ANALYSIS OF SYSTEM RELIABILITY OF AN ILLUMINATED DARK ROOM BY TIT2FNS FUZZY DATA WITH SOME OF ITS ARITHMETIC OPERATIONS

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Abstract:

According to modern view uncertainty is considered essential to science and technology, it is not only the unavoidable plague but also it has impact a great utility. Generally, fuzzy sets are used to analyse fuzzy system reliability. To analyse the fuzzy system reliability, the reliability of each component of the system is considered as a Triangular intuitionistic Type 2 fuzzy number (TIT2FN). At first, TIT2FN and their arithmetic operations are introduced. Expressions for computing the fuzzy reliability of a series system and a parallel system following TIT2FN have been described. Here, an imprecise reliability model of an electric network model of a dark room is taken. To compute the imprecise reliability of the above said system, the reliability of each component of the systems is represented by TIT2FN. A corresponding numerical example is presented.

Keywords: *Triangular intuitionistic Type 2 fuzzy number (TIT2FN), System reliability, Parallel system, Series system.*

1. INTRODUCTION:

1.1 Necessity of reliability:

Reliability is one of the most important attributes of performance in the arrival of the optimal design of a system since it directly and significantly influences the system performance and its life cycle costs. Poor reliability would greatly increase life-cycle costs of the system and reliability based design must be carried out of it the system is to achieve its desired performance. An optimal reliability design is one in which all possible means available to a designer have been explored to enhance the reliability of the system with minimum cost under constrains imposed on the development of a system.

1.2 Review:

1.2.1 Review on reliability:

It is known that conventional reliability analyses using probabilities have been found be inadequate in handling uncertainty of failure data and modelling. To overcome thproblem, Onisawa and Kacprzyk (1995) used the concept of fuzzy approach f evaluation of the reliability of a system. Kaufmann and Gupta (1988) pointed out that the discipline of the reliability engineering encompasses a number of different activities, out of which reliability modelling is the most important activity. For a long period of time, efforts have been made in the design and development of reliability large-scale systems. In that period of time, considerable work has been done by researchers to build a systematic theory of reliability based on the probability theory. Cai et al. (1991) pointed out that there are two fundamental assumptions in the conventional reliability theory, i.e.,

a. **Binary state assumptions:** The system is precisely defined as functioning or failing.

b. **Probability assumptions:** The system behaviour is fuzzy characterised in the context of probability measures.

Because of the inaccuracy and uncertainties of data, the estimation of precise values of probability becomes very difficult in many systems. Cai et al. (1993) introduced system failure engineering and its use of the fuzzy methodology. Cheng and Mon (1993) presented a method for analysing fuzzy system reliability using fuzzy number arithmetic operations. Chen (1994) used interval of confidence for analysing fuzzy system reliability. Singer (1990) presented a fuzzy set approach for fault trees and reliability analysis. Verma et al. (2004) presented dynamic reliability evaluation of deteriorating systems using the concept of probist reliability as a triangular fuzzy number.

1.2.2 Review on type-2 fuzzy number:

Liang and Mendel [2000] had presented the theory and design of interval type-2 fuzzy logic systems (FLSs). They had proposed an efficient and simplified method to compute the input and antecedent operations for interval type-2 FLSs: one that is based on a general inference formula for them. Tao and Jian [2012] was introduced the concept of type-2 intuitionistic fuzzy sets under type-2 fuzzy sets and intuitionistic fuzzy sets. Furthermore, they proved that type-2 intuitionistic fuzzy sets are the generalized forms of type-1 fuzzy sets, intuitionistic fuzzy sets, interval-valued fuzzy sets and interval-valued intuitionistic fuzzy sets. Hu et. al [2013] had proposed a new approach based on possibility degree to solve multi-criteria decision making (MCDM) problems in which the criteria value takes the form of interval type-2 fuzzy number. Mazandarani and Najariyan[2014] had defined a differentiability of the type-2 fuzzy number-valued functions. The definition is based on type-2 Hukuhara difference which is defined in the paper as well. Wang et. al [2015] had applied a new approach is presented for solving multi-criteria group decision-making (MCGDM) problems, which is based on new arithmetic operations and the ranking rules of trapezoidal interval type-2 fuzzy numbers (IT2FNs). Chhibber et. al[2019] had described incentre of centroids has been employed to convert trapezoidal fuzzy transportation problem of type 1 and type-2 both into crisp one, which is easy to approach and is applicable on existent problems of transportation.

