TRAPEZOIDAL INTUITIONISTIC FUZZY NUMBER WITH SOME ARITHMETIC OPERATIONS AND ITS APPLICATION ON RELIABILITY EVALUATION

[1]Rahul Kar, [2]A.K. Shaw

[1]Department of Mathematics, Springdale high school, Kalyani, India

[2]Department of Mathematics, Regent Education and Research Foundation, Kolkata

Email: rkar997@gmail.com

Abstract

In fuzzy mathematics fuzzy system reliability can be analysed by fuzzy sets. We can use various types of fuzzy sets for that analyzing the fuzzy system reliability but here we specially used intuitionistic fuzzy set theory. At first, TrIFNs and their arithmetic operations are introduced. Expressions for computing the fuzzy reliability of a series system, parallel system, series-parallel and parallel-series system following TrIFNs have been described. Here an imprecise failure to start of a truck is taken. To compute the imprecise failure of the above said system, failure of each component of the systems is represented by Trapezoidal Intuitionistic Fuzzy Number. This process can be utilise to measure the failure is various aspects like portfolio in stock market etc. The numerical expression also calculated and presented in this paper for the failure to start of a truck using TrIFN.

Keywords: *fuzzy set, IFN, intuitionistic fuzzy number, system reliability, TrIFN, trapezoidal intuitionistic fuzzy number.*

1. INTRODUCTION

A fuzzy set in a universe X is defined by membership function that maps X to the interval [0, 1] and therefore implies a linear, i.e. total ordering of the elements of X, one could argue that this makes them inadequate to deal with incomparable information. A possible solution, however, was already implicit in Zadeh's [40] seminal paper in a footnote; he mentioned that "in a more general setting, the range of the membership function can be taken to be a suitable partially ordered set P." In 1967, Goguen[17] formally introduced the notion of an L- fuzzy set with a membership function taking values in a lattice L. Another generalizations of fuzzy sets called interval valued fuzzy sets, apparently first studied by Sambue[27] who called them φ - flou functions, serve to capture a feature of uncertainty with respect to the assignment of membership degrees by intervals in [0, 1] because assigning an exact number to an expert's opinion is too restrictive, and that the assignment of an interval of values is more realistic. Finally intuitionistic fuzzy sets (IFS) were introduced in 1983 by K. T. Atanassov[1] as generalization of fuzzy sets. IFS theory basically defies the claim that from the fact that an element x belongs to a given degree say $\mu_A(x)$ to a fuzzy set A, naturally follows that x should not belong to A to the extent $1-\mu_A(x)$, an assertion implicit in the concept of a fuzzy set. On the contrary, IFSs assign to each element of the universe both a degree of membership $\mu_A(x)$

and one of non – membership $\nu_A(x)$ such that $\mu_A(x)+\nu_A(x)\le 1$, thus relaxing the enforced duality $\nu_A(x)=1-\mu_A(x)$ from fuzzy set theory. For each intuitionistic fuzzy set in X, we will call $\pi_A(x)=1-\mu_A(x)-\nu_A(x)$ a hesitation margin or intuitionistic fuzzy index of $x\in A$ and it expresses lack of knowledge of whether x belongs to A or not. It is obvious that $0\le\pi_A(x)\le 1$, for each $x\in X$. The application of intuitionistic fuzzy sets instead of fuzzy sets means the introduction of another degree of freedom into a set description. Such a generalization of fuzzy sets gives us an additional possibility to represent imperfect knowledge what leads to describing many real problems in a more adequate way.

In this paper we represent the basic concept of IFSs and IFNs, arithmetic operations between two Trapezoidal Intuitionistic Fuzzy Numbers (TrIFNs), expressions for finding the fuzzy reliability of a series and a parallel system using arithmetic operations on TrIFNs, presentation and calculation of the failure to start of a truck using intuitionitic fuzzy fault tree.

2. BASIC CONCEPT OF INTUITIONISTIC FUZZY SETS

Atanassov (1983)[1] presented the concept of IFS, and pointed out that this single value combines the evidence for $x_i\in X$, but does not indicate evidence against $x_i\in X$ .An IFS $\tilde{A}^i$ in X is characterised by a membership function $\mu_{\tilde{A}^i}(x)$ and a non -membership function $\nu_{\tilde{A}^i}(x)$. Here, $\mu_{\tilde{A}^i}(x)$ and $\nu_{\tilde{A}^i}(x)$ are associated with each point in *X*, a real number in $[0,1]$ with the value of $\mu_{\tilde{A}^i}(x)$ and $\nu_{\tilde{A}^i}(x)$ at *X* representing the grade of membership and non-membership of *x* in $\tilde{A}^i$.Thus, the closer the value of $\mu_{\tilde{A}^i}(x)$ to unity and the value of $\nu_{\tilde{A}^i}(x)$ to zero, the higher the grade of membership is, and the lower the grade of non-membership of *x*. When $\tilde{A}^i$ is an ordinary set, its membership function (non-membership function) can take on only two values, 0 and 1. If $\mu_{\tilde{A}^i}(x)=1$ and $\nu_{\tilde{A}^i}(x)=0$, the element *x* does not belong to $\tilde{A}^i$, similarly, if $\mu_{\tilde{A}^i}(x)=0$ and $\nu_{\tilde{A}^i}(x)=1$, the element *x* does not belong to $\tilde{A}^i$. An IFS becomes a fuzzy set $\tilde{A}^i$ when $\nu_{\tilde{A}^i}(x)=0$, but $\mu_{\tilde{A}^i}(x)\in[0,1]\forall x\in\tilde{A}^i$.

