A NUMERICAL STUDY OF REACTIVE MHD FLOW OF THIRD GRADE FLUID

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Abstract

The computational analysis of reactive MHD third grade fluid in an electrically conductive cylindrical channel with axial Magnetic field is carried out in this study. The weighted residual collocation method is used as a computational method of solution and a parametric study of the influence of various thermos-physical parameter is conducted and it was observed that critical values of the Frank–Kamenetskii and the third grade parameter exist for which the solution seizes to be unique.

Keywords: *Collocation, weighted residual, reactive, MHD, third grade.*

INTRODUCTION

The problem of reactive flow in cylindrical geometry has become subject of much research due to its applications in varied scientific, industrial and engineering devices, such applications and devices include Nuclear reactors, chemical kinetics, biomass, explosive and internal combustion engines. Due to these applications several authors had contributed to this field of research, they include *Adesanya et al* (2016) who carried out a mathematical analysis of a reactive viscous flow through a channel filled with porous media. They developed an Adomian decomposition solution to the flow equations and to gain insight into the heat transfer characteristic carried out an entropy generation analysis. Meanwhile Okedayo *et al* (2014) had carried out a computational study of reactive flow of an electrically conducting fluid with temperature dependent viscosity and axial magnetic field using the semi-implicit finite difference scheme. They determine the influence of the frank-Kamenetskii parameter and observed that at certain critical values of the parameter the solution sizes to be unique.

Rundora & Makinde (2018) studied the Unsteady MHD flow of a Non-Newtonian fluid in a channel filed with saturated porous media, a chemically exothermic third grade fluid was used as the base fluid while a finite difference method as their solution procedure.

Hassan *et al* (2015) analyzed the entropy generation of a reactive magnetohydrodynamic flow in a channel and concluded that a reactive material which undergoes an exothermic reaction generate heat in accordance with Arrhenius law if reactant consumption is neglected. Similar studies can be found in Gbadeyan and Hassan (2012) Makinde and Eegunjobi(2013) and Jha *et al* (2015).

The field of MHD flow have become a subject of much focus by scientist and practitioners due to its wide range of applications in scientific and industrial devices such as MHD generators, metal purifications, plasma studies, polymer technology Opanuga *et al* (2017).While in astrophysics and geophysics MHD is applied to the study of the stellar and solar structure, interstellar matter and radio propagation through the ionosphere. Other literatures on MHD flow abounds such as Barikbin *et al* (2014), Makinde (2005) etc.

Due to the non-linear and coupled nature of the governing equations of the flow and heat transfer, there have been a continuous search for fast and accurate solution method both analytic and numerical. Such method includes the regular perturbation, Homotopy permutation method, Adomian decomposition method and the weighted residual methods. For example Barikbin *et al* (2014) applied the Ritz–Galerkin method using Bernstein polynomial as basis functions for the couette flow of non-Newtonian fluid, they compared their result with the collocation method and they found a good agreement between the two methods.

Okedayo *et al* (2018)carried out a Galerkin weighted residual method for flow in a vertical channel, they compared their result with those obtained by the Runge-Kuta method and found the absolute error to be between 1%- 3%.

Motivated by the above studies, we proceed to carry out a Numerical study of the flow and heat transfer analysis of the reactive flow of a third grade fluid using the collocation weighted residual method. The effect of thermal critically was also determined. Effect of the third grade parameter on flow stability is also analysed graphically.

