# A MATHEMATICAL MODEL TO STUDY THE EFFECT OF POROUS PARAMETER ON BLOOD FLOW THROUGH AN ATHEROSCLEROTIC ARTERIAL SEGMENT HAVING SLIP VELOCITY

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## Abstract

This theoretical investigation focusses on blood flow through a multiple stenosed human artery under porous effects. A mathematical model is developed for estimating the effect of porous parameter on blood flow taking Harschel-Bulkley fluid model (to account for the presence of erythrocytes in plasma) and artery as circular tube with an axially non-symmetric but radially symmetric mild stenosis. The mathematical expression for the geometry of the artery with stenoses is given by the polynomial function model. The velocity slip condition is also given due weightage in the investigation. It is necessary to study the blood flow through such type of stenosis to improve the arterial system. An extensive quantitative analysis is carried out by performing large scale numerical computations of the measurable flow variables having more physiological significance. The variations of velocity profile, volumetric flow rate and pressure gradient with porous parameter are calculated numerically by developing computer codes. Their graphical representations with appropriate scientific discussions are presented at the end of the paper.

**Key words***: stenoses, stenotic geometry, stenotic height, H-B fluid model, Darcy's number, fluid index parameter, slip velocity, flow flux*

## INTRODUCTION

Among all the fatal diseases of the human body, circulatory disorders are still a major cause of morbidity or death. A systematic study on the rheological and hemodynamic properties of the streaming blood with the mechanical behavior of blood vessel walls could play a significant role in the basic understanding, diagnosis and treatment of many cardiovascular, cerebrovascular and arterial diseases. Arteries and veins are narrowed by abnormal and unusual deposition of cholesterols, fats, plaques and other suspended matters in their inner lining leading to the cardiovascular disease – atherosclerosis (medically termed stenosis). This narrowing of body passage, tube or orifice is a frequently occurring phenomenon in human artery. One of its most serious consequences is the increasing resistance to blood flow bringing about significant alterations in pressure distribution, wall shear stress and the flow resistance. The tragedy of aging is that plaques build up within narrow arteries and this phenomenon makes them stiffer (less elastic) that restricts (sometimes blocks) the regular flow of blood in the human physiological system. This type of constriction causes insufficient flow of blood in heart and may lead to stroke, ischemia and heart attack.

A wide variety of analytical as well as experimental studies on blood flow through the arterial segments having a single or multiple stenosis were carried out by several investigators applying different blood models (Newtonian or non-Newtonian) and various geometry of the stenosis (polynomial, smooth cosine or exponential). Experimental observations confirmed that blood is predominantly a suspension of erythrocytes (red cells) in plasma and may be better represented by non-Newtonian model when it flows through narrow arteries at low shear rate, particularly in diseased state. A good number of mathematicians, physicists and medical professionals have contributed their experimental and theoretical research works on cause and remedies of restriction of blood flow in the constricted artery of the human body.

It is a fact that all the phenomena in the real world show non-linear attitude. Therefore, for evaluating a phenomenon in our non-real world, the scientists should make their efforts to develop a mathematical model that exhibits the physical system or phenomena’s nearly exact behavior. So, for any investigation on blood flow in stenotic region, the rheological and physical nature of the blood together with the geometry of the stenosis were modeled mathematically by the researchers.

In the recent past, quite a good number of analytical as well as experimental investigations were performed by several researchers to explore the effect of arterial constriction on the flow characteristics of blood applying different fluid models for blood and geometry of the stenosis. It is accepted that blood may be fairly closely represented by Harschel-Bulkley (H-B) model-a non-Newtonian blood model at low shear rates when flowing through a tube of diameter 0.095 mm or less. In order to understand the effect of stenosis on blood flow through and beyond the narrowed segment of the artery many investigations have been undertaken both experimentally as well as analytically.

Sankar and Hemlatha (2007) presented an analytical study on the pulsatile flow of blood through catherized artery by modeling blood as Harschel-Bulkley fluid and the catheter and artery as rigid coaxial cylinders. The output of the study established decrease of velocity and flow rate and increase of wall shear stress and longitudinal impedance for the increase in value of yield stress with other parameters fixed. The pulsatile flow of blood through mild stenosed artery was investigated by Sankar and Lee (2009) considering the HB model of blood. They observed that the plug core radius, pressure drop and wall shear stress increase with the increase of yield stress and the stenosis height.

