STATIC BUCKLING ANALYSIS OF A FINITE IMPERFECT COLUMN RESTING ON A NON-LINEAR ELASTIC FOUNDATION BUT PRESSURIZED BY A STEP LOAD

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Abstract

In this investigation, regular perturbation procedures in asymptotic expansions of the relevant variables are employed to discuss the static buckling analysis of a finite deterministically imperfect but viscously damped column resting on some quadratic–cubic nonlinear elastic foundations, but struck by a step load. The governing equation for the system under discussion is fully nonlinear, so that a closed form and easy solution to the problem is not possible. An approximate analytical solution to the problem is obtained using asymptotic and perturbation techniques and numerical results obtained show that increase in imperfection factors lower the static buckling loads of the column.

Keywords: *Static Buckling, Asymptotic and perturbation methods, Light and Viscous damping, Step Load.*

INTRODUCTION

Most often in the dailies we hear of collapse of buildings, bridges and other material structures. All or some of these are forms of buckling. Buckling is catastrophic in nature and should be avoided at all costs. Enormous resources and efforts have already been put by Engineers and Applied Mathematicians in order to get the optimal loads (buckling loads) that structures can carry before buckling occurs, yet buckling of elastic material structures still occurs from time to time. There is already in existence a substantial quantum of investigations related to the stability (or otherwise) of columns (finite or infinite) when subjected to either a static load or dynamic loads. Some of these earlier studies include investigations by Amazigo and Ette (1987), Amazigo and Frank (1973), Elishakoff and Gue′de′ (2001) and Ette (1992), among others. We however, remark that the static problem of infinitely long columns was investigated by Amazigo et al. (1971) by using the method of equivalent linearization as well as a perturbation expansion involving double scaling in the spatial variable. Most of the earlier works done on this subject matter had used the numerical methods (Finite Element Methods) for the buckling analysis. In this study, we are investigating, using purely analytical methods, the static buckling and stability of a finite deterministically imperfect but viscously damped column resting on some quadratic–cubic nonlinear elastic foundations, but struck by a step load.

FORMULATION OF THE PROBLEM

The usual differential equation satisfied by the deflection W(X, T) of the column under consideration satisfies the following partial differential equation, as in Amazigo and Frank (1973), Amazigo and Oyesanya (1986) and Ette (2003):

$$m_0 W_{,TT} + c_0 W_{,T} + EIW_{,XXXX} + 2P(T)W_{,XX} + k_1 W - k_2 W^2 - k_3 W^3 = -2P(T)\frac{\partial^2 \bar{W}}{\partial X^2}, \quad T > 0, \qquad 0 < X < \pi \qquad (2.1a)$$

$$W = W_{,XX} = 0, at\ X = 0, \pi, \quad t \geq 0 \qquad (2.1b)$$

$$W(X,0) = W_{,T}(X,0) = 0, \ 0 < X < \pi \qquad (2.1c)$$

where, a comma preceding a subscript indicates partial differentiation. Here, m_0 is the mass per unit length, c_0 is the damping coefficient, EI is the bending stiffness, where E and I are the Young's modulus and the moment of inertia respectively. Here, the nonlinear elastic foundation exerts a force per unit length given by $k_1 W - k_2 W^2 - k_3 W^3$ on the column, where k_1, k_2 and k_3 are constants such that k_1>0, $k_2 > 0\ and\ k_3 > 0$. In this formulation, we have excluded all nonlinearities higher than cubic, while all nonlinear derivatives of W(X, T) are also excluded. $\bar{W}$ is the stress – free time independent twice – differentiable initial imperfection displacement. All aspects of axial inertia are neglected. Equation (2.1a) is a dimensional partial differential equation, and so in order to render it non – dimensional, we adopt the following quantities as in Chukwuchekwa (2017):

$$x = \left(\frac{k_1}{EI}\right)^{\frac{1}{4}} X, \quad w = \left(\frac{k_2}{k_1}\right)^{\frac{1}{2}} W, \quad \lambda f(t) = \frac{P(T)}{2(EIk_1)^{\frac{1}{2}}}, \quad \epsilon\varpi = \left(\frac{k_3}{k_1}\right)^{\frac{1}{2}} \bar{W} \quad (2.2a)$$