Senthil Kumar [2020] had designed a transportation problem in which supplies, demands are crisp numbers and cost is intuitionistic fuzzy number. This type of problem is termed as type-2 intuitionistic fuzzy transportation problem (type-2 IFTP).

1.3 Motivation

The history of reliability field may be traced back to the early 1930's when probability concepts were applied to problems associated with the electric power generation. However, generally the real beginning of reliability field is regarded as World War II, when German applied basic reliability concepts to improve the reliability of their V1 and V2 rockets. Today reliability engineering is well developed discipline and has branched out into specialised areas such as software reliability, mechanical reliability and human reliability. After that many researchers used many ways to solved human reliability problems in fuzzy environment. But in this paper we mainly take a particular topic of human reliability to solve by type-2 fuzzy number.

1.4 Novelties

Some new interest and new work have done by our self which is mentioned below:

i. Try to utilise the properties of Triangular intuitionistic Type 2 fuzzy number to solve a reliability problem.

ii. Described imprecise reliability both of series and parallel systems using Triangular intuitionistic Type 2 fuzzy number.

Frame a problem of reliability of an illuminated dark room with imprecise reliability components by Triangular intuitionistic Type 2 fuzzy number.

We used all the allocation of number in the problem by Triangular intuitionistic type 2 fuzzy number.

2. PRELIMINARIES:

In this section we discuss the definition and arithmetic operation on Triangular intuitionistic Type 2 fuzzy number. After that we have explained the imprecise reliability of series and parallel systems and also give a flow chart given to show the construction part of imprecise reliability both of series and parallel systems.

2.1 Triangular intuitionistic Type 2 fuzzy number

2.1.1. Definition: A TIT2FN $\overset{\sim i}{A}{}^{\psi}_{IFN}$ is an IFN in R with the following membership function $\mu_{\overset{\sim i}{A}{}^{\psi}_{IFN}}\left(\hat{x}\right)$ and non membership function $\nu_{\overset{\sim i}{A}{}^{\psi}_{IFN}}\left(\hat{x}\right)$

$$\mu_{\overset{\sim i}{A}{}^{\psi}_{IFN}}\left(\hat{x}\right)=\begin{cases}\varpi^{\psi}\dfrac{\hat{x}-a^{\psi}{}_{1}}{b^{\psi}{}_{1}-a^{\psi}{}_{1}}, & a^{\psi}{}_{1}\le\hat{x}\le b^{\psi}{}_{1}\\ \varpi^{\psi}, & \hat{x}=b^{\psi}{}_{1}\\ \varpi^{\psi}\dfrac{c^{\psi}{}_{1}-\hat{x}}{c^{\psi}{}_{1}-b^{\psi}{}_{1}}, & b^{\psi}{}_{1}\le\hat{x}\le c^{\psi}{}_{1}\\ 0 & ,otherwise\end{cases}$$

and

$$\nu_{\overset{\sim i}{A}{}^{\psi}_{IFN}}\left(\hat{x}\right)=\begin{cases}\varpi^{\psi}\dfrac{b^{\psi}{}_{1}-\hat{x}}{b^{\psi}{}_{1}-a^{\psi/}{}_{1}}, & a^{\psi/}{}_{1}\le\hat{x}\le b^{\psi}{}_{1}\\ 0, & \hat{x}=b^{\psi}{}_{1}\\ \varpi^{\psi}\dfrac{\hat{x}-b^{\psi}{}_{1}}{c^{\psi/}{}_{1}-b^{\psi}{}_{1}}, & c^{\psi/}{}_{1}\le\hat{x}\le c^{\psi}{}_{1}\\ \varpi^{\psi} & ,otherwise\end{cases}$$

Where $a^{\psi/}_1 < a^{\psi}_1 < b^{\psi}_1 < c^{\psi}_1 < c^{\psi/}_1$ and $\mu_{\tilde{A}^{\psi}_{IFN}}\left(\hat{x}\right), \nu_{\tilde{A}^{\psi}_{IFN}}\left(\hat{x}\right) \leq 0.5$ for $\mu_{\tilde{A}^{\psi}_{IFN}}\left(\hat{x}\right) = \nu_{\tilde{A}^{\psi}_{IFN}}\left(\hat{x}\right) \forall \hat{x} \in \Re$.

