# COMPARISON OF NUMERICAL METHODS IN PREDICTING THE GROWTH OF NORMAL AGRICULTURAL ASSETS

**Godspower C. Abanum**

*Department of Mathematics/Statistics,*

*Ignatius Ajuru University of Education, Port Harcourt, Nigeria*

*Corresponding Author: godspower.abanum@iaue.edu.ng*

## Abstract

In this paper, I considered the comparison of numerical methods in predicting the biodiversity gain due to the variation of $\alpha_1$ and $\alpha_2$ together on biodiversity scenario. However, when the model parameter values $\alpha_1$ and $\alpha_2$ are increase, the normal agricultural variable also changes. By comparing the patterns of growth in these two interacting normal agricultural data, we have finite instance of biodiversity due to the application of four numerical methods such as ODE45, ODE23, ODE23tb and ODE15s. We have found the numerical prediction upon using these four numerical methods which are similar and robust, hence we have considered ODE45 numerical simulation to be computationally more efficient than the other three methods. The novel result we have obtained in this study have not been seen elsewhere.

**Keywords:** *ODE45, ODE23, ODE23tb, ODE15s, Normal Agriculture*

## 1. INTRODUCTION

Modeling and simulation of dynamical systems has always been an important issue in Science, Engineering, Biological sector, Medical sector and Agricultural sector. Most physical phenomena such as biological model, Agricultural model, fluid dynamics, control theory, e.t.c are often transformed into ordinary differential equation to give an explicit interpretation of the physical attributes in the model equation. The effectiveness of these equations in the world of modeling has prompted researchers in developing methods for seeking the solutions to these equations.

Modeling is the bridge between the subject and real-life situation for student realization. Differential equations model real-life situations and provide real-life answers with the help of computer calculations and mathematical simulations ([8],[9]). The impact of agricultural assets in the development of nation economy depends on the provision of more foods to the expanding population, increasing the demands for industrial assets, supplying additional foreign exchange earnings for import of capital goods, increasing rural incomes to be mobilized by the state, giving productive employment and improve the well-being of the rural people ([30]). Agriculture increases and generates employment opportunities in the rural areas. As the product in agriculture increase, the rural employment opportunity expands. The notion to apply ordinary differential systems or equations based on system dynamics to investigate the interactions between ecospheric assets, industrial assets and agricultural wealth or assets begins in 1994 with the pioneering work of Apedaile et al. It was then followed by the work of Solomon ovich et al ([6], [1], [2]). [1] model the interaction between auxiliary agricultural, industry and ecosphere using a system of ordinary differential equation for non-linear case, they studied the long-term effect of each of the assets on each other, local,global analysis of equilibra of systems and dissipativity solutions are checked and the conditions interior equilibrium using the theory of uniform persistence were established. The condition for extinctions and local persistence to occur were proved. ([31]) model the impact of diseases and pest on agricultural systems they provided a brief overview of the recent state of development in coupling disease and pest models and explained the scientific and technical challenges. They proposed a five-stage road-map to improve the simulation of the impacts caused by plant pest and diseases. ([4],[5]) proposed a computational and mathematical modelling of ecosphere. They applied the numerical method of ordinary differential equations of order 45 called ODE45 numerical simulation. A high volume of biodiversity gain of ecospheric assets was presented. From the result shown, it is clear that there is a biodiversity gain of ecospheric assets when the per asset degradation rate coefficient of the ecospheric has the volume of 0.10, 0.20 and 0.30. the results also that the larger the per asset degradation rate coefficient of the ecospheric asset, the smaller the opportunity of getting a good gain in biodiversity. In the same manner, the smaller the per asset degradation rate coefficient of ecospheric asset, the bigger the volume which measures the biodiversity gain. They also worked on biodiversity scenario of ecospheric assets using Kalabari Kingdom as a case study. A method of a semi-stochastic analysis was used in the research at the variation of a low, a mild and a relatively high environmental perturbation otherwise called the per asset degradation rate of ecospheric assets to check and predict the level of biodiversity. The results obtained shows that there is change from a biodiversity gain to a biodiversity loss with respect to the growth of ecospheric assets. The work was done to predict the effect of per asset degradation rate coefficient of ecospheric asets with the inclusion of an additive semi-stochastic random perturbation on biodiversity scenarios. Other related contributions to knowledge in the context of environmental modelling of the interaction of ecosphere, industry and agriculture can be seen in the work of [1-31]