$$2\delta = \frac{c_0}{(m_0 k_1)^{\frac{1}{2}}}, \quad t = \left(\frac{k_1}{m_0}\right)^{\frac{1}{2}} T, \qquad \alpha = \frac{k_2}{\sqrt{k_1 k_2}}, \quad \beta = \left(\frac{k_3}{k_1}\right)^{\frac{3}{2}} \bar{W} \qquad (2.2b)$$

Here, we shall assume the following inequalities

$$0 < \delta < 1, \qquad 0 < \epsilon \ll 1\ and\ 0 < \lambda < 1 \qquad (2.3)$$

On substituting (2.2a, b) into (2.1a) and simplifying, we obtain

$$w_{,tt} + 2\delta w_{,t} + w_{,xxxx} + 2\lambda f(t) w_{,xx} + w - \alpha w^2 - \beta w^3 = -2\epsilon\lambda f(t)\frac{d^2\varpi}{dx^2}, \quad t > 0, \quad 0 < X < \pi \qquad (2.4a)$$

$$w = w_{,XX} = 0, at\ x = 0, \pi, \ \ t \geq 0 \qquad (2.4b)$$

$$w(x,0) = w_{,t}(x,0) = 0, \qquad 0 < x < \pi \tag{2.4c}$$

In the dynamic case, we shall assume the step function for f(t), i.e,

$$f(t) = \begin{cases} 1, & t > 0 \\ 0, & t < 0 \end{cases} \tag{2.4d}$$

In the case where there is no preload, the relevant equation is

$$w_{,tt} + 2\delta w_{,t} + w_{,xxxx} + 2\lambda w_{,xx} + w - \alpha w^2 - \beta w^3 = -2\epsilon\lambda \frac{d^2\varpi}{dx^2} \tag{2.5}$$

Here, we are assuming δ and ϵ to be two small but unrelated parameters that satisfy the inequalities as in (2.3). Our ultimate aim is to determine the static buckling load λ_S. According to Budiansky and Hutchison (1966) and Ette and Onwuchekwa (2007), the condition for static buckling is

$$\frac{d\lambda}{dw} = 0 \tag{2.6}$$

where, w is the displacement.

STATIC DEFORMATION OF THE COLUMN

We shall let ω (x) be the displacement during the static loading. Here, there is no time dependence on the displacement. In this case, we must neglect the inertia term, the damping term and all other time dependent terms and set $f(t) \equiv 1$ in (2.5), so that the resultant equation from (2.5) is

$$\frac{d^4\omega}{dx^4} + 2\lambda\frac{d^2\omega}{dx^2} + \omega - \alpha\omega^2 - \beta\omega^3 = -2\epsilon\lambda\frac{d^2\varpi}{dx^2}, \quad 0 < x < \pi \tag{3.1}$$

$$\omega = \frac{d^2\omega}{dx^2} = 0, \qquad at\ x = 0, \pi \tag{3.2}$$

For solution, we assume the asymptotic expansion,

$$\omega(x) = \sum_{i=1}^{\infty} \omega^i \epsilon^i \tag{3.3}$$

and, following (3.2), we let

$$\varpi = \bar{a}_m sinmx, m\ fixed, and\ |\bar{a}_m| \ll 1 \tag{3.4}$$

On substituting (3.4) on the right hand side of (3.1), we obtain

$$\frac{d^4\omega}{dx^4} + 2\lambda\frac{d^2\omega}{dx^2} + \omega - \alpha\omega^2 - \beta\omega^3 = 2\epsilon\lambda\bar{a}_m m^2 sinmx \tag{3.5}$$