2.1 Definition: Intuitionistic Fuzzy Set:

Let a set X be fixed. An IFS $\tilde{A}^i$ in X is an object having the for $\tilde{A}^i=\left\{\left\langle x,\mu_{\tilde{A}^i}0,1(x),\nu_{\tilde{A}^i}(x)\right\rangle : x\in X\right\}$ where $\mu_{\tilde{A}^i}(x):X\to[0,1]$ and $\nu_{\tilde{A}^i}(x):X\to[0,1]$ define the degree of membership and degree of non-membership respectively, of the element $x\in X$ to the set $\tilde{A}^i$, which is a subset of *X*, for every element of $x\in X$, $0<\mu_{\tilde{A}^i}(x)+\nu_{\tilde{A}^i}(x)<1$.

2.2 Definition: (α,β)-cuts:

A set of (α,β)-cuts, generated by IFS $\overset{i}{A}$, where $\alpha,\beta\in[0,1]$ is a set of fixed numbers such that $\alpha+\beta\le 1$ is defined as

$$\overset{i}{A}_{\alpha,\beta}=\left\{\begin{array}{c}\left(x,\mu_{\overset{i}{A}}(x),\nu_{\overset{i}{A}}(x)\right):x\in X\\ \mu_{\overset{i}{A}}(x)\ge\alpha,\nu_{\overset{i}{A}}(x)\le\beta,\alpha,\beta\in[0,1]\end{array}\right\}$$

(α,β)-cuts denoted by $\overset{i}{A}_{\alpha,\beta}$,is defined as the crisp set of elements *x* which belong to $\overset{i}{A}$, at least to the degree α and which does belong $\overset{i}{A}$ to the degree β.

2.3 Definition: Intuitionistic Fuzzy Number:

An IFN $\overset{i}{A}$ is

- an intuitionistic fuzzy sub-set of the real line
- normal, i.e., there is an $x_0\in\Re$ such that $\mu_{\overset{i}{A}}(x_0)=1\left(\nu_{\overset{i}{A}}(x_0)=0\right)$
- Convex for the membership function $\mu_{\overset{i}{A}}(x)$ i.e.

$$\mu_{\overset{i}{A}}(\lambda x_1+(1-\lambda)x_2)\ge\min\left(\mu_{\overset{i}{A}}(x_1),\mu_{\overset{i}{A}}(x_2)\right)\forall x_1,x_2\in\Re,\lambda\in[0,1]$$

- concave for the non-membership function $\nu_{\overset{i}{A}}(x)$ i.e.

$$\nu_{\overset{i}{A}}(\lambda x_1+(1-\lambda)x_2)\le\max\left(\nu_{\overset{i}{A}}(x_1),\nu_{\overset{i}{A}}(x_2)\right)\forall x_1,x_2\in\Re,\lambda\in[0,1]$$

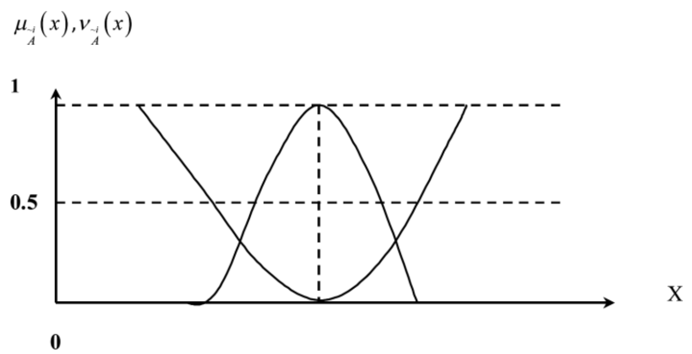

Fig. 1 Membership and non-membership functions of $\overset{i}{A}$

2.4 Definition: Trapezoidal intuitionistic fuzzy number:

A TrIFN $\tilde{A}^{i}$ is an IFN in R with the following membership function $\left(\mu_{\tilde{A}^{i}}(x)\right)$ and non membership function $\left(\nu_{\tilde{A}^{i}}(x)\right)$

$$\mu_{\tilde{A}^{i}}(x)=\begin{cases}\dfrac{x-a_1}{b_1-a_1} & a_1\le x\le b_1\\ 1 & b_1\le x\le c_1\\ \dfrac{d_1-x}{d_1-c_1} & c_1\le x\le d_1\\ 0 & otherwise\end{cases}\quad \text{and}\quad \nu_{\tilde{A}^{i}}(x)=\begin{cases}\dfrac{b_1-x}{b_1-a_1} & a_1^{/}\le x\le b_1\\ 0 & b_1\le x\le c_1\\ \dfrac{x-c_1}{d_1^{/}-c_1} & c_1\le x\le d_1^{/}\\ 1 & otherwise\end{cases}$$

Where $a_1^{/}<a_1<b_1<c_1<d_1<d_1^{/}$ and $\mu_{\tilde{A}^{i}}(x),\nu_{\tilde{A}^{i}}(x)\le 0.5$ for $\mu_{\tilde{A}^{i}}(x)=\nu_{\tilde{A}^{i}}(x)\,\forall x\in\Re$.