This TIT2FN is denoted by $\tilde{A}^{\psi}_{TIT2FN} = \left(a^{\psi}_1, b^{\psi}_1, c^{\psi}_1; \varpi^{\psi}\right)\left(a^{\psi/}_1, b^{\psi}_1, c^{\psi/}_1; \varpi^{\psi}\right)$.

2.2. Some arithmetic operations of Type-2 Intuitionistic Fuzzy Number based on cuts method:

❖ **Properties 1 Multiplication of TIT2FN by crisp number:**

- If TIT2FN $\tilde{A}^{\psi}_{TIT2FN} = \left(a^{\psi}_1, b^{\psi}_1, c^{\psi}_1; \varpi^{\psi}_1\right)\left(a^{\psi/}_1, b^{\psi}_1, c^{\psi/}_1; \varpi^{\psi}_1\right)$ and $\hat{y} = ka^{\psi}\left(k > 0\right)$, then $\tilde{Y}_{TIT2FN} = k\,\tilde{A}^{\psi}_{TIT2FN}$ is a TIT2FN $\left(ka^{\psi}_1, kb^{\psi}_1,, kc^{\psi}_1; \varpi^{\psi}\right)\left(ka^{\psi/}_1, kb^{\psi}_1, kc^{\psi/}_1; \varpi^{\psi}\right)$.
- If $\hat{y} = ka^{\psi}\left(k < 0\right)$, then $\tilde{Y}_{TIT2FN} = k\,\tilde{A}^{\psi}_{TIT2FN}$ is a TT2IFN $\left(kc^{\psi}_1, kb^{\psi}_1, ka^{\psi/}_1; \varpi^{\psi}\right)\left(kc^{\psi}_1, kb^{\psi}_1, ka^{\psi/}_1; \varpi^{\psi}\right)$.

❖ **Properties 2 Addition of two TIT2FN:**

If $\tilde{A}^{\psi}_{TIT2FN} = \left(a^{\psi}_1, b^{\psi}_1, c^{\psi}_1; \varpi^{\psi}_1\right)\left(a^{\psi/}_1, b^{\psi}_1, c^{\psi/}_1; \varpi^{\psi}_1\right)$ and $\tilde{B}^{\psi}_{TIT2FN} = \left(a^{\psi}_2, b^{\psi}_2, c^{\psi}_2; \varpi^{\psi}_2\right)\left(a^{\psi/}_2, b^{\psi}_2, c^{\psi/}_2; \varpi^{\psi}_2\right)$ are two TIT2FN then $\tilde{C}^{\psi}_{TIT2FN} = \tilde{A}^{\psi}_{TIT2FN} \oplus \tilde{B}^{\psi}_{TIT2FN}$ is also TIT2FN.

$$\tilde{A}^{\psi}_{TIT2FN} \oplus \tilde{B}^{\psi}_{TIT2FN} = \left(a^{\psi}_1 + a^{\psi}_2, b^{\psi}_1 + b^{\psi}_2, c^{\psi}_1 + c^{\psi}_2; \varpi^{\psi}\right)\left(a^{\psi/}_1 + a^{\psi/}_2, b^{\psi}_1 + b^{\psi}_2, c^{\psi/}_1 + c^{\psi/}_2; \varpi^{\psi}\right)$$

Where $0 < \varpi^{\psi} \leq 1; \varpi^{\psi} = \min\left(\varpi^{\psi}_1, \varpi^{\psi}_2\right)$

❖ **Properties 3 Subtraction of two TIT2FN:**

If $\tilde{A}^{\psi}_{TIT2FN} = \left(a^{\psi}_1, b^{\psi}_1, c^{\psi}_1; \varpi^{\psi}_1\right)\left(a^{\psi/}_1, b^{\psi}_1, c^{\psi/}_1; \varpi^{\psi}_1\right)$ and $\tilde{B}^{\psi}_{TIT2FN} = \left(a^{\psi}_2, b^{\psi}_2, c^{\psi}_2; \varpi^{\psi}_2\right)\left(a^{\psi/}_2, b^{\psi}_2, c^{\psi/}_2; \varpi^{\psi}_2\right)$ are two TIT2FN then $\tilde{C}^{\psi}_{TIT2FN} = \tilde{A}^{\psi}_{TIT2FN}\, \Theta\, \tilde{B}^{\psi}_{TIT2FN}$ is also TIT2FN.