MATHEMATICAL FORMULATION

Consider the flow of a reactive, incompressible, electrically conducting third grade fluid through an isothermally heated horizontal cylinder, in the presence of an applied uniform magnetic field B_0.With an applied pressure gradient $\frac{dp}{dz}$ .The induced magnetic field is neglected due to low or very small magnetic Reynolds number. Therefore, the governing equations of momentum and energy transfer is given by:

$$\frac{\mu d}{rdr}\left(r\frac{du}{dr}\right)+\frac{\beta_3 d}{rdr}\left(r\left(\frac{du}{dr}\right)^3\right)-\sigma B_0^2 u=\frac{dp}{dz}$$
$$\frac{kd}{rdr}\left(r\frac{dT}{dr}\right)+\left(\frac{du}{dr}\right)^2\left[\mu+\beta_3\left(\frac{du}{dr}\right)^2\right]+\sigma B_0 u^2+QC_0Ae^{\frac{-E}{RT}}=0 \quad (1)$$
$$\frac{du}{dr}(0)=\frac{dT}{dr}(0)=0, u(a)=0, T(a)=T_0$$

Where u is the velocity, T is the absolute temperature, μ is the dynamic viscosity β is the third grade parameter, σ is the electrical conductivity, k is the thermal conductivity, T_0 is the ambient temperature, a is the radius of the pipe, r is the radial distance ϵ is the rate constant, E is the activation Energy, R is the universal gas constant, C_0 is the specie concentration, A is the rate constant, and Q is the heat of reaction. In order to solve the above equation (1), the following dimensionless parameters are introduced.

$$\bar{u}=\frac{u}{\frac{a^2}{\mu}\frac{dp}{dz}}, \bar{r}=\frac{r}{a}, \theta=\frac{E}{R{T_0}^2}(T-T_0), \epsilon=\frac{RT_0}{E}, Br=\frac{Ea^4}{RT_0K}(\frac{dp}{dz})^2, \delta=\frac{Ea^2QC_0Ae^{\frac{-E}{RT_0}}}{RT_0^2K}$$

$$\beta=\frac{\beta_3 a^4}{\mu^3}\left(\frac{dp}{dz}\right)^2, M=\frac{\sigma\beta_0 a^2}{\mu} M_1=\frac{Ea^4\sigma\beta_0}{RT_0K\mu}\left(\frac{dp}{dz}\right)^2$$

We therefore obtain the dimensionless equation (2)

$$\frac{d}{rdr}\left(r\frac{du}{dr}\right)+\frac{\beta d}{rdr}\left(r\left(\frac{du}{dr}\right)^3\right)-M^2u=-1$$
$$\frac{d}{rdr}\left(r\frac{d\theta}{dr}\right)+\left(\frac{du}{dr}\right)^2\left[Br+\beta\left(\frac{du}{dr}\right)^2\right]+M_1u^2+\delta e^{\frac{\theta}{\epsilon\theta+1}}=0 \quad (2)$$
$$\frac{du}{dr}(0)=\frac{d\theta}{dr}(0)=0, u(1)=0, \theta(1)=0$$

Where$\beta, \theta, Br, M, \delta, \epsilon, M_1$, are the third grade parameter, dimensionless temperature, viscous heating parameter, magnetic field parameter, Frank-kamenetskii parameter, activation energy parameter, and the joule heating parameter.

PROBLEM SOLUTION

Solutions to the systems of equation (2) are obtained by the weighted residual collocation procedure. The weighted residual collocation method is a function approximation procedure such that the set of differential equations is reduced to a set of algebraic equation that can be solved using any method of fixed point iteration.

The function approximation procedure is to find a solution of the form

$$u(r) = \sum_{i=0}^{n} a_i \emptyset_i(r)$$

Called a trial or basis function over the domain which satisfies the boundary conditions, the function $\emptyset_i\ (r)$may be a polynomial, trigonometric function or a power series. It is required that the residual be orthogonal to a set of weight functions and in this case the Dirac delta function. In order to ascertain the convergence of the method the one, two and three terms coefficients are employed and the maximum velocity and temperature were obtained. The functions are $u_0(r) = a_0(1-r^2), u_1(r) = a_0(1-r^2) + a_1(1-r^3), u_1(r) = a_0(1-r^2) + a_1(1-r^3) + a_2(1-r^4)\ and\ \theta_0(r) = b_0(1-r^2), \theta_1(r) = b_0(1-r^2) + b_1(1-r^3), \theta_2(r) = b_0(1-r^2) + b_1(1-r^3) + b_2(1-r^4)$.