## 2. MODEL ASSUMPTIONS:

The model formulation in the sequel are defined as follows:

- The growth of the normal agricultural assets over time between the ecospheric assets and the normal agricultural assets provides an enhancing factor hereby called the growth rate coefficient of normal agricultural activity. Improving the production capacity of agriculture in developing countries through productivity increase over time is an important policy goal where agriculture represents an important asset in economy.
- Thus, a growing normal agricultural assets contributes to both overall growth and poverty alleviation.
- The growth of the normal agricultural assets also depends on the self-interaction between normal agricultural assets and itself providing an imparting factor hereby called the per assets diminishing returns coefficient for normal agriculture and the auxiliary agriculture.

- The growth of the normal agricultural assets depends on the positive interaction between the normal agricultural assets and the industrial assets in which the industrial assets contributes the per assets terms of the trade coefficient between normal agriculture and industry. The interdependence of normal agricultural assets and industrial assets help the development of both the assets. The most important aspect of this inter dependence is that the product of one serve as important inputs for the other. Growth of one assets, thus means large supply of inputs for others. The situation such that a greater is such that a greater flow of products from one asset to the other simultaneously ensures a greater return flow of inputs itself, though with some time lag.
- The growth of normal and agricultural assets and auxiliary agricultural assets, the auxiliary agriculture compete with the normal agriculture which is called the per asset competitive rate coefficient of auxiliary agriculture acting on normal agriculture.
- The growth rate of normal agricultural assets depends the interaction between the net cost rates to normal agriculture to restore the ecosphere.

## 3. MATHEMATICAL FORMULATION

Following Ibrahim A and Freedom H.I (2009), we have consider the four system of nonlinear ordinary differential equation.

$$\frac{dx_1(t)}{dt} = \alpha_1 x_1 z - \beta_1 x_1^2 + \gamma_1 x_1 y - \rho_1 x_1 x_2 - \theta x_1 + \theta x_1 z$$

$$\frac{dx_2(t)}{dt} = \alpha_2 x_2 z - \beta_2 x_2^2 - \gamma_2 x_2 y - \rho_2 x_1 x_2$$

$$\frac{dy(t)}{dt} = -\xi y - \eta y^2 + \delta x_1 y$$

$$\frac{dz(t)}{dt} = -\kappa x_1 z + \kappa_1 z - \kappa_1 z^2 + \kappa_2 x_1 - \kappa_2 x_1 z$$

where all parameters are assumed to be positive constants except $\gamma_1$ which can be any real constant. $\alpha_1(\alpha_2)$ is inter competitive growth rate coefficient of normal (auxiliary) agriculture due to normal (auxiliary) agricultural activity for fixed *z*; $\beta_1(\beta_2)$ is the per asset diminishing returns rate coefficient for normal (auxiliary) agriculture in the absence of industry and auxiliary (normal) agriculture, $\gamma_1(\gamma_2)$ is the per asset terms of trade coefficient between normal (auxiliary) agriculture and industry, $\xi$ is the constant depreciation rate coefficient of industry, $\eta$ is the per asset (linear) depreciation rate coefficient of industry, $\delta$ is the per asset growth rate for industry in dealing with normal agriculture, $\rho_1(\rho_2)$ is the per asset competitive rate coefficient of auxiliary (normal) agriculture acting on normal (auxiliary) agriculture, $\kappa$ is the per asset degradation rate coefficient of the ecosphere due to normal agricultural activities, $\kappa_1$ is the natural restoration rate coefficient for the ecosphere, $\kappa_2$ is the rate of effort input to restore the ecosphere by normal agriculture and $\theta$ is the net cost rate to normal agriculture to restore the ecosphere.