On substituting (3.3) into (3.5), and equating coefficients of powers of ϵ, we get

$$L\omega^{(1)} \equiv \omega^{(1)}_{,xxxx} + 2\lambda\omega^{(1)}_{,xx} + \omega^{(1)} = 2\lambda\bar{a}_m m^2 sinmx \tag{3.6}$$

$$L\omega^{(2)} = \alpha\left(\omega^{(1)}\right)^2 \tag{3.7}$$

$$L\omega^{(3)} = 2\,\alpha\omega^{(1)}\omega^{(2)} + \beta{\omega^{(1)}}^3 \tag{3.8}$$

etc.

$$\omega^{(i)} = \omega^{(i)}_{,xx} = 0, at\ x = 0, \pi \tag{3.9}$$

At this stage, we have let $\frac{d\omega}{dx} \equiv \omega_{,x}$

For solution of the systems of equations above, we assume $\omega^{(i)}\ in$ the form,

$$\omega^{(i)} = \sum_{n=1}^{\infty} \omega^{(i)}_n sinnx \tag{3.10}$$

We now substitute (3.10) into (3.6) and get

$$\sum_{n=1}^{\infty}\left\{(n^4 - 2\lambda n^2 + 1)\omega^{(1)}_n\right\}sinnx = 2\lambda\bar{a}_m m^2 sinmx \tag{3.11}$$

If we multiply (3.11) by sinmx and integrate from 0 to π, we obtain, for n = m,

$$\frac{\pi}{2}\left\{(m^4 - 2\lambda m^2 + 1)\omega^{(1)}_m\right\} = \frac{\pi}{2}(2\lambda\bar{a}_m m^2) \tag{3.12}$$

$$\therefore \quad \omega^{(1)}_m = \frac{2\lambda\bar{a}_m m^2}{\Omega^2_m} = D_m \tag{3.13a}$$

where,

$$\Omega^2_m = (m^4 - 2\lambda m^2 + 1\,) > 0,\ \forall\ m \tag{3.13b}$$

$$\therefore\ \omega^{(1)} = \omega^{(1)}_m sinmx \tag{3.14}$$

On substituting for $\omega^{(1)}_m$ from (3.14) in (3.7), we get

$$L\omega^{(2)} = \alpha\left(\omega^{(1)}_m\right)^2 \sin^2 mx \tag{3.15}$$

Using (3.10) 0n (3.15), we get

$$\sum_{n=1}^{\infty}\left\{(n^4 - 2\lambda n^2 + 1)\omega^{(2)}_n\right\} sinnx = \alpha\left(\omega^{(1)}_m\right)^2 \sin^2 mx \tag{3.16}$$

We multiply (3.16) by sinmx and integrate from 0 to π and obtain for n = m,

$$\frac{\pi}{2}\left\{(m^4 - 2\lambda m^2 + 1)\omega_m^{(2)}\right\} = \frac{4\alpha\left(\omega_m^{(1)}\right)^2}{3m} \tag{3.17a}$$

$$\therefore \quad \omega_m^{(2)} = \frac{8\alpha\left(\omega_m^{(1)}\right)^2}{3m\pi\Omega_m^2}, \text{(for m odd)} \tag{3.17b}$$

$$\therefore \ \omega^{(2)} = \omega_m^{(2)} sinmx \tag{3.18}$$

We now substitute in (3.8) and get,

$$L\omega^{(3)} = 2\alpha\omega_m^{(1)}\omega_m^{(2)}\sin^2 mx + \beta\left(\omega_m^{(1)}\right)^3 \sin^3 mx \tag{3.19}$$

On using (3.10) in (3.19), we get

$$\sum_{n=1}^{\infty}\left\{(n^4 - 2\lambda n^2 + 1)\omega_n^{(3)}\right\} sinnx = 2\alpha\omega_m^{(1)}\omega_m^{(2)}\sin^2 mx + \beta\left(\omega_m^{(1)}\right)^3 \sin^3 mx$$

On further simplification, we have

$$\sum_{n=1}^{\infty}\left\{(n^4 - 2\lambda n^2 + 1)\omega_n^{(3)}\right\} sinnx$$

$$= \alpha\omega_m^{(1)}\omega_m^{(2)}(1 - cos2mx) + \frac{\beta\left(\omega_m^{(1)}\right)^3}{4}(3sinmx - sin3mx) \tag{3.20}$$