RESULTS AND DISCUSSION

By specifying the values of the thermos-physical parameters the coefficients are tabulated below where N is the number of coefficients.Specific values of the thermos-physical parameters are $\beta = 0.3, \lambda = 0.2, Br = 0.5, M_1 = 10, M = 10, \delta = 0.1, \epsilon = 0.001$

N	a_0	a_1	a_2	b_0	b_1	b_2
1	0.08681995			0.05407722		
2	-0.00949418	-0.06947820		-0.05432014	0.00045590	
3	-0.05630128	0.03350407	- 0.05949099	-0.05538160	-.00257133	- .000121724

Table.1 Coefficients of Collocation.

N	u_{max}	θ_{max}
1	0.08681994651	0.05407221242
2	0.07897237559	0.05386422971
3	0.08228820685	0.05402750684

Table.2 Maximum Velocity and Temperature

In order to determine the thermal critically property, the relationship between the maximum temperature and the Frank-Kamenetskii parameter is obtained as given in equation (3) using the given values of the thermophysical parameters.

$$-4.5\theta_{max} + 0.0059334703366\delta e^{\frac{0.875\theta_{max}}{1+0.000875\theta_{max}}}=0 \qquad (3)$$

A plot of (3) can be seen in Figure.1, it is seen that there is a critical value of the Frank-Kamenetskii parameter for which multiple solutions exist. From Figure.1thecritical valueis$\delta_c = 1.8896675$, plots of the two solutions i.e. upper and lower solutions can be seen in figure.6 the upper solution denotes high activation energy while the lower solution represents lower activation. It could be noted that for $\delta > \delta_c$ steady state solutions do not exist indicating thermal runaway which could be prevented by setting$\delta \leq 1.8896675$.

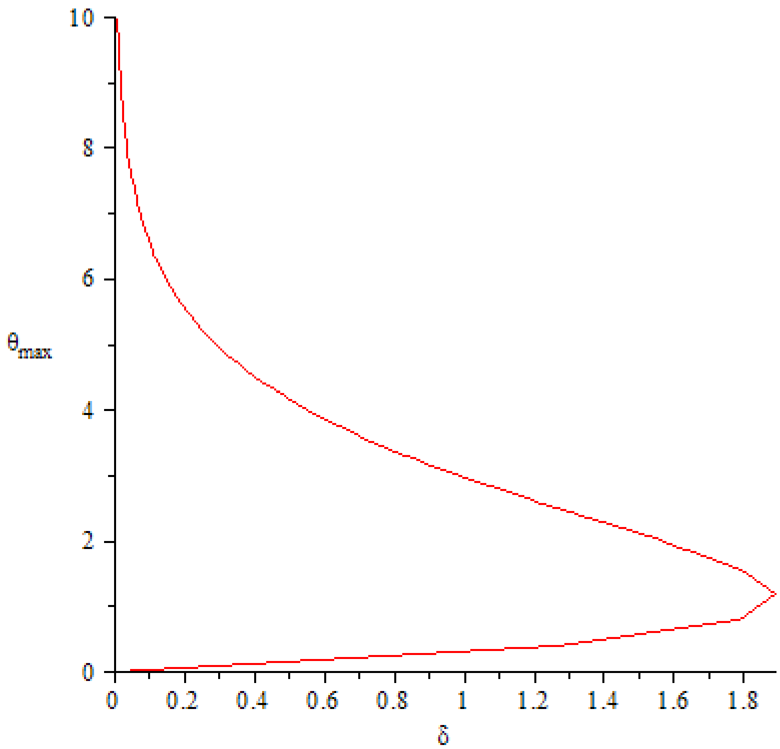

Figure.1:Bifurcation diagram

Rewriting equation (1) terms of the shear stress we

$$\frac{1d(r\tau)}{rdr} - \sigma B_0^2 u = \frac{dp}{dz} \quad (1b)$$

Wherein dimensionless form.

