GEOMETRY OF THE TETRAHEDRON AND COMPUTATION OF THE FACE ANGLES OF A TETRAHEDRON WITH THE HELP OF C-LANGUAGE

[1]Abhishek Banerjee, [2]Ritwik Pal & [3]Santanu Das

[1]*Department of Basic Science and Humanities*
Institute of Engineering & Management
Salt Lake Management Building, Kolkata-700091. India
Email: abhishek.banerjee18@outlook.com

[2] *Department of Basic Science and Humanities*
Institute of Engineering & Management
Salt Lake Management Building, Kolkata-700091. India
Email: ritwikpal20@gmail.com

[3]*Department of Basic Science and Humanities*
Institute of Engineering & Management
Salt Lake Management Building, Kolkata-700091. India
Email: santanu.das@iemcal.com

Abstract

In this paper we have tried to study the geometry of tetrahedron as a member of polyhedron family and hence calculate the face angles on each face of a tetrahedron with help of "C" Programming. We will also be looking into the applications of tetrahedron structure in chemistry in the structure of various molecules, in aviation it is used as a Wind Sock which serves as a reference to pilots indicating the direction of the wind as well as Geology where the 'Tetrahedral Hypothesis" was used to explain the formation of earth.

Keywords: *Euclidean Geometry, Riemannian Geometry, Tetrahedron, Dihedral Angle, Face Angle, Circumsphere.*

INTRODUCTION

Geometry is from the Greek word "geo" meaning earth and "metria" meaning measurement. Geometry basically measures components on Earth. Math models the world. However, we can look at the world differently and we get different models. Geometry, for example, deals with deduction of properties, measurements, and relationships of points, lines, angles, and figures in space**. Euclidean geometry** deals with a flat world that states that parallel lines are ways parallel and never intersect. Euclidean geometry is our way of measuring on Earth. On the other hand, **Riemannian geometry** deals with a spherical world that says parallel lines will intersect. This takes into consideration; the Universe may be a closed surface.

From the ancient past geometry was discovered and used to build modern civilisation, whose era was started by Egyptians. Egyptian and Greek geometry was masterpiece of Applied mathematics. The original motivation of geometry problem was the need "To tax land accurately and fairly and to erect buildings. As often happens mathematics that developed had the significance that transcend the PHARAOH"s original revenue problem, for geometry is at the heart of mathematical thinking. It is a field in which intuition abounds and new discoveries are within the compass of non-specialists.

It is popularly held that Euclid's chief contribution to geometry is his exposition of the axiomatic method of proof, a nation that we will not dispute. More relevant in the Euclidean construction there was a scheme which consists of an algorithm, it is remarkable as it defines a collection of allowable instruments and a set of legal operations (ruler & compass) that can be performed by them. Moreover it is unambiguous, correct and terminating. After that the method was changed as mathematicians thought that it is easier to prove the existence of an object by contradiction rather than by giving an explicit construction for algorithm.

In modern terms this is a computer science question: Do the Euclidean primitives suffice to perform all geometric computations? In attempt to answer this question various alternative models of computations were covered by allowing the primitives and the instruments themselves to vary.The influence of Euclid’s elements was so profound that it was not until Descartes that another formulation of geometry was proposed. It was coordinates enabled geometric problem to be expressed algebraically paving the way to study of higher plane curves and Newton's calculus. Coordinates permitted a vast increase in computational power., bridged the gulf between two great areas of mathematics and led to a renaissance in constructive thinking Then Gauss armed with algebraic tools. returned to the problem of which regular polygons with a prime number of sides could be constructed using Euclidean instruments and solved it completely. At this point a close connection between ruler and compass constructions, field extensions and algebraic equations become apparent. We analysed two dimensional regions using collections of triangles. In three dimensions, the corresponding approach uses collections of tetrahedrons. Triangles and tetrahedrons are the 2D and 3D examples of the simplex family. The useful properties of triangles and tetrahedrons are typical of the simplex family and so this gives an indication of how abstract N-dimensional problems can be treated as well. This lab is intended to introduce the basic geometry and representation of tetrahedrons. We will measure "face angles" of the tetrahedron. Of course, once we are comfortable with a single tetrahedron, we will want to look at collections of them.