$$\tilde{A}^{\psi}_{TIT2FN} \Theta \tilde{B}^{\psi}_{TIT2FN} = \left(a^{\psi}{}_1 - a^{\psi}{}_2, b^{\psi}{}_1 - b^{\psi}{}_2, c^{\psi}{}_1 - c^{\psi}{}_2; \varpi^{\psi}\right)\left(a^{\psi/}{}_1 - a^{\psi/}{}_2, b^{\psi}{}_1 - b^{\psi}{}_2, c^{\psi/}{}_1 - c^{\psi/}{}_2; \varpi^{\psi}\right)$$

Where $0 < \varpi^{\psi} \leq 1; \varpi^{\psi} = \min\left(\varpi^{\psi}_1, \varpi^{\psi}_2\right)$

- **Properties 4 Multiplication of two TIT2FN:**

If $\tilde{A}^{\psi}_{TIT2FN} = \left(a^{\psi}{}_1, b^{\psi}{}_1, c^{\psi}{}_1; \varpi^{\psi}_1\right)\left(a^{\psi/}{}_1, b^{\psi}{}_1, c^{\psi/}{}_1; \varpi^{\psi}_1\right)$ and $\tilde{B}^{\psi}_{TIT2FN} = \left(a^{\psi}{}_2, b^{\psi}{}_2, c^{\psi}{}_2; \varpi^{\psi}_2\right)\left(a^{\psi/}{}_2, b^{\psi}{}_2, c^{\psi/}{}_2; \varpi^{\psi}_2\right)$ are two TIT2FN then $\tilde{P}^{\psi}_{TIT2FN} = \tilde{A}^{\psi}_{TIT2FN} \otimes \tilde{B}^{\psi}_{TIT2FN}$ is an approximated TIT2FN.

$$\tilde{A}^{\psi}_{TIT2FN} \otimes \tilde{B}^{\psi}_{TIT2FN} = \left(a^{\psi}{}_1 a^{\psi}{}_2, b^{\psi}{}_1 b^{\psi}{}_2, c^{\psi}{}_1 c^{\psi}{}_2; \varpi^{\psi}\right)\left(a^{\psi/}_1 a^{\psi/}_2, b^{\psi}{}_1 b^{\psi}{}_2, c^{\psi/}_1 c^{\psi/}_2; \varpi^{\psi}\right)$$

Where $0 < \varpi^{\psi} \leq 1; \varpi^{\psi} = \min\left(\varpi^{\psi}_1, \varpi^{\psi}_2\right)$

- **Properties 5 Division of two TIT2FN:**

If $\tilde{A}^{\psi}_{TIT2FN} = \left(a^{\psi}{}_1, b^{\psi}{}_1, c^{\psi}{}_1; \varpi^{\psi}_1\right)\left(a^{\psi/}{}_1, b^{\psi}{}_1, c^{\psi/}{}_1; \varpi^{\psi}_1\right)$ and $\tilde{B}^{\psi}_{TIT2FN} = \left(a^{\psi}{}_2, b^{\psi}{}_2, c^{\psi}{}_2; \varpi^{\psi}_2\right)\left(a^{\psi/}{}_2, b^{\psi}{}_2, c^{\psi/}{}_2; \varpi^{\psi}_2\right)$ are two TIFN then $\tilde{P}^{\psi}_{TIT2FN} = \tilde{A}^{\psi}_{TIT2FN} \div \tilde{B}^{\psi}_{TIT2FN}$ is an approximated TIT2FN.

$$\tilde{A}^{\psi}_{TIT2FN} \div \tilde{B}^{\psi}_{TIT2FN} = \left(a^{\psi}{}_1 / a^{\psi}{}_2, b^{\psi}{}_1 / b^{\psi}{}_2, c^{\psi}{}_1 / c^{\psi}{}_2; \varpi^{\psi}\right)\left(a^{\psi/}{}_1 / a^{\psi/}{}_2, b^{\psi}{}_1 / b^{\psi}{}_2, c^{\psi/}_1 / c^{\psi/}_2; \varpi^{\psi}\right)$$

Where $0 < \varpi^{\psi} \leq 1; \varpi^{\psi} = \min\left(\varpi^{\psi}_1, \varpi^{\psi}_2\right)$

2.3 Imprecise reliability of series and parallel systems using arithmetic operations or Triangular Intuitionistic Type2 Fuzzy Numbers:

The imprecise reliability of a series and a parallel system present here. Triangular Intuitionistic Type2 Fuzzy numbers are used to represent the reliability of each component of the systems.