$$\tau = \frac{du}{dr} + \beta(\frac{du}{dr})^3 \quad (1c)$$

which in terms of the maximum velocity we have

$$\tau = -2u_{max} - 8\beta u_{max}^3 \quad (4)$$

And

$$4u_{max} + 8\beta u_{max}^3 + 0.75Mu_{max} = 1 \quad (5)$$

Graphs of shear stress at the wall against the maximum velocity is shown in Figure.6, it is worth noting that increase in maximum temperature increases the shear thinning behavior thereby reducing the wall shear stress, while the maximum velocity against the third grade parameter β is shown in Figure.7.is observed that a turning point exist, which indicates the multiple solutions could exist for critical values of the third grade parameter. This turning point is critical value β_c of the third grade parameter for which instabilities may set in and for values of $\beta < \beta_c$ steady state solutions does not exist.

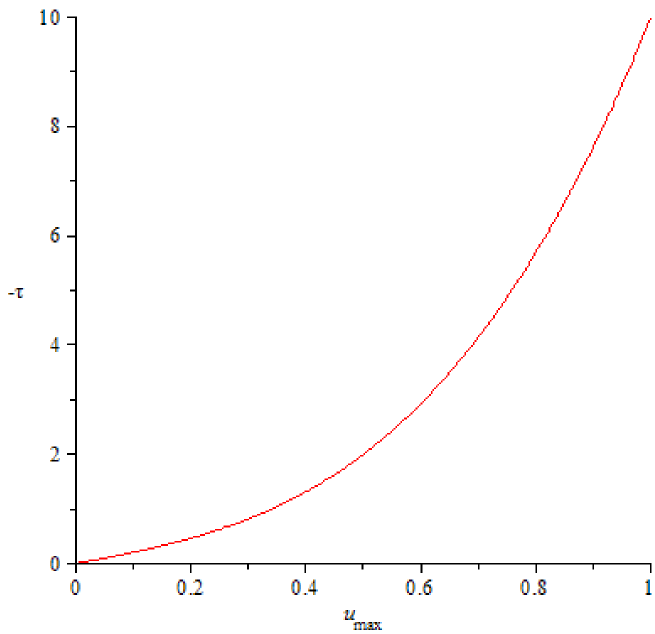

Figure.6:Plot of Wall Shear Stress against Maximum Velocity

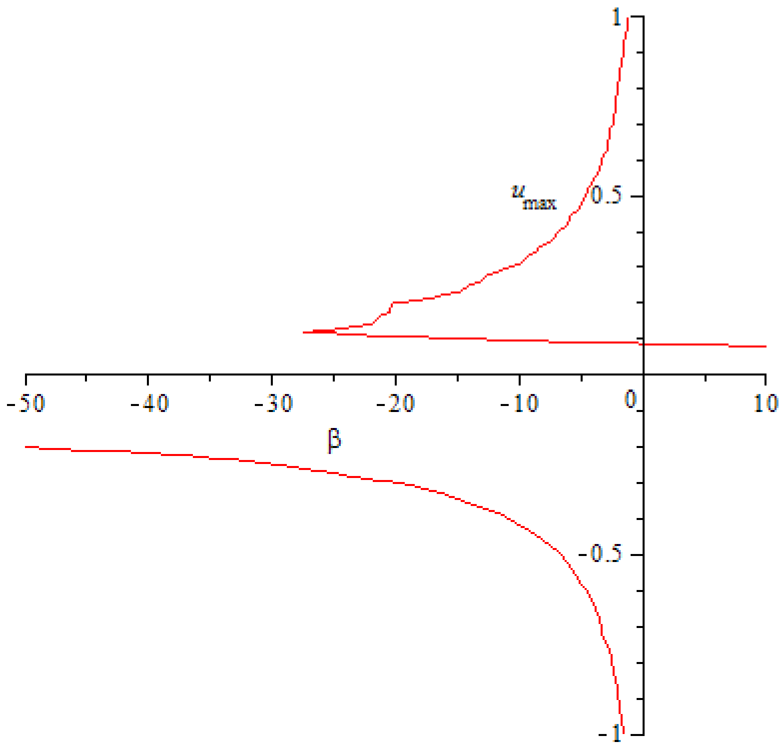

Figure.7: Maximum Velocity profile versus third grade parameter

In figures: 2-5 we depicts the velocity and temperature profile for the various values of the thermo-physical parameters that influence the flow and heat transfer, in figure.2 the impact of the magnetic parameter on the velocity is shown and it is seen that an increase in the parameter leads to a decrease in the velocity profile which is due to Laurent forces generated by the magnetic fields which opposes the fluid motion. While in figure.3 the influence of the third grade parameter is depicted and it seen that increase in the third grade parameter leads to a decrease in the velocity. In figure.4 the influence of the third grade parameter on the temperature profile is shown and it is seen that the temperature profile increases with the rise in in the third grade parameter due to internal fluid friction which leads to a rise in temperature. A similar phenomenon is also observed in figure.5 where increase the Brinkmann number leads to increase in the temperature.