It has been observed that in case of some arterial diseases, a porous structure is formed in the lumen of an artery by fatty substances – cholesterol as well as due to blood clots. Walls are porous and generally Darcy's law is used to investigate the problem analytically. Krishna *et. al*. (2012) investigated analytically the effects of various parameters like Darcy number (for porous medium), slip parameter and radius on velocity and frictional force and obtained results for volumetric flow rate and frictional forces. Singh (2012) presented a mathematical analysis on the effect of stenosis shape parameter and height on the flow resistance. The study reveals that flow resistance decreases as shape parameter increases and it shows increasing trend with the increase of stenosis height and length. In the analysis, he adopted two phase macroscopic model for blood and the polynomial model to represent the geometry of the stenosis. Shah (2013) developed a mathematical model for the analysis of blood flow through diseased blood vessels under the influence of porous parameter. It was observed that the wall shear stress increases with the increase of porous parameter, stenosis size and length. On the other hand, the wall shear stress decreases as the stenosis shape parameter increases. The Harschel-Bulkley fluid model for blood with flow model represented by Navier-Stokes and the continuity equations were adopted in the study. In the analysis, the traditional no slip boundary condition was employed. But a number of theoretical and experimental studies on blood flow have suggested the likely presence of slip (a velocity discontinuity) at the flow boundaries (or in their immediate neighborhood) [Biswas (2000)]. Thus in blood flow modeling, consideration of a velocity slip at the stenosed vessel wall will be quite relevant. Hematocrit-a representation of the oxygen carrying capacity of the blood (its normal values for an adult male is 40-54% and for an adult female is 36-46%) was not involved in the investigation.

Nanda *et. al*. (2017) developed a mathematical model for studying blood flow through an elastic artery with the consideration of slip velocity at the inner wall of the artery. The study reveals considerable alterations in flow characteristics due to the presence of elastic property of blood vessel wall and the presence of velocity slip at the wall.

It gives us an opportunity to develop a mathematical model for estimating the effect of porous parameter on blood flow taking Harschel-Bulkley fluid model and artery as circular tube with an axially non-symmetric but radially symmetric mild stenosis. The velocity slip condition is also given due weightage in the investigation. It is necessary to study the blood flow through such type of stenosis to improve the arterial system. An extensive quantitative analysis is carried out by performing large scale numerical computations of the measurable flow variables having more physiological significance. The variations of velocity profile, volumetric flow rate and pressure gradient with porous parameter are calculated numerically by developing computer codes. Their graphical representations with appropriate scientific discussions are presented at the end of the paper.

## MATHEMATICAL FORMULATION OF THE PROBLEM

Let us consider the axisymmetric flow of blood through a a uniform circular artery with an axially non-symmetric but radially symmetric mild stenosis specified at the position shown in Fig. 1.

The geometry of the stenosis assumed to be manifested in the arterial segment is given by

$$\frac{R(z)}{R_0} = \begin{cases} 1 - A\left[L_0^{\,m-1}(z-d) - (z-d)^m\right], & d \le z \le d + L_0 \\ 1 & , otherwise \end{cases} \qquad \text{\_\_\_\_\_\_(1)}$$

where

$R(z)$: Radius of the tube with stenosis $R_0$ : Radius of the tube without stenosis

$L$: Length of the artery $L_0$ : Length of the stenosis

$\delta$: Max. height of the stenosis in the lumen $(\delta << R_0)$

$m$: Stenosis shape parameter $(m \ge 2)$

$d$: Location of the stenosis in the artery and $A = \frac{\delta}{R_0 L_0^{\,m}} \cdot \frac{m^{m/(m-1)}}{m-1}$

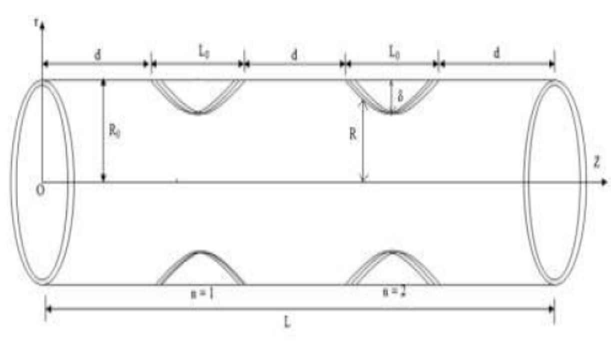

Fig-1: Geometry of the Artery with stenoses

## METHOD OF SOLUTION

The constitutive equation in one dimensional form for Herschel-Bulkley fluid in terms of the axial velocity of blood $(u)$ with the shearing stress $\tau,$ is given by

$$f(\tau) = -\frac{du}{dr} = \begin{cases} \frac{1}{\mu}\left(\tau - \tau_0\right)^n, \tau \geq \tau_0 \\ 0, \qquad \tau < \tau_0 \end{cases} -----(2)$$

where $\tau = \frac{r}{2}\left(-\frac{dp}{dz}\right),$ measure of yield stress $\tau_0 = \frac{R_c}{2}\left(-\frac{dp}{dz}\right),$ the strain rate $e = -\frac{du}{dr}$