With the following precise model parameter values from Agyemang and Freedman (2009)

$$\alpha_1 = 3, \alpha_2 = 1\,, \beta_1 = \frac{1}{10}, \beta_2 = \frac{1}{10}, \gamma_1 = -\frac{1}{49}, \gamma_2 = \frac{1}{10}, \rho_1 = \frac{1}{10}, \rho_2 = \frac{1}{5}, \delta = \frac{1}{4},$$
$$\theta = \frac{6}{5}, \eta = \frac{1}{20}, \xi = 1, \kappa = 2, \kappa_1 = 2, \kappa_2 = 1$$

## 4. METHOD OF ANALYSIS

To tackle this environmental problem, we have fully explore the application of four numerical methods, namely ODE45, ODE23, ODE15s, ODE23tb to model and predict the effect of increasing $\alpha_1$ *and* $\alpha_2$ by 101%, 102%, 103%, 104% and 105% on a biodiversity gain scenario for a fixed initial condition consisting of a point of values.

## 5. RESULT

On the application of the above mention four numerical methods, we have obtain the following novel empirical results which are presented as displayed as shown in table 1.1- table 4.5

**Table 1.1: Quantifying the effects of increasing $\alpha_1$ *and* $\alpha_2$ together by 101% on biodiversity gain of a normal Agriculture using ODE45 numerical method**

| Example | LDS TIME (WEEKLY) | $x_1(old)$ | $x_1(new)$ | EBG(%) |
|---|---|---|---|---|
| 1 | 1 | 1.0000 | 1.0000 | 0 |
| 2 | 8 | 3.1975 | 3.2735 | 2.3739 |
| 3 | 15 | 3.6804 | 3.7598 | 2.1568 |
| 4 | 22 | 3.7410 | 3.8160 | 2.0059 |
| 5 | 29 | 3.7470 | 3.8218 | 1.9944 |
| 6 | 36 | 3.7485 | 3.8235 | 1.9983 |
| 7 | 43 | 3.7490 | 3.8239 | 1.9977 |
| 8 | 50 | 3.7492 | 3.8242 | 2.0005 |
| 9 | 57 | 3.7499 | 3.8247 | 1.9963 |
| 10 | 64 | 3.7498 | 3.8251 | 2.0086 |
| 11 | 71 | 3.7501 | 3.8257 | 2.0165 |
| 12 | 78 | 3.7499 | 3.8255 | 2.0161 |

| | | | | |
|---|---|---|---|---|
| 13 | 85 | 3.7503 | 3.8255 | 2.0047 |
| 14 | 92 | 3.7500 | 3.8257 | 2.0192 |
| 15 | 99 | 3.7498 | 3.8257 | 2.0232 |
| 16 | 106 | 3.7499 | 3.8259 | 2.0269 |
| 17 | 113 | 3.7502 | 3.8257 | 2.0138 |
| 18 | 120 | 3.7501 | 3.8260 | 2.0240 |
| 19 | 127 | 3.7499 | 3.8259 | 2.0257 |
| 20 | 134 | 3.7497 | 3.8257 | 2.0293 |
| 21 | 141 | 3.7501 | 3.8258 | 2.0204 |

LGS= Length of growing season

$x_1(old)$= Measures the normal agricultural volume when all model parameters are fixed at 100%

$x_1(new)$= Measures the normal agricultural volume when $\alpha_1$ $and$ $\alpha_2$ only are varied.