- **Series system**

Let us consider a series system consisting of n components. Here we want to show the intuitionistic fuzzy reliability of $\tilde{R}^{\tau_{ss}}_{ss}$. This can be evaluated by using the expression as follows:

$$\tilde{R}^{i^{/}\tau_{ss}}_{ss} = \tilde{R}^{i^{/}\tau_{ss}}_{1} \otimes \tilde{R}^{i^{/}\tau_{ss}}_{2} \otimes ....\otimes \tilde{R}^{i^{/}\tau_{ss}}_{n} = \left\{ \left(\hat{r}_{11}, \hat{r}_{12}, \hat{r}_{13}; \omega_1 \right)\left(\hat{r}^{/}_{11}, \hat{r}_{12}, \hat{r}^{/}_{13}; \omega_1 \right) \otimes \left(\hat{r}_{21}, \hat{r}_{22}, \hat{r}_{23}; \omega_2 \right)\left(\hat{r}^{/}_{21}, \hat{r}_{22}, \hat{r}^{/}_{23}; \omega_2 \right) \right.$$

$$\left. \otimes ....\otimes \left(\hat{r}_{n1}, \hat{r}_{n2}, \hat{r}_{n3}; \omega_n \right)\left(\hat{r}^{/}_{n1}, \hat{r}_{n2}, \hat{r}^{/}_{n3}; \omega_n \right) \right\}$$

The approximated to a TIT2FN can be written as;

$$\cong \left(\prod_{j^{/}=1}^{n} \hat{r}_{j^{/}1}, \prod_{j^{/}=1}^{n} \hat{r}_{j^{/}2}, \prod_{j^{/}=1}^{n} \hat{r}_{j^{/}3}; \omega \right)\left(\prod_{j^{/}=1}^{n} \hat{r}^{/}_{j^{/}1}, \prod_{j^{/}=1}^{n} \hat{r}_{j^{/}2}, \prod_{j^{/}=1}^{n} \hat{r}^{/}_{j^{/}3}; \omega \right)$$

Where $\tilde{R}^{i^{/}\tau_{ss}}_{j^{/}} = \left(\hat{r}_{j^{/}1}, \hat{r}_{j^{/}2}, \hat{r}_{j^{/}3}; \omega \right)\left(\hat{r}^{/}_{j^{/}1}, \hat{r}_{j^{/}2}, \hat{r}^{/}_{j^{/}3}; \omega \right)$ represent the intuitionistic fuzzy reliability of the jth component for $j^{/}=1,2,......,n$.

- **Parallel system:**

Let us consider a parallel system consisting of n components. The evaluation of fuzzy reliability $\tilde{R}^{i\,\tau_{ps}}_{ps}$ of the parallel system is described by the following expression as follows:

$$\tilde{R}^{i^{/}\tau_{ps}}_{ps} = 1\Theta \prod_{j^{/}=1}^{n} \left(1 - \tilde{R}^{i^{/}\tau_{ps}}_{j^{/}} \right) = 1\Theta \left[\left(1\Theta \left(\hat{r}_{11}, \hat{r}_{12}, \hat{r}_{13}; \omega_1 \right)\left(\hat{r}^{/}_{11}, \hat{r}_{12}, \hat{r}^{/}_{13}; \omega_1 \right) \right) \otimes .... \right.$$

$$\left. \left(1\Theta \left(\hat{r}_{n1}, \hat{r}_{n2}, \hat{r}_{n3}; \omega_n \right)\left(\hat{r}^{/}_{n1}, \hat{r}_{n2}, \hat{r}^{/}_{n3}; \omega_n \right) \right) \right]$$

It is an approximated to a TIT2FN, Where $\tilde{R}^{i\,\tau_{ps}}_{j^{/}} = \left(\hat{r}_{j^{/}1}, \hat{r}_{j^{/}2}, \hat{r}_{j^{/}3}; \omega \right)\left(\hat{r}^{/}_{j^{/}1}, \hat{r}_{j^{/}2}, \hat{r}^{/}_{j^{/}3}; \omega \right)$ is an intuitionistic fuzzy reliability of the jth component for $j^{/}=1,2,......,n$.