At this stage, the buckling mode splits into two, i.e, those in the shape sinmx, and those in the shape sin3mx. On multiplying (3.20) by sinmx and integrating from 0 to π, we get for n = m,

$$\frac{\pi}{2}\Omega_m^2\omega_n^{(3)} = \frac{8\alpha}{3m}\omega_m^{(1)}\omega_m^{(2)} + \frac{3\pi\beta\left(\omega_m^{(1)}\right)^3}{8} \tag{3.21a}$$

hence,

$$\omega_n^{(3)} = \frac{16\alpha\omega_m^{(1)}\omega_m^{(2)}}{3m\pi\Omega_m^2} + \frac{3\beta\left(\omega_m^{(1)}\right)^3}{4\Omega_m^2} \tag{3.21b}$$

$$\therefore \quad \omega_m^{(3)} = \frac{128\alpha^2 D_m^2}{(3m\pi\Omega_m^2)^2} + \frac{3\beta\left(\omega_m^{(1)}\right)^3}{4\Omega_m^2} \tag{3.21c}$$

On the other hand, if we multiply (3.20) by sin3mx and integrate from 0 to π, we get for

$n = m,$

$$\omega_{3m}^{(3)} = \frac{-2\alpha\omega_m^{(1)}\omega_m^{(2)}}{3m\pi\Omega_{3m}^2} - \frac{\beta\omega_m^{(1)}}{4\Omega_{3m}^2} \qquad (3.22a)$$

where,

$$\Omega_{3m}^2 = 81m^4 - 18\lambda m^2 + 1 > 0, \ \forall \ m \qquad (3.22b)$$

Hence, we get

$$\omega^{(3)} = \omega_m^{(3)} sinmx + \omega_{3m}^{(3)} sin3mx \qquad (3.23)$$

Thus, following (3.3), we get

$$\omega(x) = \omega^{(1)}\epsilon + \omega^{(2)}\epsilon^2 + \omega^{(3)}\epsilon^3 + \ldots$$

$$\Rightarrow \omega(x) = \epsilon\omega_m^{(1)} sinmx + \epsilon^2\omega_m^{(2)} sinmx + \epsilon^3\left(\omega_m^{(3)} sinmx + \omega_{3m}^{(3)} sin3mx\right) + \ldots \ (3.24)$$

For simplification, we shall determine (3.24) at its maximum point, where

$$x_a = \frac{\pi}{2m} \qquad (3.25)$$

where, x_a is the value of x for $\omega(x)$ to attain its maximum, thus, $\omega_a = \omega(x_a)$. Then,

$$\omega_a = \epsilon\omega_m^{(1)} + \epsilon^2\omega_m^{(2)} + \epsilon^3\left(\omega_m^{(3)} - \omega_{3m}^{(3)}\right) + \ldots \qquad (3.26)$$

If we substitute for $\omega_m^{(1)}$, $\epsilon^2\omega_m^{(2)}$, $\omega_m^{(3)}$and $\omega_{3m}^{(3)}$in (3.26), we have

$$\omega_a = D_m\epsilon + \frac{8\alpha D_m^2}{3m\pi\Omega_m^2}\epsilon^2 + \epsilon^3\left(\frac{128\alpha^2 D_m^2}{(3m\pi\Omega_m^2)^2} + \frac{3\beta {D_m}^3}{4\Omega_m^2} + \frac{2\alpha^2 D_m^3}{3m\pi\Omega_m^2} + \frac{\beta D_m^3}{4\Omega_{3m}^2}\right) + \ldots \quad (3.27)$$