GEOMETRY OF TETRAHEDRON

The regular tetrahedron is the most basic of all polyhedra. It has 4 faces, which are all equilateral triangles. It has 6 edges and 4 vertices. As a general form, the tetrahedron is the only polyhedron with 4 faces, it is the only polyhedron with 6 edges and it is the only polyhedron with 4 vertices, and no polyhedron can be built with fewer faces, with fewer edges, or with fewer vertices; that means that any effort to build any nomenclature of polyhedra starting from the smallest number of faces, of edges or of vertices always starts with a tetrahedron.

The regular tetrahedron is part of the family of 5 platonic solids, which are the most regular polyhedra. It is also part of the family of 8 convex decahedra (because it is convex and all its faces are equilateral triangles), and it is the first member of the infinite family of pyramids (one of the vertices shares identical edges with all the other vertices, which are arranged as a regular polygon).

The tetrahedron's simplicity gives it a few other unusual or unique properties. It is the only convex polyhedron in which the straight line between any two vertices is always an edge (mathematically, that means that the tetrahedron has no polyhedron diagonals, just like a triangle in two dimensions). If you join the centre of the faces that share a common edge (mathematically, that's called creating the **dual polyhedron**), you get another tetrahedron, and the faces of the dual have the same shape as the faces of the original, the tetrahedron is therefore called self-dual. The regular tetrahedron defines two sets of symmetry axes: one set of 4 3-fold axes that join each vertex to its opposite face, and one set of 3 2-fold axis that join the middle of opposite edges.

Tetrahedra occur in nature, and especially in chemistry, the 4 hydrogen atoms of a methane molecule are exactly located at the vertices of a tetrahedron. The same is true for the hydrogen atoms in the ammonium ion, and for the atoms surrounding the central carbon in tetrafluoromethane, tetrachloromethane, tetra bromomethane and triiodo methane. The same molecular geometry occurs in perchlorate, sulphate and phosphate ions. Finally, the atoms in diamond and in silicon crystals follow a tetrahedral geometry.

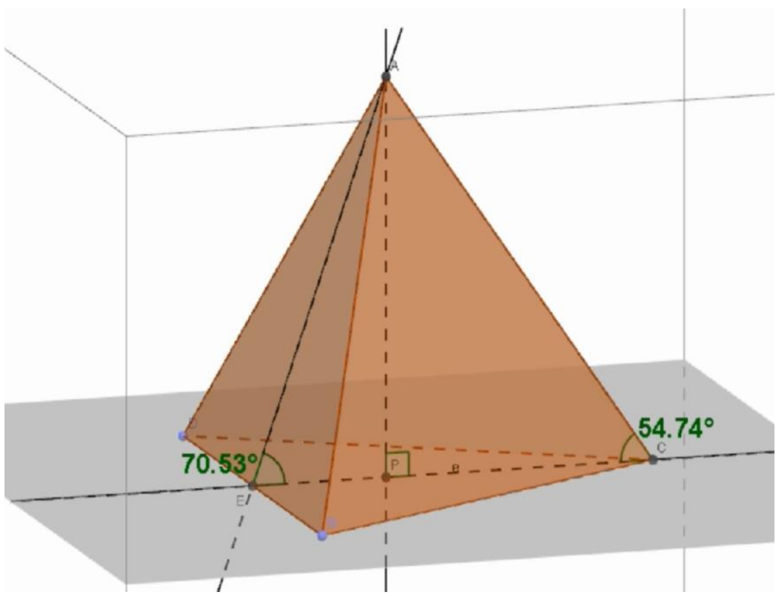
The following table lists the various dimensions in a regular tetrahedron, assuming that the edge length is 1:

Single edge length	1
Total edge length	6
Face inradius	0.288
Face circumradius	0.577
Polyhedron inradius	0.204

Polyhedron mid radius	0.354
Polyhedron circumradius	0.612
Single face area	0.433
Total face area	1.732
Volume	0.118

The following table lists various angles in a regular tetrahedron:

Angles between vertices(from centre)	109.5^0
Angles between edges	60^0
Angles between faces	70.53^0



METHODOLOGY

If the co-ordinates are given in a plane then we will verify if the figure formed is a tetrahedron. If it is, then we will find the Face angles using the help of “C” programming.