This TrIFN is denoted by $A_{TrIFN}=\left(a_1,b_1,c_1,d_1;a_1^{/},b_1,c_1,d_1^{/}\right)$.

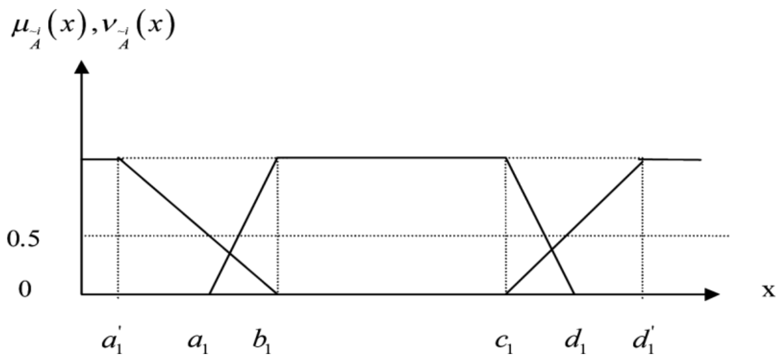

Fig. 2 Membership and non-membership function of TrIFN

3. SOME ARITHMETIC OPERATIONS OF INTUITIONISTIC FUZZY NUMBER BASED ON CUTS METHOD:

❖ **Properties 3.1**

- If TrIFN $\tilde{A}^{i}=\left(a_1,b_1,c_1,d_1;a_1^{/},b_1,c_1,d_1^{/}\right)$ and $y=ka\,(k>0)$, then $\tilde{Y}^{i}=k\tilde{A}^{i}$ is a TrIFN $\left(ka_1,kb_1,kc_1,kd_1;ka_1^{/},kb_1,kc_1,kd_1^{/}\right)$.
- If $y=ka\,(k<0)$, then $\tilde{Y}^{i}=k\tilde{A}^{i}$ is a TrIFN $\left(kd_1,kc_1,kb_1,ka_1^{/};kd_1,kc_1,kb_1,ka_1^{/}\right)$.

❖ **Properties 3.2**

If $\tilde{A}^{i}=\left(a_1,b_1,c_1,d_1;a_1^{/},b_1,c_1,d_1^{/}\right)$ and $\tilde{B}^{i}=\left(a_2,b_2,c_2,d_2;a_2^{/},b_2,c_2,d_2^{/}\right)$ are two TrIFN then $\tilde{C}^{i}=\tilde{A}^{i}\oplus\tilde{B}^{i}$ is also TrIFN.

$$\tilde{A}^{i}\oplus\tilde{B}^{i}=\left(a_1+a_2,b_1+b_2,c_1+c_2,d_1+d_2;a_1^{/}+a_2^{/},b_1+b_2,c_1+c_2,d_1^{/}+d_2^{/}\right)$$

Example 3.2.1: let us consider two TrIFNs $\tilde{A}^{i} = (3,4,5.5,6;2,4,5.5,7)$ and $\tilde{B}^{i} = (2,4.5,5,6;1,4.5,5,7)$. Then addition is defined by $\tilde{A}^{i} \oplus \tilde{B}^{i} = (5,8.5,10.5,12;3,8.5,10.5,14)$ with membership and non-membership functions as follows,

$$\mu_{\tilde{A}^{i}\oplus\tilde{B}^{i}}(x) = \begin{cases} \frac{x-5}{3.5} & 5 \le x \le 8.5 \\ 1 & 8.5 \le x \le 10.5 \\ \frac{12-x}{1.5} & 10.5 \le x \le 12 \\ 0 & otherwise \end{cases} \quad \text{and} \quad \nu_{\tilde{A}^{i}\oplus\tilde{B}^{i}}(x) = \begin{cases} \frac{8.5-x}{5.5} & 3 \le x \le 8.5 \\ 0 & 8.5 \le x \le 10.5 \\ \frac{x-10.5}{3.5} & 10.5 \le x \le 14 \\ 1 & otherwise \end{cases}$$

- **Properties 3.3**

If $\tilde{A}^{i} = (a_1,b_1,c_1,d_1;a_1',b_1,c_1,d_1')$ and $\tilde{B}^{i} = (a_2,b_2,c_2,d_2;a_2',b_2,c_2,d_2')$ are two TrIFN then $\tilde{P}^{i} = \tilde{A}^{i} \odot \tilde{B}^{i}$ is an approximated TrIFN.

$$\tilde{A}^{i} \odot \tilde{B}^{i} = (a_1a_2, b_1b_2, c_1c_2, d_1d_2; a_1'a_2', b_1b_2, c_1c_2, d_1'd_2')$$