On further simplification, we get

$$\omega_a = e_1\epsilon + e_2\epsilon^2 + e_3\epsilon^3 + \ldots \qquad (3.28a)$$

where,

$$e_1 = D_m, \quad e_2 = \frac{8\alpha D_m^2}{3m\pi\Omega_m^2}, \qquad e_3 = \frac{3\beta {D_m}^3\rho_1}{4\Omega_m^2} \qquad (3.28b)$$

where,

$$\rho_1 = 1 + \frac{512}{27(m\pi)^2 \Omega_m^2}\left(\frac{\alpha^2}{\beta}\right) + \frac{1}{3}\left(\frac{\Omega_m^2}{\Omega_{3m}^2}\right)\left(\frac{\alpha^2}{\beta}\right) + \frac{8}{9m\pi}\left(\frac{\varphi_m^2}{\varphi_{3m}^2}\right) \qquad (3.28c)$$

In order to use the condition, $\frac{d\lambda}{d\omega_a} = 0$, we need to reverse the series (3.28a), as in Ette and Onwuchekwa (2007), and get

$$\epsilon = f_1\omega_a + f_2{\omega_a}^2 + f_3{\omega_a}^3 + \ldots \qquad (3.29)$$

We need to determine f_i, $i = 1, 2, 3, \ldots$. By substituting for ω_a from (3.28a) into (3.29) and equating the coefficients of $\epsilon, \epsilon^2, \epsilon^3$, etc., we get

$$f_1 = \frac{1}{e_1}, \quad f_2 = \frac{-f_1 e_2}{e_1^2} = \frac{-e_2}{e_1^3}, \quad f_3 = \frac{-(f_1 e_3 + 2f_2 e_1 e_2)}{e_1^3} = \frac{2e_1^2 - e_1 e_3}{e_1^5} \qquad (3.30)$$

The maximization, $\frac{d\lambda}{d\omega_a} = 0$ is accomplished by knowing that each e_i is a function of λ. The maximization is better done using (3.29) to yield,

$$\frac{d\epsilon}{d\omega_a} = \left\{\left(\frac{df_1}{d\lambda}\frac{d\lambda}{d\omega_a}\right)\omega_a + f_1\right\} + \left\{\left(\frac{df_2}{d\lambda}\frac{d\lambda}{d\omega_a}\right)\omega_a + 2f_2\omega_a\right\} + \left\{\left(\frac{df_3}{d\lambda}\frac{d\lambda}{d\omega_a}\right)\omega_a + 3f_3\omega_a^2\right\},$$

but,

$$\frac{df_1}{d\lambda}\frac{d\lambda}{d\omega_a} = \frac{df_2}{d\lambda}\frac{d\lambda}{d\omega_a} = \frac{df_3}{d\lambda}\frac{d\lambda}{d\omega_a} = 0$$

Hence, we get

$$f_1 + 2f_2\omega_a + 3f_3\omega_a^2 = 0 \qquad (3.31)$$

where, (3.31) is determined at $\lambda = \lambda_S$, and on solving for $\omega_a(\lambda_S)$ in (3.31), we get,

$$\omega_a(\lambda_S) = \frac{1}{3f_3}\left(-f_2 \pm (f_2^2 - 3f_1 f_3)^{\frac{1}{2}}\right) \qquad (3.32)$$

We shall now simplify (3.32). Now, we have,

$$(f_2^2 - 3f_1 f_3)^{\frac{1}{2}} = \sqrt{\frac{3e_3}{e_1^5}\left(1 - \frac{5e_2^2}{3e_1 e_3}\right)} \qquad (3.33a)$$

However, we shall take only the negative sign in (3.32). Thus, we get

$$-f_2 - (f_2^2 - 3f_1 f_3)^{\frac{1}{2}} = -(f_2^2 - 3f_1 f_3)^{\frac{1}{2}}\left[1 + \frac{f_2}{(f_2^2 - 3f_1 f_3)^{\frac{1}{2}}}\right] \qquad (3.33b)$$