- **Parallel-series System :**

Consider a parallel-series system consisting of m branches connected in parallel. The evaluation of fuzzy reliability $\tilde{R}^{i\,\tau_{pss}}_{pss}$ of the parallel-series system can be evaluated as follows:

$$\tilde{R}_{pss}^{\tau_{pss}} =$$
$$\left[1-\prod_{k'=1}^{m}\left(1-\left(\prod_{i'=1}^{n} x_{k'i'}\right)\right),1-\prod_{k'=1}^{m}\left(1-\left(\prod_{i=1}^{n} y_{k'i'}\right)\right),1-\prod_{k'=1}^{m}\left(1-\left(\prod_{i=1}^{n} z_{k'i'}\right)\right);\omega\right]\left[1-\prod_{k'=1}^{m}\left(1-\left(\prod_{i'=1}^{n} x'_{k'i'}\right)\right),1-\prod_{k'=1}^{m}\left(1-\left(\prod_{i'=1}^{n} y_{k'i'}\right)\right),1-\prod_{k'=1}^{m}\left(1-\left(\prod_{i'=1}^{n} z'_{k'i'}\right)\right);\omega\right]$$

Where $\tilde{R}_{i'k'}^{\tau_{sps}} = \left(x_{i'k'}, y_{i'k'}, z_{i'k'};\omega\right)\left(x'_{i'k'}, y_{i'k'}, z'_{i'k'};\omega\right)$ represents the reliability of the ith component at kth branch.

- **Series-parallel System:**

Consider a series-parallel system consisting of n stages connected in series and each stage contains m components. The fuzzy reliability $\tilde{R}_{sps}^{\tau_{sps}}$ of the series-parallel system can be evaluated as follows:

$$\tilde{R}_{sps}^{\tau_{sps}} =$$
$$\left[\left(\prod_{k'=1}^{n}\left(1-\left(\prod_{i'=1}^{m} x_{i'k'}\right)\right),\prod_{k'=1}^{n}\left(1-\left(\prod_{i'=1}^{m} y_{i'k'}\right)\right),\prod_{k'=1}^{n}\left(1-\left(\prod_{i'=1}^{m} z_{i'k'}\right)\right)\right);\omega\right]\left[\left(\prod_{k'=1}^{n}\left(1-\left(\prod_{i'=1}^{m} x'_{i'k'}\right)\right),\prod_{k'=1}^{n}\left(1-\left(\prod_{i'=1}^{m} y_{i'k'}\right)\right),\prod_{k'=1}^{n}\left(1-\left(\prod_{i'=1}^{m} z'_{i'k'}\right)\right)\right);\omega\right]$$

Where $\tilde{R}_{i'k'}^{\tau_{sps}} = \left(x_{i'k'}, y_{i'k'}, z_{i'k'};\omega\right)\left(x'_{i'k'}, y_{i'k'}, z'_{i'k'};\omega\right)$ represent the reliability of the ith component at kth stage.

3. APPLICATION WITH DISCUSSION:

Reliability of an illuminated dark room with imprecise reliability components

An electric network model presented by Dhillon (2007) of a dark room is considered A windowless room has a switch and four light bulbs. Develop a success tree for the desired event (i.e., the top event): an illuminated dark room. Thus, in this case, the room can only be brightened if there is availability of electricity, the switch is OK, and all four light bulbs are not burnt out. Each success event in the success tree diagram is considered as TIT2FN.

$\tilde{R}_1$ represents the reliability of no fuse failure of electric supply of the dark room

$\tilde{R}_2$ represents the reliability of no power failure of electric supply of the dark room

$\tilde{R}_3$ represents the reliability of switch of the dark room

$\tilde{R}_4$ represents the reliability of bulb no. A of the dark room

$\tilde{R}_5$ represents the reliability of bulb no. B of the dark room

$\tilde{R}_6$ represents the reliability of bulb no. C of the dark room

$\tilde{R}_7$ represents the reliability of bulb no. D of the dark room

$\tilde{R}_8$ represents the reliability of electric supply of the dark room

$\tilde{R}_9$ represents the resultant reliability of bulbs connecting in parallel

$\tilde{R}_{10}$ represents the reliability of the desired event (i.e. illustrated dark room).