$\mu$ : viscosity coefficient of blood $n$ : flow behavior index

$r$ : radius of artery and $p$ : pressure gradient

[The values of $'n'$ for blood flow problems are generally taken to lie between 0.9 and 1.1]

(Basu Mallik & Nanda S. [9])

The Darcy's equation governing the flow of blood through a porous media is given by

$$Q = -k.\frac{p}{\mu} ------(3)$$

where $k$ : porous parameter, $\mu$ : viscosity coefficient of blood

The present mathematical model is devoted to estimate the effect of porous parameter on blood (treated as an incompressible non-Newtonian fluid) flow.

The governing Navier- Stokes equation in cylindrical polar co-ordinates is given by

$$\frac{\partial u}{\partial t} + v\frac{\partial u}{\partial r} + u\frac{\partial u}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial z} + \frac{\mu}{\rho}\left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{\partial^2 u}{\partial z^2}\right) ------(4)$$

$$\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial r} + u\frac{\partial v}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial r} + \frac{\mu}{\rho}\left(\frac{\partial^2 v}{\partial r^2} + \frac{1}{r}\frac{\partial v}{\partial r} - \frac{v}{r^2} + \frac{\partial^2 v}{\partial z^2}\right) ----(5)$$

The continuity equation is $\frac{1}{r}\frac{\partial (rv)}{\partial r} + \frac{\partial u}{\partial z} = 0 -------------(6)$

## BOUNDARY CONDITIONS:

The slip (a velocity discontinuity) at the flow boundaries gives the following boundary conditions:

$u = u_s$ (slip velocity) at $r = R_0$ and $u = 0$ at $r = R(z)$ $-----(7)$

## ANALYTICAL SOLUTION:

Applying the governing equation of motion for steady incompressible blood flow with pressure gradient through the mid stenosis in an artery is reduced to the following form:

$P=\frac{1}{r}\frac{\partial}{\partial r}(r\tau)$, where, $\frac{\partial p}{\partial z}=-P,\ P$ being a constant. Integrating with respect to $r,\ \tau=P\frac{r}{2}$

The equation of velocity gradient can be obtained as:

$$\frac{dv}{dr}=\left(\frac{P}{2\mu}\right)^{1/n}\left(r-r_p\right)^{1/n}$$

Now, integrating the above equation by considering slip velocity $u_s$ we will have,

$$V=U_S+\left(\frac{P}{2\mu}\right)^{1/n}\times\frac{n}{n+1}\left\{\left(r-r_p\right)^{\frac{n+1}{n}}-\left(R-r_p\right)^{\frac{n+1}{n}}\right\}$$

Now the total flow flux can be calculated via the following equation: $Q=\int_0^R 2\pi r\times V\,dr$

Or, $Q=\pi R^2U_S+2\pi\left(\frac{P}{2\mu}\right)^{1/n}\times\frac{n}{n+1}\left[\frac{(R-r_p)^{\left(\frac{3n+1}{n}\right)}}{\left(\frac{3n+1}{n}\right)}-\frac{(-r_p)^{\left(\frac{3n+1}{n}\right)}}{\left(\frac{3n+1}{n}\right)}+r_p\left\{\frac{(R-r_p)^{\left(\frac{2n+1}{n}\right)}}{\left(\frac{2n+1}{n}\right)}-\frac{(-r_p)^{\left(\frac{2n+1}{n}\right)}}{\left(\frac{2n+1}{n}\right)}\right\}-\left(R-r_p\right)^{\frac{n+1}{n}}\times R\right]$ and the pressure P from the above equation can be calculated as:

$$P=\frac{2\mu\times\left(\frac{n+1}{n}\right)^n\times\left[Q-\pi R^2U_S\right]^n}{(2\pi)^n\left[\frac{\left(R-r_p\right)^{\frac{3n+1}{n}}}{\left(\frac{3n+1}{n}\right)}-\frac{\left(-r_p\right)^{\frac{3n+1}{n}}}{\left(\frac{3n+1}{n}\right)}+r_p\left\{\frac{\left(R-r_p\right)^{\frac{2n+1}{n}}}{\left(\frac{2n+1}{n}\right)}-\frac{\left(-r_p\right)^{\frac{2n+1}{n}}}{\left(\frac{2n+1}{n}\right)}\right\}-\left(R-r_p\right)^{\frac{n+1}{n}}\times R\right]^n}$$