On substituting f_1, f_2 and f_3 in (3.33b), we get

$$-f_2 - (f_2^2 - 3f_1f_3)^{\frac{1}{2}} = -(f_2^2 - 3f_1f_3)^{\frac{1}{2}}\left[1 + \frac{f_2}{(f_2^2 - 3f_1f_3)^{\frac{1}{2}}}\right]$$

$$= -\sqrt{\frac{3e_3}{e_1^5}\left(1 - \frac{5e_2^2}{3e_1e_3}\right)}\left[\frac{1 - \frac{e_2}{e_1^3}}{\sqrt{\frac{3e_3}{e_1^5}\left(1 - \frac{5e_2^2}{3e_1e_3}\right)}}\right]$$

$$= -\sqrt{\frac{3e_3}{e_1^5}\left(1 - \frac{5e_2^2}{3e_1e_3}\right)}\left[1 - \frac{e_2}{\sqrt{\frac{3e_3}{e_1^5}\left(1 - \frac{5e_2^2}{3e_1e_3}\right)}}\right] \tag{3.34}$$

Now, we have

$$f_3 = \frac{2e_1^2 - e_1e_3}{e_1^5} = \frac{-e_3}{e_1^4}\left(1 - \frac{2e_2^2}{e_1e_3}\right) \tag{3.35}$$

$$\therefore\ \omega_a(\lambda_S) = \frac{1}{3f_3}\left(-f_2 - (f_2^2 - 3f_1f_3)^{\frac{1}{2}}\right)$$

$$= \frac{1}{\frac{-e_3}{e_1^4}\left(1 - \frac{2e_2^2}{e_1e_3}\right)}\left[-\sqrt{\frac{3e_3}{e_1^5}\left(1 - \frac{5e_2^2}{3e_1e_3}\right)}\left[1 - \frac{e_2}{\sqrt{\frac{3e_3}{e_1}\left(1 - \frac{5e_2^2}{3e_1e_3}\right)}}\right]\right]$$

$$= \sqrt{\frac{e_1^3\left(1 - \frac{5e_2^2}{3e_1e_3}\right)}{3e_3}}\left[\frac{1 - \frac{e_2}{\sqrt{\frac{3e_3}{e_1}\left(1 - \frac{5e_2^2}{3e_1e_3}\right)}}}{\left(1 - \frac{2e_2^2}{e_1e_3}\right)}\right] \tag{3.36}$$

We shall now simplify (3.36) by doing the following:

$$\left(\frac{e_1^3}{3e_3}\right)^{\frac{1}{2}} = \left(\frac{D_m^3}{3\ \frac{3\beta D_m^3\rho_1}{4\Omega_m^2}}\right)^{\frac{1}{2}} = \frac{2\Omega_m R_1^{\frac{1}{2}}}{3\beta^{\frac{1}{2}}} \qquad (3.37a)$$

where,

$$R_1 = \frac{1}{\rho_1} \qquad (3.37b)$$

Also,

$$\left(1 - \frac{5e_2^2}{3e_1e_3}\right) = \left(1 - \frac{5\left(\frac{8\alpha D_m^2}{3m\pi\Omega_m^2}\right)^2}{\frac{3D_m.3\beta D_m^3\rho_1}{4\Omega_m^2}}\right)^{\frac{1}{2}} = R_2^{\frac{1}{2}} \qquad (3.37c)$$

where,

$$R_2 = \left(1 - \frac{1280\left(\frac{\alpha^2}{\beta}\right)}{81(m\pi\Omega_m)^2\rho_1}\right) \qquad (3.37d)$$

We know that,

$$1 - \frac{e_2}{\sqrt{\frac{3e_3}{e_1}\left(1 - \frac{5e_2^2}{3e_1e_3}\right)}} = \left(1 - \frac{\left(\frac{8\alpha D_m^2}{3m\pi\Omega_m^2}\right)^{\frac{1}{2}}}{\sqrt{3.\frac{3\beta D_m^3}{4\Omega_m^2}..R_2}}\right) = \left(1 - \frac{512\left(\frac{\alpha^2}{\beta}\right)}{27(m\pi\Omega_m)^2\rho_1}\right) \qquad (3.37e)$$

Therefore, we get

$$\frac{1 - \frac{e_2}{\sqrt{\frac{3e_3}{e_1}\left(1 - \frac{5e_2^2}{3e_1e_3}\right)}}}{\left(1 - \frac{2e_2^2}{e_1e_3}\right)} = \left(\frac{1 - \frac{16\alpha D_m}{9m\pi\Omega_m\sqrt{BR_2}}}{1 - \frac{512\left(\frac{\alpha^2}{\beta}\right)}{27(m\pi\Omega_m)^2\rho_1}}\right) = R_3 \qquad (3.37f)$$