All imprecise components' reliability $\tilde{R}_j$ are represented by TIT2FN $\left(\hat{r}_{j'1},\hat{r}_{j'2},\hat{r}_{j'3};\omega\right)\left(\hat{r}'_{j'1},\hat{r}_{j'2},\hat{r}'_{j'3};\omega\right)$ for j = 1, 2, 3,......,7. Let us calculate the reliability of occurrence of the desired event (illuminated dark room).

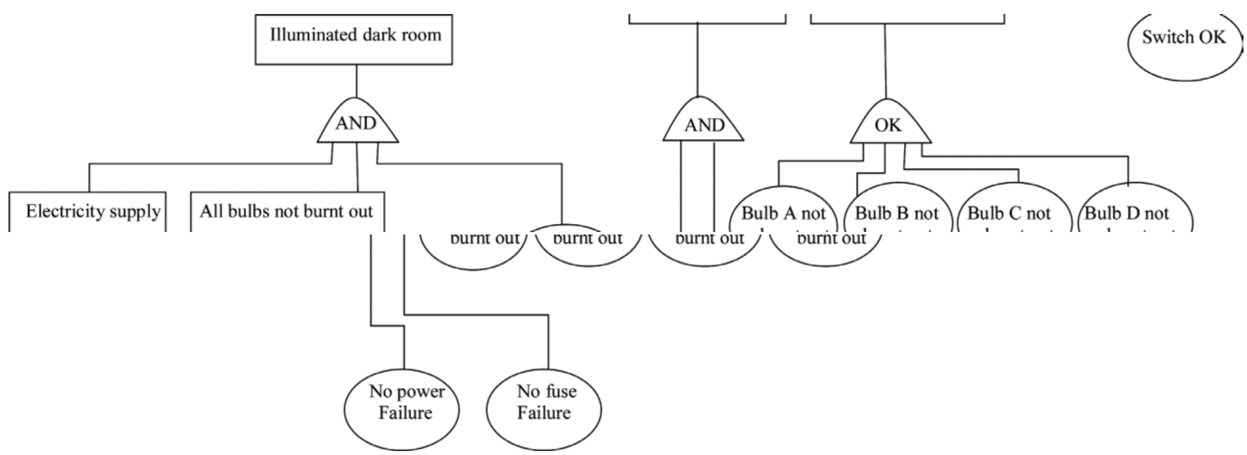

Figure 1 Success tree for the top event illuminated dark room

The reliability value, for the occurrence of event electricity supply,

$$\tilde{R}_8 = \tilde{R}_1 \otimes \tilde{R}_2 \cong \left(\hat{r}_{11},\hat{r}_{12},\hat{r}_{13};\omega_1\right)\left(\hat{r}'_{11},\hat{r}_{12},\hat{r}'_{13};\omega_1\right) \otimes \left(\hat{r}_{21},\hat{r}_{22},\hat{r}_{23};\omega_2\right)\left(\hat{r}'_{21},\hat{r}_{22},\hat{r}'_{23};\omega_2\right)$$

$$\cong \left(\hat{r}_{11}\hat{r}_{21},\hat{r}_{12}\hat{r}_{22},\hat{r}_{13}\hat{r}_{23};\omega\right)\left(\hat{r}'_{11}\hat{r}'_{21},\hat{r}_{12}\hat{r}_{22},\hat{r}'_{13}\hat{r}'_{23};\omega\right);\varpi = \min\left(\varpi_1,\varpi_2\right) \qquad \text{............(A)}$$

It is approximated to a TIT2FN as follows

$$\tilde{R}_9 \cong \left(1-\prod_{j=4}^{7}\left(1-r_{j1}\right),1-\prod_{j=4}^{7}\left(1-r_{j2}\right),1-\prod_{j=4}^{7}\left(1-r_{j3}\right),\omega\right)\left(1-\prod_{j=4}^{7}\left(1-r'_{j1}\right),1-\prod_{j=4}^{7}\left(1-r_{j2}\right),1-\prod_{j=4}^{7}\left(1-r'_{j3}\right),\omega\right) \quad ....(B)$$