THE FACE ANGLES OF A TETRAHERON

Since the faces of the tetrahedron are triangles, it is also natural to want to describe the shape of these faces in terms of the angles of these triangles. We are simply imagining that we take some face of the tetrahedron and lay it down on a plane and measure the angles in the usual way. Considering the triangular face bounded by the tetrahedral vertices A, B and C. We denote A by 1, B by 2 and C by 3 respectively. Now we denote by A_{23} the angle opposite the triangle edge E_{23} and let S_{23} indicate the length of the edge E_{23}. Then the angles of the triangle may be found by the laws:

$$A_{23}=\mathrm{ArcCos}(S_{12}^2 + S_{13}^2 - S_{23}^2 / 2 * S_{12} * S_{13})$$

$$A_{13} =\mathrm{ArcCos}(S_{23}^2 + S_{12}^2 - S_{13}^2 / 2 * S_{23} * S_{12})$$

$$A_{12} =\mathrm{ArcCos}(S_{13}^2 + S_{23}^2 - S_{12}^2 / 2 * S_{13} * S_{23})$$

“C” PROGRAMMING TO CALCULATE THE FACE ANGLE OF A TETRAHEDRON

```
#include <stdio.h>

#include<math.h>

float dist(float x1, float y1, float z1, float x2, float y2, float z2);

int main()

{

        float x1,y1,z1,x2,y2,z2,x3,y3,z3,x4,y4,z4;

        float A1,B1,C1,D1,A2,B2,C2,D2,A3,B3,C3,D3,A4,B4,C4,D4;

        float d, e1, e2, pi = 3.14159;

        float a1,b1,c1,a2,b2,c2, s12, s13, s23, a12, a13, a23;

        int flag = 1, check;


// Scanning the 4-co-ordinate input from user

        printf("Please enter 4 co-ordinates of the tetrahedron (x,y,z) x 4 : \n");

        scanf("%f%f%f%f%f%f%f%f%f%f%f%f", &x1, &y1, &z1, &x2, &y2, &z2, &x3, &y3,
&z3, &x4, &y4, &z4);
```

```
// Equation of the 4 planes
//Eq of base plane :
a1 = x2 - x1;
   b1 = y2 - y1;
   c1 = z2 - z1;
   a2 = x3 - x1;
   b2 = y3 - y1;
   c2 = z3 - z1;
   A1 = b1 * c2 - b2 * c1;
   B1 = a2 * c1 - a1 * c2;
   C1 = a1 * b2 - b1 * a2;
   D1 = (- A1 * x1 - B1 * y1 - C1 * z1);

   //Eq of second plane :
   a1 = x2 - x1;
   b1 = y2 - y1;
   c1 = z2 - z1;
   a2 = x4 - x1;
   b2 = y4 - y1;
   c2 = z4 - z1;
   A2 = b1 * c2 - b2 * c1;
   B2 = a2 * c1 - a1 * c2;
   C2 = a1 * b2 - b1 * a2;
   D2 = (- A2 * x1 - B2 * y1 - C2 * z1);
```

```
//Eq of third plane :
a1 = x3 - x2;
b1 = y3 - y2;
c1 = z3 - z2;
a2 = x4 - x2;
b2 = y4 - y2;
c2 = z4 - z2;
A3 = b1 * c2 - b2 * c1;
B3 = a2 * c1 - a1 * c2;
C3 = a1 * b2 - b1 * a2;
D3 = (- A3 * x1 - B3 * y1 - C3 * z1); \

//Eq of fourth plane :
a1 = x1 - x3;
b1 = y1 - y3;
c1 = z1 - z3;
a2 = x4 - x3;
b2 = y4 - y3;
c2 = z4 - z3;
A4 = b1 * c2 - b2 * c1;
B4 = a2 * c1 - a1 * c2;
C4 = a1 * b2 - b1 * a2;
D4 = (- A4 * x1 - B4 * y1 - C4 * z1);
```

```
    //Checking if the fourth point satisfies the base plane equation

     check = A1*x4 + B1*y4 + C1*z4 + D1;

    (check == 0)?printf("\nFigure is not a tetrahedral (4th point satisfys the equation of the base plane) ! \n"):printf("\n Figure is a Tetrahedral \n");


    // find Face Angles:


    s12 = dist(x1,y1,z1,x2,y2,z2);

    s13 = dist(x1,y1,z1,x3,y3,z3);

    s23 = dist(x2,y2,z2,x3,y3,z3);


    a12 = acos(((s13*s13) + (s23 * s23) - (s12*s12))/(2*s13*s23));

    a12 = (180/3.142857143)*a12;

    a13 = acos(((s23*s23) + (s12 * s12) - (s13*s13))/(2*s23*s12));

    a13 = (180/3.142857143)*a13;

    a23 = acos(((s12*s12) + (s13 * s13) - (s23*s23))/(2*s12*s13));

    a23 = (180/3.142857143)*a23;


printf("\nThe face angles are : %.2f\t%.2f\t%.2f\n", a12,a13,a23);



     return 0;


}
```

```
float dist(float x1, float y1, float z1, float x2, float y2, float z2)
{
    float m,d;
    m = pow((x2 - x1),2) + pow((y2-y1),2) + pow((z2-z1),2);
    d = sqrt(m);

    return d;
}
```