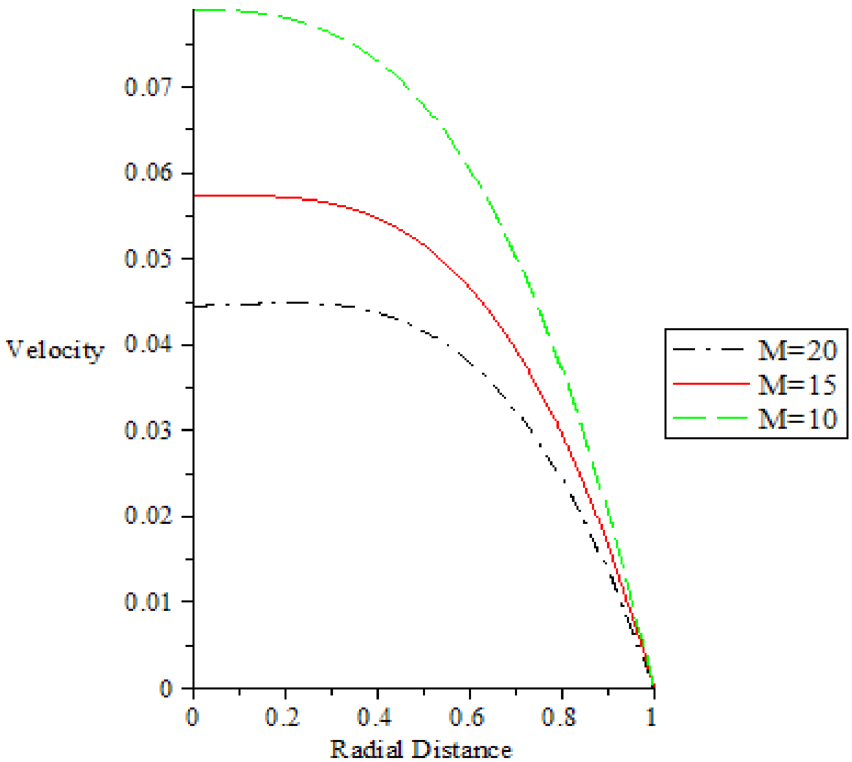

Figure.2:Velocity Profile for Various Values of M

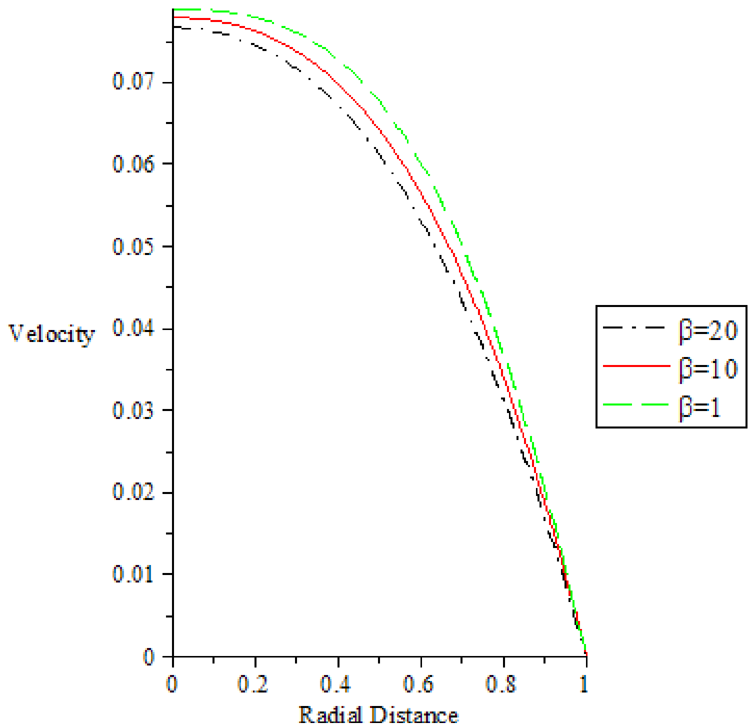

Figure.3:Velocity Profile for Various Values of β

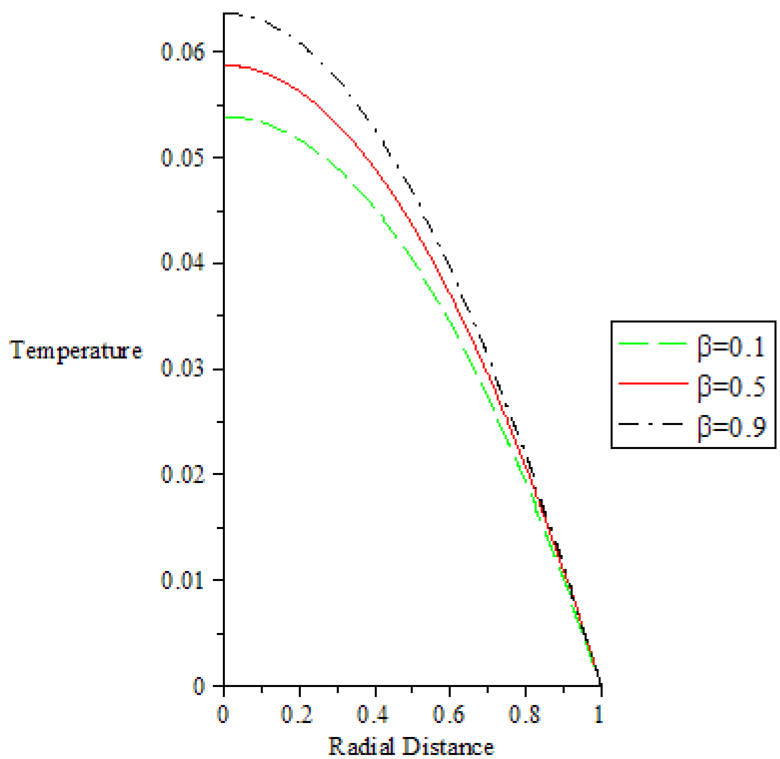

Figure.4:Temperature Profile for Various Values of β

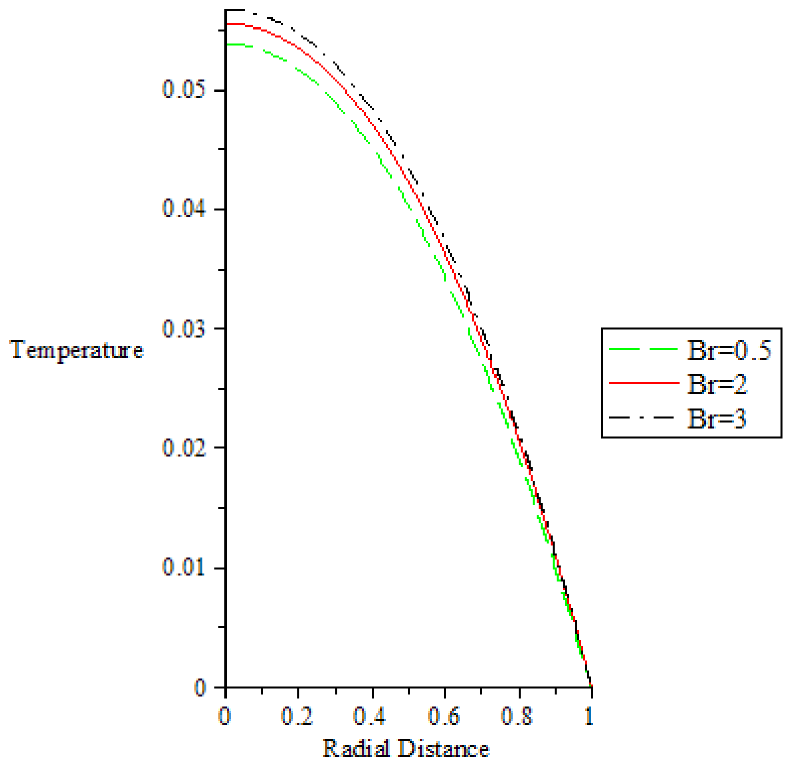