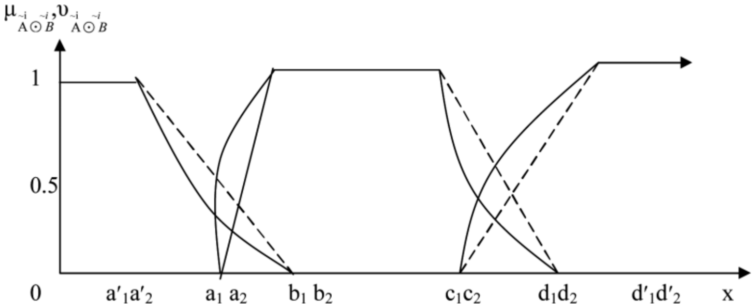

Fig. 3 Membership and non-membership functions for product of two TrIFNs

Example 3.2.1: let us consider two TrIFNs $\tilde{A}^{i} = (2,3,4.5,5;1,3,4.5,6)$ and $\tilde{B}^{i} = (3,4.5,5,6;2,4.5,5,7)$. Then addition is defined by $\tilde{A}^{i} \odot \tilde{B}^{i} = (6,13.5,22.5,30;2,13.5,22.5,42)$ with membership and non-membership functions as follows,

$$\mu_{\tilde{A}^i \otimes \tilde{B}^i}(x) = \begin{cases} \dfrac{-6+\sqrt{36-6(6-x)}}{3} & 6 \le x \le 13.5 \\ 1 & 13.5 \le x \le 22.5 \\ \dfrac{-8-\sqrt{64-0.5(30-x)}}{1} & 22.5 \le x \le 30 \\ 0 & otherwise \end{cases}$$

And

$$\nu_{\tilde{A}^i \otimes \tilde{B}^i}(x) = \begin{cases} \dfrac{9-\sqrt{81-20(13.5-x)}}{10} & 2 \le x \le 13.5 \\ 1 & 13.5 \le x \le 22.5 \\ \dfrac{-16.5+\sqrt{272.25-12(22.5-x)}}{6} & 22.5 \le x \le 42 \\ 0 & otherwise \end{cases}$$

4. IMPRECISE RELIABILITY OF SERIES AND PARALLEL SYSTEMS USING ARITHMETIC OPERATIONS OR TRAPEZOIDAL INTUITIONISTIC FUZZY NUMBERS:

Here, we present the expressions for evaluation of the imprecise reliability of a series and a parallel system where the reliability of each component of the systems is represented by a TrIFN.

- **Series system**

Let us consider a series system consisting of n components, as shown in Figure 4. The intuitionistic fuzzy reliability of $\tilde{R}^i_{ss}$ the series system shown below can be evaluated by using the expression as follows:

$$\tilde{R}^i_{ss} = \tilde{R}^i_1 \otimes \tilde{R}^i_2 \otimes \ldots\ldots. \otimes \tilde{R}^i_n = \{(r_{11}, r_{12}, r_{13}, r_{14}; r'_{11}, r_{12}, r_{13}, r'_{14}) \otimes (r_{21}, r_{22}, r_{23}, r_{24}; r'_{21}, r_{22}, r_{23}, r'_{24})$$

$$\otimes \ldots\ldots\ldots\ldots\ldots\ldots\ldots.. \otimes (r_{n1}, r_{n2}, r_{n3}, r_{n4}; r'_{n1}, r_{n2}, r_{n3}, r'_{n4})\}$$

It can be approximated to a TrIFN as

$$\cong \left(\prod_{j=1}^{n} r_{j1}, \prod_{j=1}^{n} r_{j2}, \prod_{j=1}^{n} r_{j3}, \prod_{j=1}^{n} r_{j4}; \prod_{j=1}^{n} r'_{j1}, \prod_{j=1}^{n} r_{j2}, \prod_{j=1}^{n} r_{j3}, \prod_{j=1}^{n} r'_{j4}, \right)$$

Where $\tilde{R}^i_j = (r_{j1}, r_{j2}, r_{j3}, r_{j4}; r'_{j1}, r_{j2}, r_{j3}, r'_{j4})$ is an intuitionistic fuzzy reliability of the jth component for j=1,2,......,n.

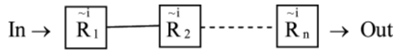

Fig. 4 Diagram of a series system

- **Parallel system:**

Let us consider a parallel system consisting of n components, as shown in Figure 5. The fuzzy reliability $\tilde{R}^i_{ps}$ of the parallel system shown below can be evaluated by using the expression as follows:

$$\tilde{R}^i_{ps} = 1\Theta\prod_{j=1}^{n}\left(1-\tilde{R}^i_j\right) = 1\Theta\left[\left(1\Theta\left(r_{11},r_{12},r_{13},r_{14};r'_{11},r_{12},r_{13},r'_{14}\right)\right)\otimes \text{..........} \left(1\Theta\left(r_{n1},r_{n2},r_{n3},r_{n4};r'_{n1},r_{n2},r_{n3},r'_{n4}\right)\right)\right]$$

It is an approximated to a TrIFN, Where $\tilde{R}^i_j = \left(r_{j1},r_{j2},r_{j3},r_{j4};r'_{j1},r_{j2},r_{j3},r'_{j4}\right)$ is an intuitionistic fuzzy reliability of the jth component for j=1,2,......,n.