EBL= Estimated Biodiversity Gain

**Table 1.2: Quantifying the effects of increasing $\alpha_1$ $and$ $\alpha_2$ together by 102% on biodiversity gain of a normal Agriculture using ODE45 numerical method**

| Example | LDS TIME (WEEKLY) | $x_1(old)$ | $x_1(new)$ | EBG(%) |
|---|---|---|---|---|
| 1 | 1 | 1.0000 | 1.0000 | 0 |
| 2 | 8 | 3.1975 | 3.3511 | 4.8030 |
| 3 | 15 | 3.6804 | 3.8392 | 4.3133 |
| 4 | 22 | 3.7410 | 3.8916 | 4.0254 |
| 5 | 29 | 3.7470 | 3.8965 | 3.9901 |
| 6 | 36 | 3.7485 | 3.8975 | 3.9729 |
| 7 | 43 | 3.7490 | 3.8985 | 3.9869 |
| 8 | 50 | 3.7492 | 3.8995 | 4.0076 |
| 9 | 57 | 3.7499 | 3.8998 | 3.9971 |
| 10 | 64 | 3.7498 | 3.9005 | 4.0197 |

| | | | | |
|---|---|---|---|---|
| 11 | 71 | 3.7501 | 3.9002 | 4.0027 |
| 12 | 78 | 3.7499 | 3.9008 | 4.0254 |
| 13 | 85 | 3.7503 | 3.9010 | 4.0188 |
| 14 | 92 | 3.7500 | 3.9014 | 4.0370 |
| 15 | 99 | 3.7498 | 3.9014 | 4.0435 |
| 16 | 106 | 3.7499 | 3.9013 | 4.0376 |
| 17 | 113 | 3.7502 | 3.9020 | 4.0489 |
| 18 | 120 | 3.7501 | 3.9020 | 4.0507 |
| 19 | 127 | 3.7499 | 3.9023 | 4.0642 |
| 20 | 134 | 3.7497 | 3.9020 | 4.0638 |
| 21 | 141 | 3.7501 | 3.9017 | 4.0444 |

**Table 1.3: Quantifying the effects of increasing $\alpha_1$ $and$ $\alpha_2$ together by 103% on biodiversity gain of a normal Agriculture using ODE45 numerical method**

| Example | LDS TIME (WEEKLY) | $x_1(old)$ | $x_1(new)$ | EBG(%) |
|---|---|---|---|---|
| 1 | 1 | 1.0000 | 1.0000 | 0 |
| 2 | 8 | 3.1975 | 3.4297 | 7.2607 |
| 3 | 15 | 3.6804 | 3.9186 | 6.4706 |
| 4 | 22 | 3.7410 | 3.9667 | 6.0325 |
| 5 | 29 | 3.7470 | 3.9703 | 5.9587 |
| 6 | 36 | 3.7485 | 3.9709 | 5.9324 |
| 7 | 43 | 3.7490 | 3.9718 | 5.9426 |
| 8 | 50 | 3.7492 | 3.9722 | 5.9475 |
| 9 | 57 | 3.7499 | 3.9724 | 5.9355 |
| 10 | 64 | 3.7498 | 3.9730 | 5.9523 |
| 11 | 71 | 3.7501 | 3.9737 | 5.9614 |

| | | | | |
|---|---|---|---|---|
| 12 | 78 | 3.7499 | 3.9740 | 5.9757 |
| 13 | 85 | 3.7503 | 3.9743 | 5.9735 |
| 14 | 92 | 3.7500 | 3.9752 | 6.0060 |
| 15 | 99 | 3.7498 | 3.9749 | 6.0033 |
| 16 | 106 | 3.7499 | 3.9750 | 6.0036 |
| 17 | 113 | 3.7502 | 3.9754 | 6.0054 |
| 18 | 120 | 3.7501 | 3.9760 | 6.0237 |
| 19 | 127 | 3.7499 | 3.9757 | 6.0214 |
| 20 | 134 | 3.7497 | 3.9760 | 6.0359 |
| 21 | 141 | 3.7501 | 3.9760 | 6.0247 |

