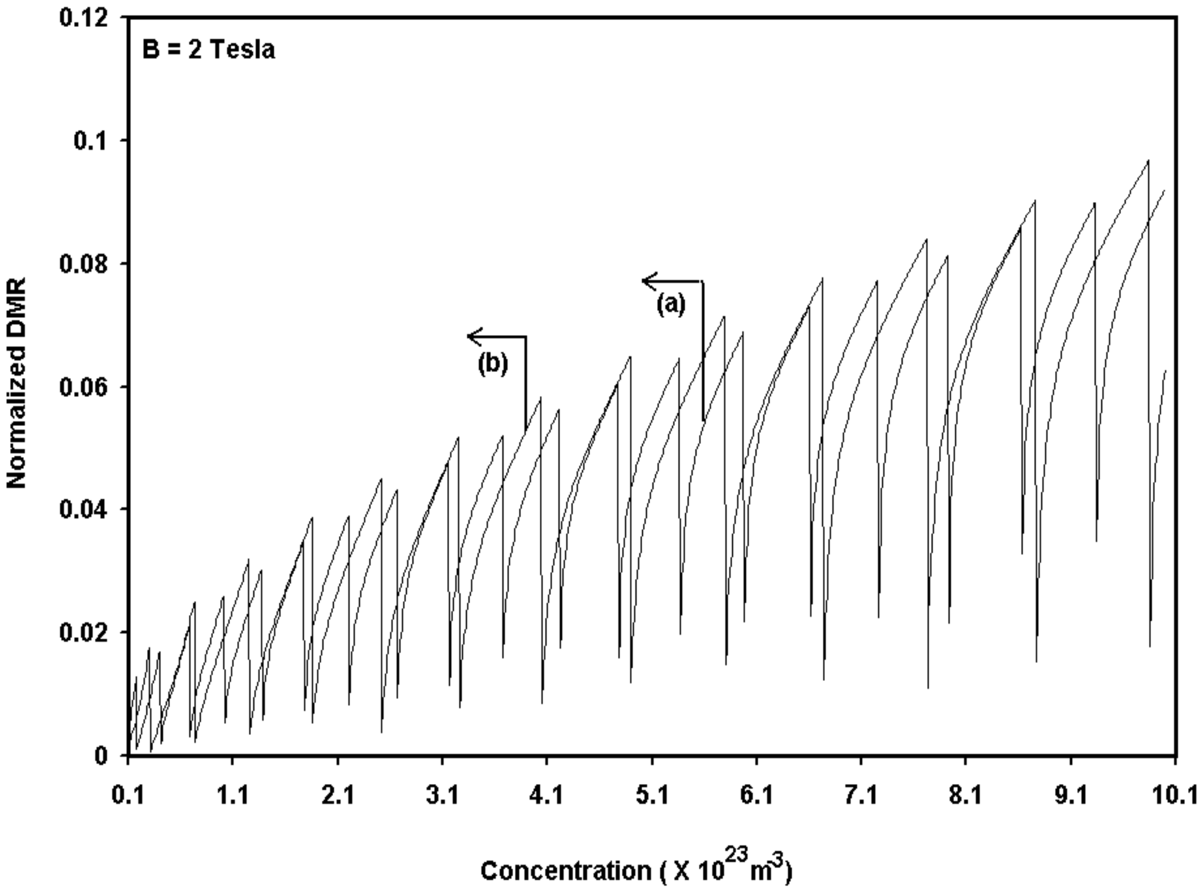

Fig 5 The plot of the DMR as a function of electron concentration for (a) GaAs/$Ga_{1-x}Al_xAs$ and (b) CdS/CdTe effective mass superlattices.