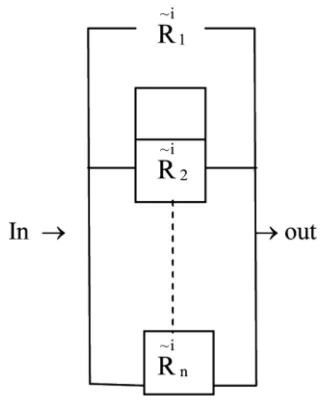

Fig. 5 Diagram of a parallel system

- **Parallel-series System :**

Consider a parallel-series system consisting of m branches connected in parallel and each branch contains n components as shown in Fig.6.The fuzzy reliability $\tilde{R}^i_{pss}$ of the parallel-series system shown in Fig.6 can be evaluated as follows:

$$\tilde{R}^i_{pss} = 1-\prod_{k=1}^{m}\left(1\Theta\left(\prod_{i=1}^{n}\tilde{R}^i_{ki}\right)\right)$$

$$=\left[1-\prod_{k=1}^{m}\left(1-\left(\prod_{i=1}^{n}x_{ki}\right)\right),1-\prod_{k=1}^{m}\left(1-\left(\prod_{i=1}^{n}y_{ki}\right)\right),1-\prod_{k=1}^{m}\left(1-\left(\prod_{i=1}^{n}z_{ki}\right)\right),1-\prod_{k=1}^{m}\left(1-\left(\prod_{i=1}^{n}w_{ki}\right)\right);1-\prod_{k=1}^{m}\left(1-\left(\prod_{i=1}^{n}x'_{ki}\right)\right),1-\prod_{k=1}^{m}\left(1-\left(\prod_{i=1}^{n}y_{ki}\right)\right),1-\prod_{k=1}^{m}\left(1-\left(\prod_{i=1}^{n}z_{ki}\right)\right),1-\prod_{k=1}^{m}\left(1\right.\right.$$

Where $\tilde{R}^{i}_{ki}=\left(x_{ki},y_{ki},z_{ki},w_{ki};x'_{ki},y_{ki},z_{ki},w'_{ki}\right)$ represents the reliability of the ith component at kth branch.

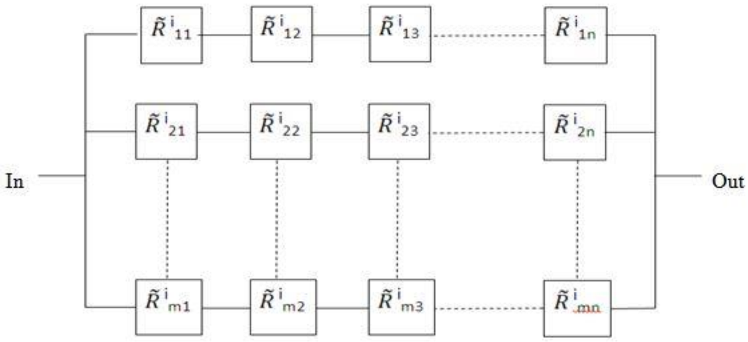

Fig. 6 Parallel-Series system

- **Series-parallel System:**

Consider a series-parallel system consisting of n stages connected in series and each stage contains m components as shown in Fig.7.The fuzzy reliability $\tilde{R}^{i}_{sps}$ of the series-parallel system shown in Fig.5 can be evaluated as follows:

$$\tilde{R}^{i}_{sps}=\prod_{k=1}^{n}\left(1\Theta\left(\prod_{i=1}^{m}\tilde{R}^{i}_{ik}\right)\right)$$

$$=\left[\left(\prod_{k=1}^{n}\left(1-\left(\prod_{i=1}^{m}x_{ik}\right)\right),\prod_{k=1}^{n}\left(1-\left(\prod_{i=1}^{m}y_{ik}\right)\right),\prod_{k=1}^{n}\left(1-\left(\prod_{i=1}^{m}z_{ik}\right)\right),\prod_{k=1}^{n}\left(1-\left(\prod_{i=1}^{m}w_{ik}\right)\right)\right);\left(\prod_{k=1}^{n}\left(1-\left(\prod_{i=1}^{m}x'_{ik}\right)\right),\prod_{k=1}^{n}\left(1-\left(\prod_{i=1}^{m}y_{ik}\right)\right),\prod_{k=1}^{n}\left(1-\left(\prod_{i=1}^{m}z_{ik}\right)\right),\prod_{k=1}^{n}\left(1-\left(\prod_{i=1}^{m}w'_{ik}\right)\right)\right)\right]$$

Where $\tilde{R}^{i}_{ik}=\left(x_{ik},y_{ik},z_{ik},w_{ik};x'_{ik},y_{ik},z_{ik},w'_{ik}\right)$ represent the reliability of the ith component at kth stage.

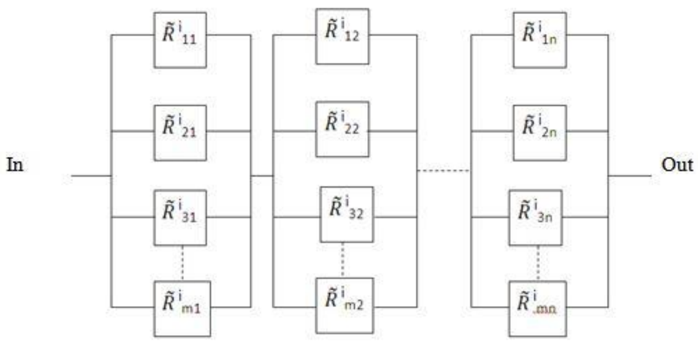

Fig. 7 Series-Parallel systems

5. Calculation of system failure using trapezoidal intuitionistic fuzzy number:

Failure to start a truck depends on different facts. The facts are battery low charge, ignition failure and fuel supply failure. There are two sub factors of each of facts. The fault-tree of failure to start of truck is shown in the figure 8.

$\tilde{F}^{i}_{fs}$ represents the system failure to start of truck.

$\tilde{F}^{i}_{blc}$ represents the failure to start of truck due to battery low charge.