**Table 1.4: Quantifying the effects of increasing $\alpha_1$ $and$ $\alpha_2$ together by 104% on biodiversity gain of a normal Agriculture using ODE45 numerical method**

| Example | LDS TIME (WEEKLY) | $x_1(old)$ | $x_1(new)$ | EBG(%) |
|---|---|---|---|---|
| 1 | 1 | 1.0000 | 1.0000 | 0 |
| 2 | 8 | 3.1975 | 3.5089 | 9.7380 |
| 3 | 15 | 3.6804 | 3.9982 | 8.6332 |
| 4 | 22 | 3.7410 | 4.0408 | 8.0126 |
| 5 | 29 | 3.7470 | 4.0428 | 7.8938 |
| 6 | 36 | 3.7485 | 4.0424 | 7.8386 |
| 7 | 43 | 3.7490 | 4.0420 | 7.8149 |
| 8 | 50 | 3.7492 | 4.0420 | 7.8085 |
| 9 | 57 | 3.7499 | 4.0412 | 7.7700 |
| 10 | 64 | 3.7498 | 4.0403 | 7.7458 |
| 11 | 71 | 3.7501 | 4.0398 | 7.7242 |
| 12 | 78 | 3.7499 | 4.0393 | 7.7184 |
| 13 | 85 | 3.7503 | 4.0388 | 7.6945 |

| | | | | |
|---|---|---|---|---|
| 14 | 92 | 3.7500 | 4.0379 | 7.6786 |
| 15 | 99 | 3.7498 | 4.0381 | 7.6886 |
| 16 | 106 | 3.7499 | 4.0379 | 7.6809 |
| 17 | 113 | 3.7502 | 4.0375 | 7.6610 |
| 18 | 120 | 3.7501 | 4.0372 | 7.6559 |
| 19 | 127 | 3.7499 | 4.0373 | 7.6625 |
| 20 | 134 | 3.7497 | 4.0372 | 7.6680 |
| 21 | 141 | 3.7501 | 4.0362 | 7.6313 |

**Table 2.1: Quantifying the effects of increasing $\alpha_1$ *and* $\alpha_2$ together by 101% on biodiversity gain of a normal Agriculture using ODE23 numerical method**

| Example | LGS TIME (WEEKLY) | $x_1(old)$ | $x_1(new)$ | EBG(%) |
|---|---|---|---|---|
| 1 | 1 | 1.0000 | 1.0000 | 0 |
| 2 | 8 | 3.1974 | 3.2733 | 2.3731 |
| 3 | 15 | 3.6802 | 3.7596 | 2.1560 |
| 4 | 22 | 3.7408 | 3.8165 | 2.0236 |
| 5 | 29 | 3.7479 | 3.8221 | 1.9790 |
| 6 | 36 | 3.7481 | 3.8234 | 2.0070 |
| 7 | 43 | 3.7485 | 3.8241 | 2.0158 |
| 8 | 50 | 3.7500 | 3.8246 | 1.9885 |
| 9 | 57 | 3.7497 | 3.8249 | 2.0047 |
| 10 | 64 | 3.7490 | 3.8251 | 2.0292 |
| 11 | 71 | 3.7501 | 3.8252 | 2.0041 |
| 12 | 78 | 3.7504 | 3.8253 | 1.9972 |
| 13 | 85 | 3.7493 | 3.8254 | 2.0275 |
| 14 | 92 | 3.7497 | 3.8254 | 2.0165 |
| 15 | 99 | 3.7507 | 3.8254 | 1.9908 |

| | | | | |
|---|---|---|---|---|
| 16 | 106 | 3.7498 | 3.8253 | 2.0141 |
| 17 | 113 | 3.7494 | 3.8253 | 2.0243 |
| 18 | 120 | 3.7505 | 3.8253 | 1.9932 |
| 19 | 127 | 3.7503 | 3.8252 | 1.9988 |
| 20 | 134 | 3.7493 | 3.8252 | 2.0252 |
| 21 | 141 | 3.7501 | 3.8252 | 2.0031 |