Figure.5:Temperature Profile for Various Values of Br

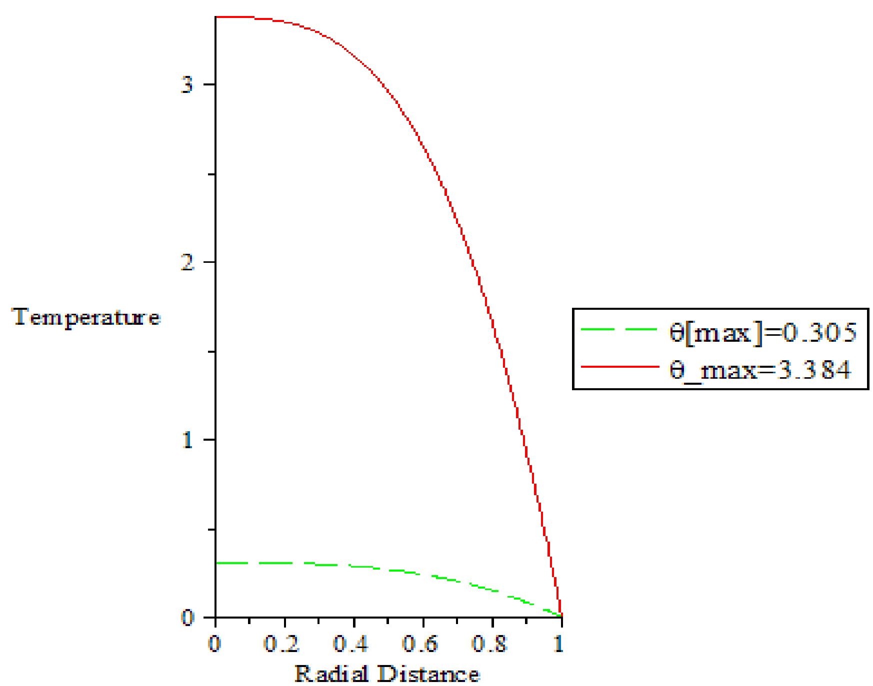

Figure.6:Multiple Solutions for Temperature Profile for values of θ_max at δ=0.794

CONCLUSION

In this paper the problem of reactive MHD third grade fluid in an electrically conductive cylindrical channel with axial Magnetic field in a cylindrical pipe is considered, the coupled nonlinear equation is solved using the collocation weighted residual method. Results obtained shows that multiple solutions exist for certain values of the Frank-Kamenetskii and third grade parameters. The increase in maximum velocity increases the shear thinning behavior.

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