$\tilde{F}^{i}_{if}$ represents the failure to start of truck due to ignition failure.

$\tilde{F}^{i}_{fsf}$ represents the failure to start of truck due to fuel supply failure.

$\tilde{F}^{i}_{lbf}$ represents the failure to start of truck due to low battery fluid.

$\tilde{F}^{i}_{bis}$ represents the failure to start of truck due to battery internal short.

$\tilde{F}^{i}_{whf}$ represents the failure to start of truck due to wire harness failure.

$\tilde{F}^{i}_{spf}$ represents the failure to start of truck due to spark plug failure.

$\tilde{F}^{i}_{fif}$ represents the failure to start of truck due to fuel injector failure.

$\tilde{F}^{i}_{fpf}$ represents the failure to start of truck due to fuel pump failure.

The intuitionstic fuzzy failure to start of a truck can be calculated when the failures of the occurrence of basic fault events are known. Failure to start of a truck can be evaluated by using the following steps.

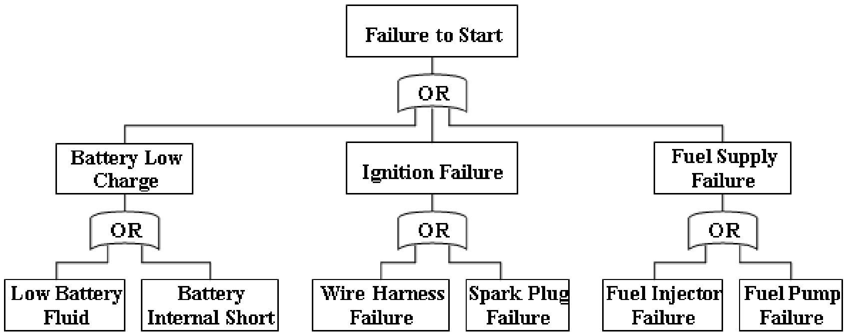

Fig. 8: Fault-tree of failure to start of a truck

- **Step 1.**

$$\tilde{F}^{i}_{blc} = 1\Theta\left(1\Theta \tilde{F}^{i}_{lbf}\right)\left(1\Theta \tilde{F}^{i}_{bis}\right)$$

$$\tilde{F}^{i}_{if} = 1\Theta\left(1\Theta \tilde{F}^{i}_{whf}\right)\left(1\Theta \tilde{F}^{i}_{spf}\right) \qquad (5.1.1)$$

$$\tilde{F}^{i}_{fsf} = 1\Theta\left(1\Theta \tilde{F}^{i}_{fif}\right)\left(1\Theta \tilde{F}^{i}_{fpf}\right)$$

- **Step 2.**

$$\tilde{F}^{i}_{fs} = 1\Theta\left(1\Theta \tilde{F}^{i}_{blc}\right)\left(1\Theta \tilde{F}^{i}_{if}\right)\left(1\Theta \tilde{F}^{i}_{fsf}\right) \qquad (5.1.2)$$

- **Result of start to failure of truck using TrIFN :**

Numerical of starting failure of a truck using fault tree analysis with intuitionistic fuzzy failure rate. The components failure rates as TrIFN are given by

$$\tilde{F}^{i}_{lbf} = \left(0.02, 0.03, 0.04, 0.05; 0.01, 0.03, 0.04, 0.06\right), \tilde{F}^{i}_{bis} = \left(0.03, 0.04, 0.06, 0.07; 0.02, 0.04, 0.06, 0.08\right)$$

$$\tilde{F}^{i}_{whf} = \left(0.03, 0.04, 0.05, 0.06; 0.02, 0.04, 0.05, 0.07\right), \tilde{F}^{i}_{spf} = \left(0.03, 0.04, 0.06, 0.07; 0.02, 0.04, 0.06, 0.08\right)$$

$$\tilde{F}^{i}_{fif} = \left(0.06, 0.07, 0.08, 0.09; 0.04, 0.07, 0.08, 0.1\right), \tilde{F}^{i}_{fpf} = \left(0.04, 0.06, 0.07, 0.08; 0.03, 0.06, 0.07, 0.09\right)$$

Using (5.1.1) in the step-1 we have the following results

$$\tilde{F}^{i}_{blc} = \left(0.0494, 0.0688, 0.0976, 0.1165; 0.0298, 0.0688, 0.0976, 0.1352\right),$$

$$\tilde{F}^{i}_{if} = \left(0.0591, 0.0784, 0.1070, 0.1258; 0.0396, 0.0784, 0.1070, 0.1444\right),$$

$$\tilde{F}^{i}_{fsf} = (0.0976, 0.1258, 0.1444, 0.1628; 0.0688, 0.1258, 0.1444, 0.1810)$$

using (5.1.2) in the second and final step ,we get the failure to start of truck, calculated fuzzy failure to start of a truck as shown in the figure 9,represented by the following TrIFN

$$\tilde{F}^{i}_{fs} = (0.1928758071, 0.2497668751, 0.3105205581, 0.353383808; 0.1323263895, 0.2497668751, 0.3105205581, 0.3940$$