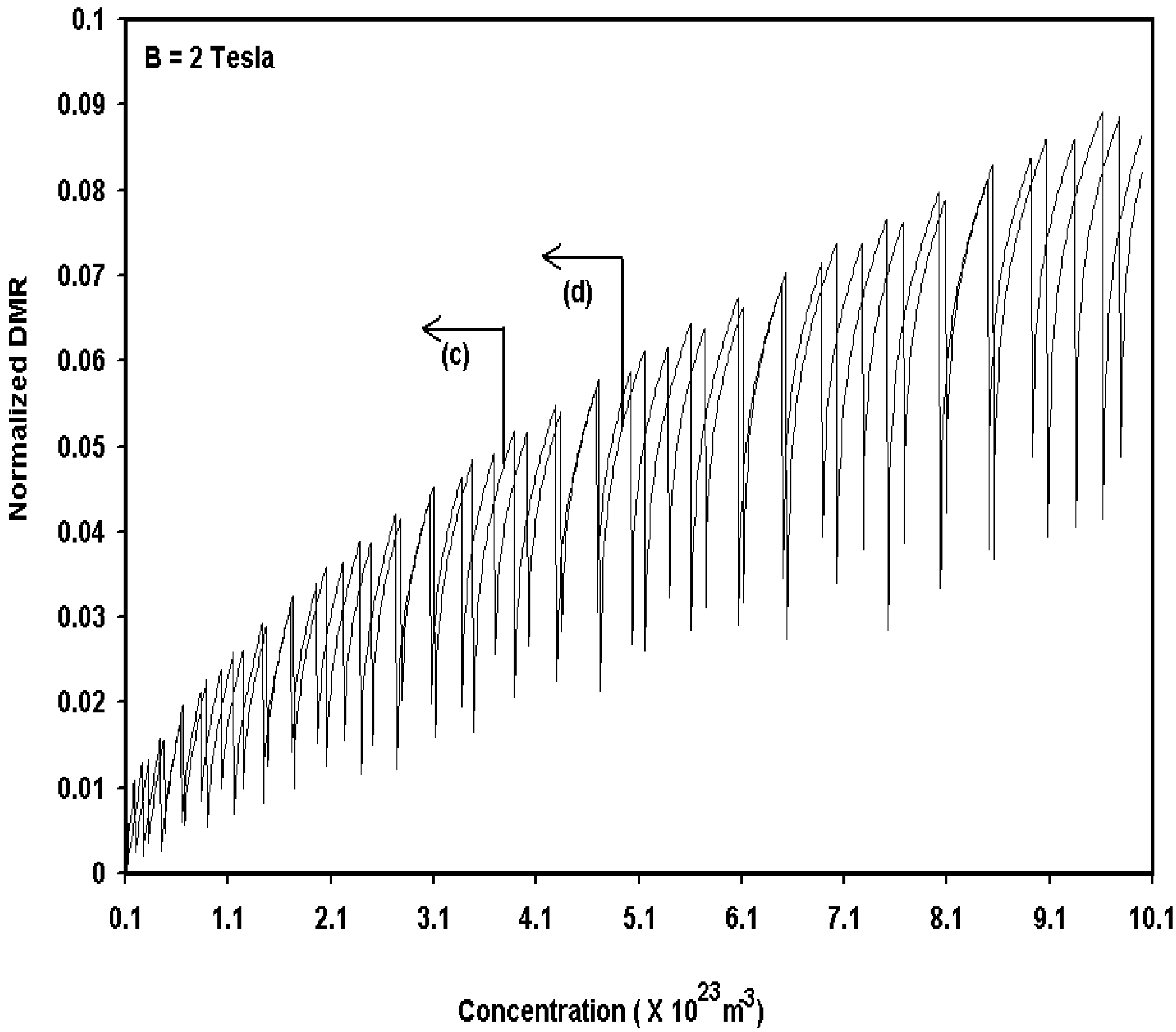

Fig 6 The plot of the DMR as a function of electron concentration for (c) PbTe/PbSnTe and (d) HgTe/CdTe effective mass superlattices.