## NUMERICAL RESULTS AND DISCUSSION:

In order to have an estimate of the qualitative and quantitative effects of the physical and rheological parameters involved in the analysis, it is necessary to quantify them. The values of different material constants and other parameters have been taken from standard literatures. $P=550,\ \mu=0.004,\ u_s=0.05,\ \gamma_p=0.001,\ \frac{\delta}{R_0}=0.2,\ L_0=0.5,\ d=0.25,\ z=0.75$ Computer codes are developed and the graphs are plotted using MATLAB 8.5.

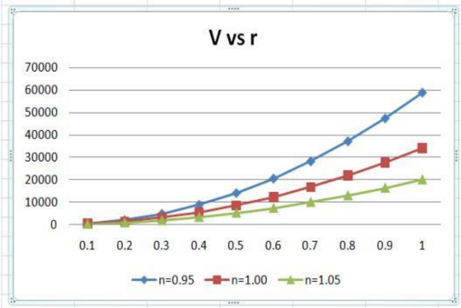


Fig. 2: Variation of flow velocity with radial distance

The variation of flow velocity $(v)$ with radial distance $(r)$ for $n$ (fluid index parameter) = 0.95, 1.00, 1.05 is exhibited in Fig. 2. In the normal stage, the flow velocity should be minimum in the vicinity of the flow boundary but the trend may be justified due to the presence of stenosis in the artererial segment. The tendency is similar for $n$ = 0.95, 1.00, 1.05.

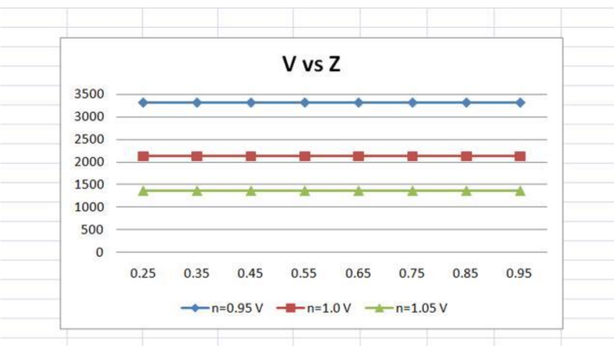


Fig. 3: Variation of flow velocity with axial distance, taking $\frac{\delta}{R_0}$ =0.2

Fig. 3 illustrates the variation of flow velocity with axial distance for $n$ = 0.95, 1.00, 1.05 taking $\frac{\delta}{R_0}$ (stenotic height) = 0.2. The velocity is proportional to $r$ that is the presence of stenosis does not influence the longitudinal velocity.

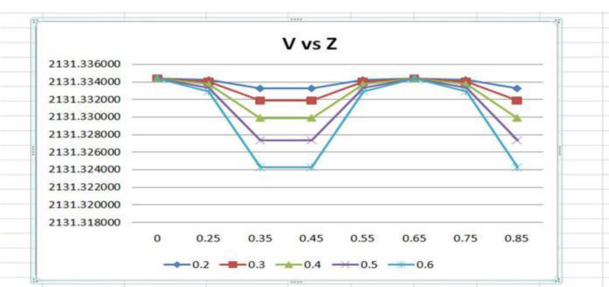

Fig. 4: Variation of flow velocity with axial distance, $\frac{\delta}{R_0}$ =0.2, 0.3, 0.4, 0.5, 0.6

The variation of flow velocity $(v)$ with radial distance $(r)$ for $n$ (fluid index parameter) = 0.95, 1.00, 1.05 is demonastrated in Fig. 4. The flow velocity sharply declines in the stenotic region and converge for the values of $\frac{\delta}{R_0}$ the under consideration.