**Table 2.2: Quantifying the effects of increasing $\alpha_1$ $and$ $\alpha_2$ together by 102% on biodiversity gain of a normal Agriculture using ODE23 numerical method**

| Example | LGS TIME (WEEKLY) | $x_1(old)$ | $x_1(new)$ | EBL(%) |
|---|---|---|---|---|
| 1 | 1 | 1.0000 | 1.0000 | 0 |
| 2 | 8 | 3.1974 | 3.3514 | 4.8166 |
| 3 | 15 | 3.6802 | 3.8399 | 4.3393 |
| 4 | 22 | 3.7408 | 3.8918 | 4.0368 |
| 5 | 29 | 3.7479 | 3.8956 | 3.9410 |
| 6 | 36 | 3.7481 | 3.8978 | 3.9940 |
| 7 | 43 | 3.7485 | 3.8989 | 4.0126 |
| 8 | 50 | 3.7500 | 3.8985 | 3.9592 |
| 9 | 57 | 3.7497 | 3.8998 | 4.0037 |
| 10 | 64 | 3.7490 | 3.9007 | 4.0460 |
| 11 | 71 | 3.7501 | 3.8999 | 3.9951 |
| 12 | 78 | 3.7504 | 3.9010 | 4.0148 |
| 13 | 85 | 3.7493 | 3.9017 | 4.0635 |
| 14 | 92 | 3.7497 | 3.9007 | 4.0258 |
| 15 | 99 | 3.7507 | 3.9017 | 4.0258 |
| 16 | 106 | 3.7498 | 3.9022 | 4.0649 |
| 17 | 113 | 3.7494 | 3.9012 | 4.0475 |

| | | | | |
|---|---|---|---|---|
| 18 | 120 | 3.7505 | 3.9021 | 4.0421 |
| 19 | 127 | 3.7503 | 3.9025 | 4.0593 |
| 20 | 134 | 3.7493 | 3.9014 | 4.0576 |
| 21 | 141 | 3.7501 | 3.9024 | 4.0618 |

**Table 2.3: Quantifying the effects of increasing $\alpha_1$ *and* $\alpha_2$ together by 103% on biodiversity gain of a normal Agriculture using ODE23 numerical method**

| Example | LGS TIME (WEEKLY) | $x_1(old)$ | $x_1(new)$ | EBL(%) |
|---|---|---|---|---|
| 1 | 1 | 1.0000 | 1.0000 | 0 |
| 2 | 8 | 3.1974 | 3.4303 | 7.2834 |
| 3 | 15 | 3.6802 | 3.9183 | 6.4678 |
| 4 | 22 | 3.7408 | 3.9672 | 6.0526 |
| 5 | 29 | 3.7479 | 3.9695 | 5.9142 |
| 6 | 36 | 3.7481 | 3.9713 | 5.9543 |
| 7 | 43 | 3.7485 | 3.9719 | 5.9602 |
| 8 | 50 | 3.7500 | 3.9715 | 5.9077 |
| 9 | 57 | 3.7497 | 3.9733 | 5.9633 |
| 10 | 64 | 3.7490 | 3.9730 | 5.9730 |
| 11 | 71 | 3.7501 | 3.9731 | 5.9479 |
| 12 | 78 | 3.7504 | 3.9746 | 5.9777 |
| 13 | 85 | 3.7493 | 3.9737 | 5.9846 |
| 14 | 92 | 3.7497 | 3.9745 | 5.9936 |
| 15 | 99 | 3.7507 | 3.9754 | 5.9913 |
| 16 | 106 | 3.7498 | 3.9744 | 5.9894 |
| 17 | 113 | 3.7494 | 3.9756 | 6.0330 |
| 18 | 120 | 3.7505 | 3.9759 | 6.0097 |
| 19 | 127 | 3.7503 | 3.9751 | 5.9943 |

| | | | | |
|---|---|---|---|---|
| 20 | 134 | 3.7493 | 3.9765 | 6.0601 |
| 21 | 141 | 3.7501 | 3.9763 | 6.0313 |

