THE FORMULA ^nC_r REVISITED

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Abstract

A formula expressing $^n\mathbf{C}_r$ in summation form is formulated by the use of algorithmic counting techniques. Initially, a general counting problem is mathematically modeled and its solution is given by a formula derived using algorithmic counting. Thus, by generalization a formula for nCr as a series is obtained.

Keywords: *^nC_r, summation, algorithm, counting, mathematical modeling, generalization, series.*

INTRODUCTION

The formula for the number of possible combinations when r objects are selected out of n different objects is given by-

$$^nC_r = \frac{n!}{r!(n-r)!}$$

This paper attempts to explore a different perspective of combinations by expressing ^nC_r in sigma form (or summation form).

DERIVATION OF ^nC_r IN SUMMATION FORM

A linear sequence of the following symbols (or objects) -1, 2, 3,4,5,6,7,8,9 is considered. Each symbol is denoted by any general symbol x such that,

Set X={x: x=1, 2, 3, 4, 5, 6, 7, 8, 9}. Hence, x can assume 9 values.

Also, ^nC_r denotes the number of ways of selecting r objects out of n different objects. It also denotes the number of unordered collection of r numbers possible out of n different numbers.

Let us construct cases by varying r=1,2,3….n for a given n=9(in this case).

CASE 1- Number of different values (x) can assume is given by

$^9C_1$= (Number of values x can assume) =9 (by counting)

CASE 2- Number of different values the unordered pair (x, x) can assume is given by

$^9C_2$= (Number of values (1, x) can assume)

+ (Number of values (2, x) can assume excluding values already considered in (1, x))

+ (Number of values (3, x) can assume excluding values already considered in (1, x) and (2, x))

+ ……………………………………………………………………………………………………

+ (Number of values (9, x) can assume excluding values already considered in (1,x),(2,x),(3,x),(4,x),(5,x),(6,x)……..(8,x))

= 8 + 7 + 6 + 5 + 4 + 3 + 2 + 1 + 0

$= \sum_{p_1=0}^{p_1=8} p_1$ where p_1 is the index of summation.

CASE 3- Number of different values the unordered triplet (x,x,x) can assume is given by

$^9C_3$= [(Number of values (1, 2, x) can assume)

+ (Number of values (1, 3, x) can assume excluding values already considered in (1, 2, x))

+ (Number of values (1,4,x) can assume excluding values already considered in (1, 2, x) and (1, 3, x))

+ ……………………………………………………………………………………………………

+ (Number of values (1, 9, x) can assume excluding values already

considered in (1, 2, x), (1, 3, x)… (1, 8, x))

]

+ [(Number of values (2, 3, x) can assume)

+ (Number of values (2, 4, x) can assume excluding values already considered in (2, 3, x))

+ (Number of values (2, 5, x) can assume excluding values already considered in (2, 3, x) and (2, 4, x))

+ ……………………………………………………………………..……………………………

+ (Number of values (2,9,x) can assume excluding values already considered in (2,3,x),(2,4,x),…(2,8,x))]

……………………………………………………………………………………………………

+ [(Number of values (8, 9, x) can assume)]

= (7+6+5+4+3+2+1+0)

+ (6+5+4+3+2+1+0)

+ (5+4+3+2+1+0)

+ (4+3+2+1+0)

+ (3+2+1+0)

+ (2+1+0)

+ (1+0)

+ (0)

= $\sum_{p_2=0}^{p_2=7} \sum_{p_1=0}^{p_2} p_1$ where p_1 and p_2 are the respective indices of summation

CASE 4- Number of different values (x,x,x,x) can assume is given by

$$ {}^{9}C_4 = \sum_{p_3=0}^{p_3=6} \sum_{p_2=0}^{p_3} \sum_{p_1=0}^{p_2} p_1 $$

where p_1, p_2, p_3 are the indices of summation.

CASE 5- Number of different values (x,x,x,x,x) can assume is given by

$$ {}^{9}C_5 = \sum_{p_4=0}^{p_4=5} \sum_{p_3=0}^{p_4} \sum_{p_2=0}^{p_3} \sum_{p_1=0}^{p_2} p_1 $$

Where p_1, p_2, p_3, p_4 are the indices of summation.

Replacing 9 with ‘n’ and generalizing for r th case we get –

$$ {}^{n}C_r = \sum_{p_{r-1}=0}^{n-r+1} \sum_{p_{r-2}=0}^{p_{r-1}} \cdots\cdots \sum_{p_2=0}^{p_3} \sum_{p_1=0}^{p_2} p_1 $$

where p_i (i=1,2,…….,r-1) is the index of summation.

CONCLUSION

Thus, the given series expresses the number of ways of selecting r objects out of n different objects. This series can be used in various fields like algebra, combinatorics and probability and statistics.

REFERENCES

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