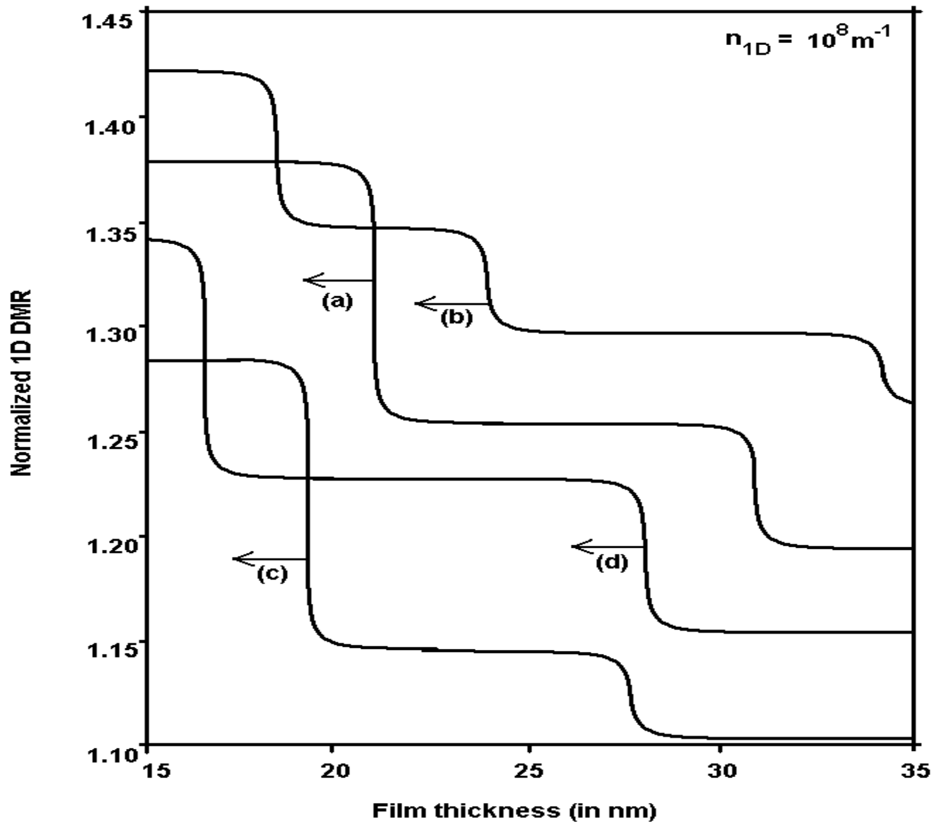

Fig 7 The plot of the 1D DMR as a function of film thickness for (a) $GaAs/Ga_{1-x}Al_xAs$ (b) CdS/CdTe (c) PbTe/PbSnTe and (d) HgTe/CdTe superlattices with graded interfaces.