**Table 2.4: Quantifying the effects of increasing $\alpha_1$ $and$ $\alpha_2$ together by 104% on biodiversity gain of a normal Agriculture using ODE23 numerical method**

| Example | LGS TIME (WEEKLY) | $x_1(old)$ | $x_1(new)$ | EBG(%) |
|---|---|---|---|---|
| 1 | 1 | 1.0000 | 1.0000 | 0 |
| 2 | 8 | 3.1974 | 3.5100 | 9.7755 |
| 3 | 15 | 3.6802 | 3.9987 | 8.6541 |
| 4 | 22 | 3.7408 | 4.0410 | 8.0257 |
| 5 | 29 | 3.7479 | 4.0426 | 7.8630 |
| 6 | 36 | 3.7481 | 4.0419 | 7.8370 |
| 7 | 43 | 3.7485 | 4.0412 | 7.8074 |
| 8 | 50 | 3.7500 | 4.0407 | 7.7521 |
| 9 | 57 | 3.7497 | 4.0404 | 7.7520 |
| 10 | 64 | 3.7490 | 4.0402 | 7.7671 |
| 11 | 71 | 3.7501 | 4.0401 | 7.7337 |
| 12 | 78 | 3.7504 | 4.0400 | 7.7207 |
| 13 | 85 | 3.7493 | 4.0397 | 7.7440 |
| 14 | 92 | 3.7497 | 4.0392 | 7.7181 |
| 15 | 99 | 3.7507 | 4.0385 | 7.6730 |
| 16 | 106 | 3.7498 | 4.0377 | 7.6780 |
| 17 | 113 | 3.7494 | 4.0370 | 7.6700 |
| 18 | 120 | 3.7505 | 4.0364 | 7.6233 |
| 19 | 127 | 3.7503 | 4.0362 | 7.6241 |
| 20 | 134 | 3.7493 | 4.0362 | 7.6534 |
| 21 | 141 | 3.7501 | 4.0364 | 7.6349 |

**Table 3.1: Quantifying the effects of increasing $\alpha_1$ $and$ $\alpha_2$ together by 101% on biodiversity gain of a normal Agriculture using ODE15s numerical method**

| Example | LGS TIME (WEEKLY) | $x_1(old)$ | $x_1(new)$ | EBG(%) |
|---|---|---|---|---|
| 1 | 1 | 1.0000 | 1.0000 | 0 |
| 2 | 8 | 3.1976 | 3.2737 | 2.3851 |
| 3 | 15 | 3.6802 | 3.7599 | 2.1584 |
| 4 | 22 | 3.7410 | 3.8163 | 2.0120 |
| 5 | 29 | 3.7472 | 3.8218 | 1.9918 |
| 6 | 36 | 3.7484 | 3.8232 | 1.9935 |
| 7 | 43 | 3.7490 | 3.8239 | 1.9980 |
| 8 | 50 | 3.7494 | 3.8245 | 2.0028 |
| 9 | 57 | 3.7496 | 3.8248 | 2.0071 |
| 10 | 64 | 3.7497 | 3.8251 | 2.0108 |
| 11 | 71 | 3.7498 | 3.8253 | 2.0138 |
| 12 | 78 | 3.7499 | 3.8255 | 2.0163 |
| 13 | 85 | 3.7499 | 3.8256 | 2.0182 |
| 14 | 92 | 3.7500 | 3.8257 | 2.0197 |
| 15 | 99 | 3.7500 | 3.8258 | 2.0209 |
| 16 | 106 | 3.7500 | 3.8258 | 2.0218 |
| 17 | 113 | 3.7500 | 3.8258 | 2.0225 |
| 18 | 120 | 3.7500 | 3.8259 | 2.0231 |
| 19 | 127 | 3.7500 | 3.8259 | 2.0235 |
| 20 | 134 | 3.7500 | 3.8259 | 2.0238 |
| 21 | 141 | 3.7500 | 3.8259 | 2.0240 |