By substituting the above two calculated values and the given data value, we get the reliability value for the occurrence of the top event, illuminated dark room, $\tilde{R}_{10}$;

$$\tilde{R}_{10} = \tilde{R}_1 \otimes \tilde{R}_2 \otimes \tilde{R}_3 \otimes \tilde{R}_4 \otimes \tilde{R}_5 \otimes \tilde{R}_6 \otimes \tilde{R}_7 \otimes \tilde{R}_8 \otimes \tilde{R}_9$$

It is approximated to a TIT2FN as follows;

$$\tilde{R}_{10} \cong$$

$$\left(r_{11}r_{21}r_{31}r_{41}\left(1-\prod_{j=4}^{7}\left(1-r_{j1}\right)\right),r_{12}r_{22}r_{32}r_{42}\left(1-\prod_{j=4}^{7}\left(1-r_{j2}\right)\right),r_{13}r_{23}r_{33}r_{43}\left(1-\prod_{j=4}^{7}\left(1-r_{j3}\right)\right),\omega\right)$$

$$\left(r'_{11}r'_{21}r'_{31}r'_{41}\left(1-\prod_{j=4}^{7}\left(1-r_{j1}\right)\right),r_{12}r_{22}r_{32}r_{42}\left(1-\prod_{j=4}^{7}\left(1-r_{j2}\right)\right),r'_{13}r'_{23}r'_{33}r'_{43}\left(1-\prod_{j=4}^{7}\left(1-r_{j3}\right)\right),\omega\right)$$

$$\omega = \min\left(\omega_i; i=1,2,3,4,5,6,7,8,9\right) \quad ....(C)$$

Let the reliability of the events be

$\tilde{R}_1 = (0.70,0.75,0.85;0.2)(0.65,0.75,0.90;0.2)$

$\tilde{R}_2 = (0.80,0.85,0.95;0.3)(0.75,0.85,0.92;0.3)$

$\tilde{R}_3 = (0.82,0.85,0.91;0.5)(0.80,0.85,0.93;0.5)$

$\tilde{R}_4 = (0.84,0.86,0.90;0.4)(0.82,0.86,0.92;0.4)$

$\tilde{R}_5 = (0.85,0.86,0.89;0.6)(0.83,0.86,0.91;0.6)$

$\tilde{R}_6 = (0.89,0.92,0.96;0.3)(0.87,0.92,0.97;0.3)$

$\tilde{R}_7 = (0.80,0.82,0.86;0.4)(0.75, 0.82,0.89;0.4)$

So, the results of equations **(A)** and **(B)** are as follows:

$\tilde{R}_8 = (0.55,0.6375,0.8075;0.2)(0.4875,0.6375,0.829;0.2)$

$\tilde{R}_9 = (0.999412,0.99971715,0.9999384;0.3)(0.9990055,0.99971715,0.99997624;0.3)$

By substituting these above two calculated values in the given data value in equation **(C),** the reliability value for the occurrence of the top event, dark room with light, $\tilde{R}_{10}$ is;

$$\tilde{R}_{10} = (0.450134812, 0.5417217, 0.7347797; 0.2)(0.3896121, 0.5417217, 0.7709517; 0.2)$$

4. CONCLUSION:

In this paper, a definition of TIT2FN is proposed. Some arithmetic operations of the proposed TIT2FN are also evaluated. Here, a method to analyse system reliability, which is based on the IFS theory, has been presented, where the components of the system are represented by TIT2FN. Some arithmetic operations over the TIT2FN are used to analyse the fuzzy reliability of the series system and the parallel system. An intuitionistic type-2 fuzzy success tree is used to analyse the imprecise reliability of a dark room with the desired event. In an imprecise situation, several real life operations research and scientific models may be described and evaluated by a TIT2FN. The major advantage of using IFSs over fuzzy sets is that IFSs separate the positive and the negative evidence for the membership of an element in a set. Our approaches and computational procedures may be efficient and simple to implement for calculation in an intuitionistic type-2 fuzzy environment for all fields of engineering and science where impreciseness occur. However in the future instead of taking component reliability as TIT2FN other more general fuzzy numbers like TrIT2FN may be used to evaluate system reliability.

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