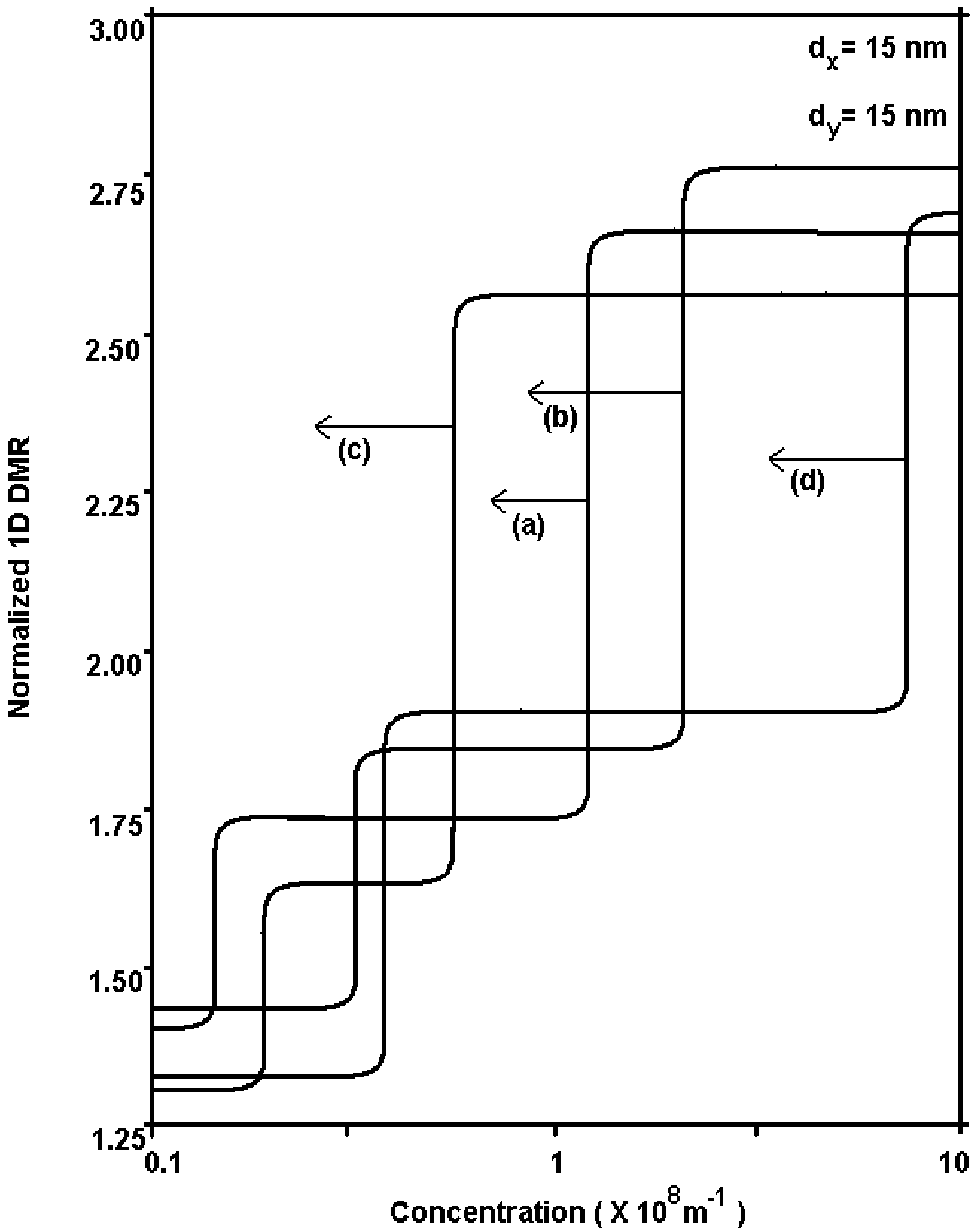

Fig 8 The plot of the 1D DMR as a function of surface electron concentration for (a) $GaAs/Ga_{1-x}Al_xAs$ (b) CdS/CdTe (c) PbTe/PbSnTe and (d) HgTe/CdTe superlattices with graded interfaces.

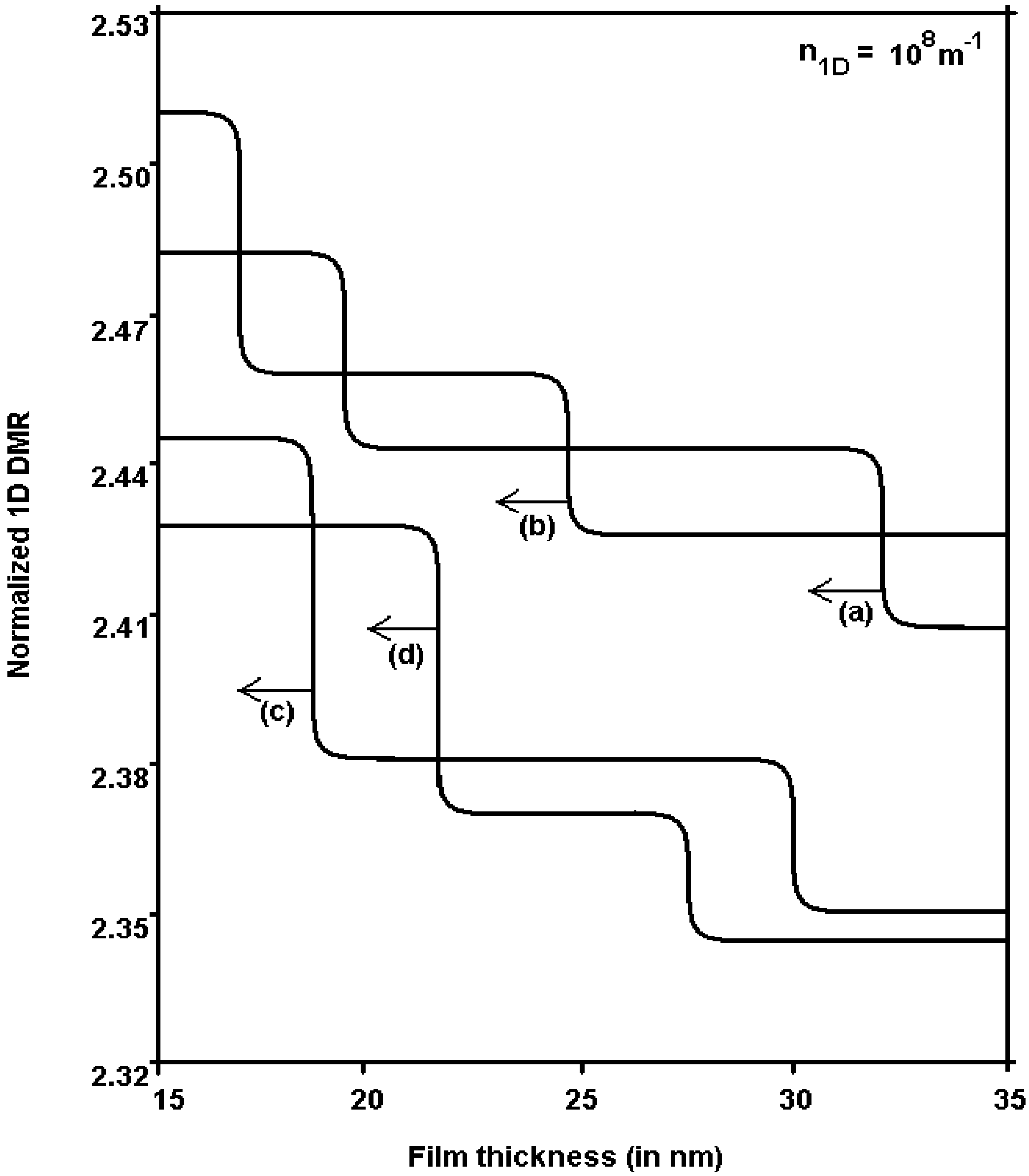

Fig 9 The plot of the 1D DMR as a function of film thickness for (a) $GaAs/Ga_{1-x}Al_xAs$ (b) CdS/CdTe (c) PbTe/PbSnTe and (d) HgTe/CdTe effective mass superlattices.