From (3.36), we get

$$\omega_a(\lambda_S) = \omega_{as} = \left(\frac{2\Omega_m R_1^{\frac{1}{2}}}{3\beta^{\frac{1}{2}}}\right) R_2^{\frac{1}{2}} R_3 \qquad (3.38)$$

where, all the terms in (3.38) are evaluated at $\lambda = \lambda_S$. Thus, we have evaluated the maximum displacement at static buckling, as in (3.38). We shall now determine the static buckling load and this is obtained by evaluating (3.29) at $\lambda = \lambda_S$.To simplify the operation, we multiply (3.29) by 3 and get,

$$3\epsilon = 3f_1\omega_{as} + 3f_2\omega_{as}^2 + 3f_3\omega_{as}^3 \tag{3.39}$$

We now evaluate terms at static buckling from (3.39), and get

$$3\epsilon = 3(f_1\omega_{as} + f_2\omega_{as}^2) + \omega_{as}(3f_3\omega_{as}^2) \tag{3.40}$$

But from (3.31),

$$3f_3\omega_{as}^2 = -f_1 - 2f_2\omega_{as} \tag{3.41}$$

On substituting (3.41) in (3.40), we get

$$\begin{aligned} 3\epsilon &= 3(f_1\omega_{as} + f_2\omega_{as}^2) + \omega_{as}(-f_1 - 2f_2\omega_{as}) \\ &= \omega_{as}(3f_1 + 3f_2\omega_{as} - f_1 - 2f_2\omega_{as}) = \omega_{as}(2f_1 + f_2\omega_{as}) \\ &= 2f_1\omega_{as}\left(1 + \frac{f_2\omega_{as}}{2f_1}\right) \end{aligned} \tag{3.42}$$

On substituting for f_1 and f_2 in (3.42) from (3.30), we obtain,

$$3\epsilon = \frac{2\omega_{as}}{e_1}\left(1 - \frac{e_2\omega_{as}}{2f_1}\right) \tag{3.43}$$

If we substitute for e_1 and e_2 in (3.43) from (3.28b), we get

$$3\epsilon = \frac{4\Omega_m(R_1R_2)^{\frac{1}{2}}R_3}{3\beta^{\frac{1}{2}}D_m}\left[1 - \frac{8\alpha D_m(R_1R_2)^{\frac{1}{2}}R_3}{9m\pi\Omega_m\beta^{\frac{1}{2}}}\right] \tag{3.44}$$

But from (3.13a), we have

$$D_m = \frac{2\lambda m^2\bar{a}_m}{\Omega_m^2}$$

If we simplify the terms in the square bracket in (3.44), we get

$$\Omega_m^3 = \frac{9\beta^{\frac{1}{2}}\lambda_S m^2\bar{a}_m\epsilon}{2(R_1R_2)^{\frac{1}{2}}R_3\left(1 - \frac{8\alpha D_m(R_1R_2)^{\frac{1}{2}}R_3}{9m\pi\Omega_m\beta^{\frac{1}{2}}}\right)} \tag{3.45}$$

On substituting for Ω_m at buckling in (3.45), we get

$$(m^4 - 2\lambda_S m^2 + 1)^{\frac{3}{2}} = \frac{9\beta^{\frac{1}{2}}\lambda_S m^2 \bar{a}_m \epsilon}{2(R_1R_2)^{\frac{1}{2}}R_3\left(1 - \frac{8\alpha D_m (R_1R_2)^{\frac{1}{2}}R_3}{9m\pi\Omega_m\beta^{\frac{1}{2}}}\right)} \tag{3.46}$$

Equation (3.46) gives an implicit equation for the evaluation of λ_S. The dominant value of λ_S is ontained for $m = 1$.