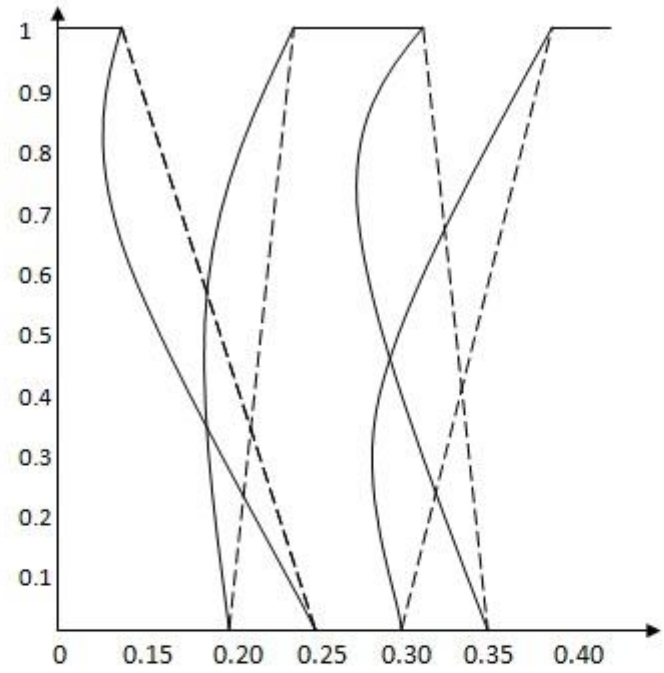

Fig 9: TrIFN representing the system failure to start of a truck

6. CONCLUSION:

In this paper, we proposed a definition of IFN according to the approach of fuzzy number presentation. On the bases of the intuitionistic fuzzy (α, β) cut method some arithmetic operations of the proposed TrIFN are also evaluated. Here, a method to analyse system reliability, which is based on the IFS theory, has been presented, where the components of the system are represented by TrIFNs. Here we analyse the fuzzy reliability of the series system and the parallel system. An intuitionistic fuzzy fault tree is used to analyse the failure of starting of a truck. The major advantage of using IFSs over fuzzy sets is that IFSs separate the positive and the negative evidence for the membership of an element in a set. Our approaches and computational procedures may be efficient and simple to implement for calculation in an intuitionistic fuzzy environment for all fields of engineering and science where impreciseness occur.

REFERENCES:

[1] Atanassov, K.T. (1983) 'Intuitionistic fuzzy sets', *VII ITKR's Session, Sofia (deposed in Central*

Science-Technical Library of Bulgarian Academy of Science, 1697/84, Bulgarian.

[2]Atanassov, K.T. (1986) 'Intuitionistic fuzzy sets', *Fuzzy Sets and Systems*, Vol. 20, pp.87–96.

[3]Atanassov, K.T. (1989) 'More on intuitionistic fuzzy sets', *Fuzzy Sets and Systems*, Vol. 33, Pp.37–46.

[4]Atanassov, K.T. (1999) *Intuitionistic Fuzzy Sets*, Physica-Verlag, Heidelberg, New York.

[5]Atanassov, K.T. (2000) 'Two theorems for Intuitionistic fuzzy sets', *Fuzzy Sets and Systems*, Vol. 110, pp.267–269.

[6]Atanassov, K.T. and Gargov, G. (1989) 'Interval-valued intuitionistic fuzzy sets', *Fuzzy Sets and Systems*, Vol. 31, No. 3, pp.343–349.

[7].Buhaesku, T. (1988) 'On the convexity of intuitionistic fuzzy sets', *Itinerant Seminar on Functional Equations, Approximation and Convexity*, Cluj-Napoca, pp.137–144.

[8]Burillo, P., Bustince, H. and Mohedano, V. (1994) 'Some definition of intuitionistic fuzzy Number, fuzzy based expert systems', *Fuzzy Bulgarian Enthusiasts*, Sofia, Bulgaria.

[9]Cai, K.Y., Wen, C.Y. and Zhang, M.L. (1991) 'Fuzzy variables as a basis for a theory of fuzzy reliability in the possibility context', *Fuzzy Sets and Systems*, Vol. 42, pp.142–145.

[10]Cai, K.Y., Wen, C.Y. and Zhang, M.L. (1993) 'Fuzzy states as a basis for a theory of fuzzy reliability', *Microelectronics and Reliability*, Vol. 33, No. 15, pp.2253–2263.

[11]Cheng, C.H. and Mon, D.L. (1993) 'Fuzzy system reliability analysis by interval of confidence', *Fuzzy Sets and Systems*, Vol. 56, No. 1, pp.29–35.

[12]Chen, S.M. (1994) 'Fuzzy system reliability analysis using fuzzy number arithmetic operations', *Fuzzy Sets and Systems*, Vol. 64, No. 1, pp.31–38.

[13]Cornelis, C., Deschrijver, G. and Kerre, E.E. (2004) 'Implication in intuitionistic fuzzy and Interval–valued fuzzy set theory: construction, application', *International Journal of Approximate Reasoning*, Vol. 35, pp.55–95.

[14]Deschrijver, G. and Kerre, E.E. (2002) On the relationship between intuitionistic fuzzy sets and some other extensions of fuzzy set theory', *Journal of Fuzzy Mathematics*, Vol. 10, No. 3, Pp.711–724.

[15]Dhillon, B.S. (2007) *Human Reliability and Error in Transportation Systems – (Springer Series in Reliability Engineering)*, Springer-Verlag, London.

[16]Gerstenkorn, T. and Manko, J. (1991) 'Correlation of intuitionistic fuzzy sets', *Fuzzy Sets and Systems*, Vol. 44, pp.39–43.