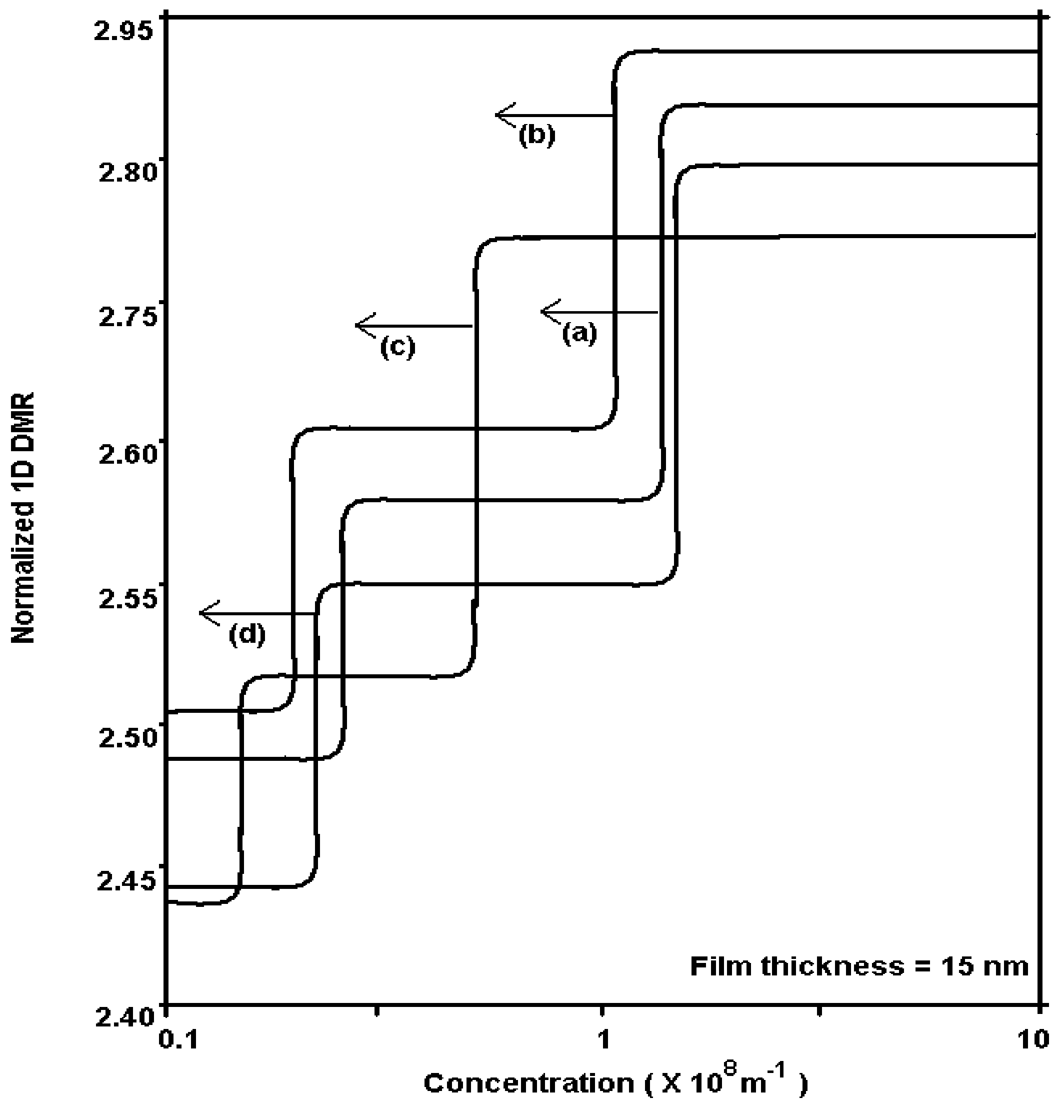

Fig 10 The plot of the 1D DMR as a function of surface electron concentration for (a) $GaAs/Ga_{1-x}Al_xAs$ (b) CdS/CdTe (c) PbTe/PbSnTe and (d) HgTe/CdTe effective mass superlattices.