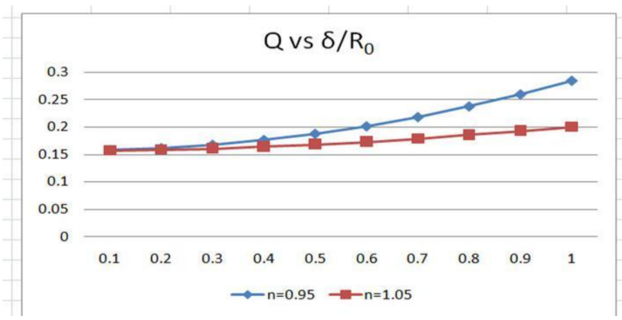


Fig. 5: Variation of flow flux with stenotic height

Taking $n$ (fluid index parameter) = 0.95 and 1.05, the variation of flow flux with stenotic height is exhibited in Fig. 5. The Flow flux shows an increasing tendency for increase in the values of stenotic height for both values of $n$.

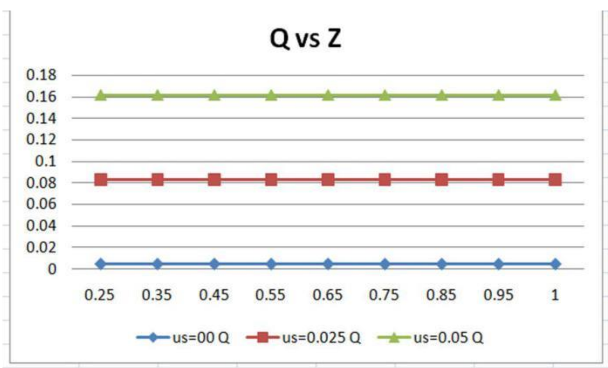


Fig. 6: Variation of flow flux with axial distance

Taking slip velocity 0.0 (no slip), 0.025 and 0.05, the variation of flow flux with axial distance is demonastrated in the above figure (6). The flow flux is minimum for minimum slip velocity. So, velocity discontinuity has an important role in the calculation of flow flux.

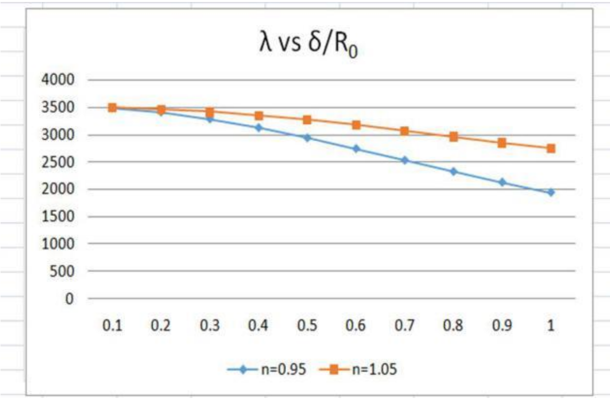


Fig. 7: Flow resistance $\lambda = \dfrac{P}{Q}$ vs. Stenotic height

The variation of flow resistance with stenotic height for $n = 0.95, 1.05$ are presented.

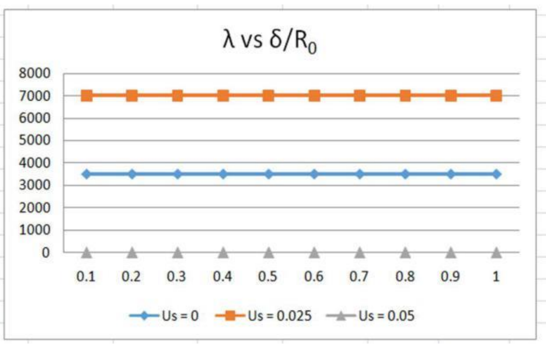


Fig. 8: Flow resistance $\lambda = \dfrac{P}{Q}$ vs. Stenotic height

The variation of flow resistance with stenotic height for $u_s = 0.0, 0.025, 0.05$ are examined.

## CONCLUSION

It is now established that lifesaving medicines applied for treatment of many fatal diseases like cancer, diabetes, heart failure damage some other organs /cells of the human body. If the location of the stenosis is detected without any surgery, then appropriate medicine may be sent to the affected area/cells with the help of Nano technology. This may open a new dimension in the treatment of some fatal diseases and more and more theoretical as well as experimental research in this field are suggested to overcome the restrictions imposed on the analysis. Also, the outcome of this analysis may be shared for further investigation in this emerging field and for providing primary support to the deceased where adequate medical/surgical support is not available.

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