[17] GOGUEN, J. A. L-Fuzzy sets. J. Mat/l. Anal. Appl. 18 (1967), 145-174.

[18]Hajeeh, M.A. (2011) 'Reliability and availability of a standby system with common cause failure', *International Journal of Operational Research*, Vol. 11, No. 3, pp.343–363.

[19]Kaufmann, A. and Gupta, M. (1988) *Fuzzy Mathematical Models in En Engineering And Management Science*, North Holland Publishers, Amsterdam.

[20]Kumar, M., Yadav, S.P. and Kumar, S. (2011) 'A new approach for analyzing the fuzzy system reliability using intuitionistic fuzzy number', *International Journal of Industrial and Systems Engineering*, Vol. 8, No. 2, pp.135–156.

[21]Mahapatra, G.S. and Roy, T.K. (2009) 'Reliability evaluation using triangular intuitionistic fuzzy numbers arithmetic operations', *Proceedings of World Academy of Science, Engineering and Technology*, Vol. 38, pp.587–595.

[22]Mitchell, H.B. (2004) 'Ranking-intuitionistic fuzzy numbers', *International Journal of Uncertainty, Fuzziness and Knowledge-Based Systems*, Vol. 12, No. 3, pp.377–386.

[23]Nikolaidis, E. and Mourelatos, Z.P. (2011) 'Imprecise reliability assessment when the type of the probability distribution of the random variables is unknown', *International Journal of Reliability and Safety*, Vol. 5, No. 2, pp.140–157.

[24]Onisawa, T. and Kacprzyk, J. (1995) *Reliability and Safety Under Fuzziness*, 1st ed., Physica Verlag, Heidelberg.

[25]Persona, A., Sqarbossa, F. and Pham, H. (2009) 'Systemability function to optimization reliability in random environment', *International Journal of Mathematics in Operational Research*, Vol. 1, No. 3, pp.397–417.

[26]Prabha, B., Sujatha, R. and Srikrishna, S. (2011) 'Posfust reliability of a unified fuzzy markov model', *International Journal of Reliability and Safety*, Vol. 5, No. 1, pp.83–94.

[27] Sambue, R. Fonctions Φ foues. Application I'Aide au Diagnostic en Patholologie Thyroidienne. PhD thesis, Univ. Marseille, France (1975).

[28]Shaw,A.K and Roy,T.K (2013),Trapezoidal Intuitionistic Fuzzy Number with some arithmetic Operations and its application on reliability evaluation, International Journal of Mathematics in Operational Research (IJMOR), Vol.5, No. 1 (Switzerland, 2013), pp.55-73

[29] Shaw, A.K. and Roy, T.K. Fuzzy Reliability Optimization based on Fuzzy Geometric Programming Method using different operators, The Journal of Fuzzy Mathematics (USA) Vol.23, No.1, pp.79-88, 2015

[30] Shaw, A.K. and Roy, T.K. Reliability Analysis of the System with Imprecise Constant Failure Rate of the Components, IAPQR Transaction, *Vol.40, No.1,* 2015

[31] Shaw,A.K, Jana,D.K. and Roy, T.K. Multi-stage Mixed System Reliability via Random fuzzy, Possibility and Credibility Measure, The Journal of Fuzzy Mathematics (USA) *Vol.22, No.4, pp.879-890*, 2014

[32] Shaw,A.K, and Roy, T.K. Some arithmetic operations on Triangular Intuitionistic Fuzzy Number and its application on reliability evaluation, International Journal of Fuzzy Mathematics and System (IJFMS), Vol.2, No. 4(2012), pp.363-382

[33] Shaw,A.K, and Roy, T.K. , Generalized Trapezoidal Triangular Intuitionistic Fuzzy Number and its application on reliability evaluation, Fuzzy Number with its arithmetic Operations and its application in fuzzy system reliability analysis, International Journal of Pure Applied Science and Technology , 5(2) (2011), 60-76.

[34]Singer, D. (1990) 'A fuzzy set approach to fault tree and reliability analysis', *Fuzzy Sets And Systems*, Vol. 34, No. 2, pp.145–155.

[35]Stoyanova, D. and Atanassov, K.T. (1990) 'Relation between operators, defined over intuitionistic fuzzy sets', *IM-MFAIS*, Vol. 1, Sofia, Bulgaria, pp.46–49.

[36]Stoyanova, D. (1993) 'More on cartesian product over intuitionistic fuzzy sets', *BUSEFAL*, Vol. 54, pp.9–13.

[37]Szmidt, E. and Kacrzyk, J. (2000) 'Distances between intuitionistic fuzzy sets', *Fuzzy Sets And Systems*, Vol. 114, No. 3, pp.505–518.

[38]Verma, A.K., Srividya, A. and Gaonkar, R.P. (2004) 'Fuzzy dynamic reliability evaluation of a deteriorating system under imperfect repair', *International Journal of Reliability, Quality and Safety Engineering*, Vol. 11, No. 4, pp.387–398.

[39]Wang, Y. (2010) 'Imprecise probabilities based on generalized intervals for system reliability assessment', *International Journal of Reliability and Safety*, Vol. 4, No. 4, pp.319–342.

[40]Zadeh, L.A. (1965) 'Fuzzy sets', *Information and Control*, Vol. 8, pp.338–353.