Having initiated the static deformation which finally resulted in the determination of the static buckling load, λ_S, we shall, in our next research work, discuss the dynamic deformation of the same column.

RESULTS AND DISCUSSION

Equation (3.46) gives an implicit equation for the evaluation of λ_S. Sample codes written in Q-basic programming language were able to obtain numerical values for the static buckling loads of the finite imperfect column, as we vary the imperfection factors, as shown in Table 1. Using Table 1, a graph of the static buckling load against imperfection factors is shown in Figure 1. We observe from Figure 1 that imperfection is a key factor in determining the buckling load of the column because as the imperfection factors increase, the static buckling load decreases and is in agreement with Chukwuchekwa (2017).

Imperfection factor, $\bar{a}\epsilon$	Static Buckling Load, λ_S
0.01	0.895588
0.02	0.866607
0.03	0.846223
0.04	0.829973
0.05	0.816239
0.06	0.804232
0.07	0.793499
0.08	0.783737
0.09	0.774766

0.1	0.766442

Table 1: Table showing the relationship between the Static buckling load, $\boldsymbol{\lambda_S}$ and the imperfection parameter, $\bar{a}\epsilon$ and using equation (3.46).

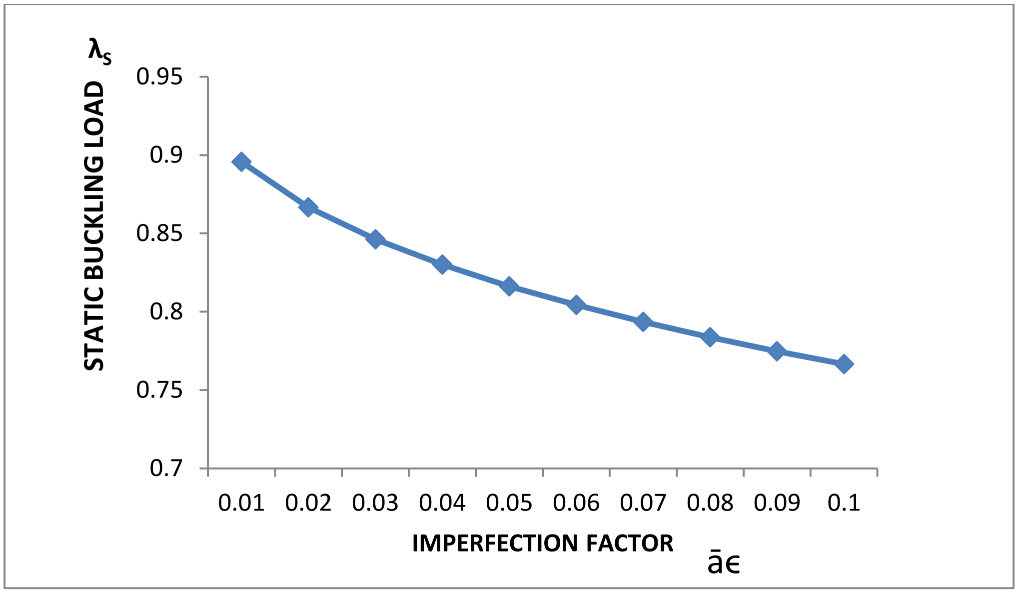

Fig.1: Graphical plot showing the relationship between the Static buckling load, λ_S and the imperfection parameter, $\bar{a}\epsilon$ using equation (3.46).

Results show that imperfection is a key factor in determining the buckling load of the column because as the imperfection factors increase, the static buckling load decreases.

CONCLUSION

The regular perturbation technique in asymptotic expansion of the relevant variables has been effectively utilized in analyzing the static buckling of a finite deterministically imperfect but viscously damped column resting on some quadratic–cubic nonlinear elastic foundations, but struck by a step load. With the help of Q-BASIC codes, numerical results obtained show that increase in imperfection of the column lowers the static buckling loads of the column. It is our contention that this same procedure can be used in the stability analysis of the dynamic buckling of the same column to ascertain the effects of light viscous damping and pre-static loads on the dynamic loads of the column.

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