RESULT

In the following two examples, we have calculated the face angles of a tetrahedron with help of "C" programming where the four co-ordinates of a tetrahedron are given. Here, the co-ordinates of a tetrahedron satisfies some sphere equation.

Example 1:

Points and Their Co-Ordinates

Vertices	x	Y	Z
A	-1	-2	0
B	1	0	-2
C	-1	2	0
D	1	0	2

Face Angles :

Face	1st	*2nd*	*3rd*
ABC	*54.71*	*70.50*	*54.71*

ABD	*54.71*	*54.71*	*70.50*
BCD	*54.71*	*70.50*	*54.71*
ACD	*70.50*	*54.71*	*54.71*

Example 2:

Points And Their Co-Ordinates

Vertices	X	Y	Z
A	*0*	*-0.6*	*-0.8*
B	*0.64*	*-0.024*	*-0.768*
C	*-0.64*	*-0.024*	*-0.768*
D	*0*	*0.0352*	*0.936*

Face Angles :

Face	1st	*2nd*	*3rd*
ABC	*42.01*	*42.01*	*95.90*
ABD	*25.68*	*85.00*	*69.24*
BCD	*40.27*	*69.83*	*69.83*
ACD	*25.68*	*85.00*	*69.24*

CONCLUSION

If all the points of a tetrahedron satisfies a sphere equation, then when the face angles of the triangles of the tetrahedron is less than 90 degree then the circumcentre of those faces lies interior of those faces and the circumcentre of the circumsphere lies interior of the tetrahedron AND when face angles of the triangles of the tetrahedron is greater than 90 degree then the circumcentre of the circumsphere lies exterior of the tetrahedron.

Now from the above two examples we observe that in EXAMPLE 1 the circumcentre of the circumsphere lies interior of the tetrahedron and in EXAMPLE 2 the circumcentre of the circumsphere lies exterior of the tetrahedron.

REFERENCES

[1]. D.Eppstein, J.M.Sullivan and A.Ungor, Tilling space and slabs with acute tetrahedra, ***Computational Gerometry: Theory and Application,*** 27(3) (2004), 237-255

[2]. F.P. Preparate and Michael Ian Shamos, Computational Geometry An Introduction @1985 Springer-Verlag New York.Inc ISBN: 978-81-8489-212-3