**Table 3.2: Quantifying the effects of increasing $\alpha_1$ $and$ $\alpha_2$ together by 102% on biodiversity gain of a normal Agriculture using ODE15s numerical method**

| Example | LGS TIME (WEEKLY) | $x_1(old)$ | $x_1(new)$ | EBG(%) |
|---|---|---|---|---|
| 1 | 1 | 1.0000 | 1.0000 | 0 |
| 2 | 8 | 3.1976 | 3.3511 | 4.8041 |
| 3 | 15 | 3.6802 | 3.8395 | 4.3201 |
| 4 | 22 | 3.7410 | 3.8916 | 4.0237 |
| 5 | 29 | 3.7472 | 3.8963 | 3.9800 |
| 6 | 36 | 3.7484 | 3.8976 | 3.9797 |
| 7 | 43 | 3.7490 | 3.8985 | 3.9866 |
| 8 | 50 | 3.7494 | 3.8992 | 3.9951 |
| 9 | 57 | 3.7496 | 3.8997 | 4.0039 |
| 10 | 64 | 3.7497 | 3.9002 | 4.0123 |
| 11 | 71 | 3.7498 | 3.9006 | 4.0199 |
| 12 | 78 | 3.7499 | 3.9009 | 4.0267 |
| 13 | 85 | 3.7499 | 3.9012 | 4.0327 |
| 14 | 92 | 3.7500 | 3.9014 | 4.0379 |
| 15 | 99 | 3.7500 | 3.9016 | 4.0423 |
| 16 | 106 | 3.7500 | 3.9017 | 4.0461 |
| 17 | 113 | 3.7500 | 3.9018 | 4.0494 |
| 18 | 120 | 3.7500 | 3.9019 | 4.0521 |
| 19 | 127 | 3.7500 | 3.9020 | 4.0544 |
| 20 | 134 | 3.7500 | 3.9021 | 4.0564 |
| 21 | 141 | 3.7500 | 3.9022 | 4.0581 |

**Table 3.3: Quantifying the effects of increasing $\alpha_1$ $and$ $\alpha_2$ together by 103% on biodiversity gain of a normal Agriculture using ODE15s numerical method**

| Example | LGS TIME (WEEKLY) | $x_1(old)$ | $x_1(new)$ | EBG(%) |
|---|---|---|---|---|
| 1 | 1 | 1.0000 | 1.0000 | 0 |
| 2 | 8 | 3.1976 | 3.4295 | 7.2580 |
| 3 | 15 | 3.6802 | 3.9190 | 6.4816 |
| 4 | 22 | 3.7410 | 3.9666 | 6.0289 |
| 5 | 29 | 3.7472 | 3.9703 | 5.9526 |
| 6 | 36 | 3.7484 | 3.9711 | 5.9401 |
| 7 | 43 | 3.7490 | 3.9717 | 5.9398 |
| 8 | 50 | 3.7494 | 3.9722 | 5.9440 |
| 9 | 57 | 3.7496 | 3.9727 | 5.9504 |
| 10 | 64 | 3.7497 | 3.9731 | 5.9581 |
| 11 | 71 | 3.7498 | 3.9736 | 5.9661 |
| 12 | 78 | 3.7499 | 3.9739 | 5.9742 |
| 13 | 85 | 3.7499 | 3.9743 | 5.9821 |
| 14 | 92 | 3.7500 | 3.9746 | 5.9897 |
| 15 | 99 | 3.7500 | 3.9749 | 5.9969 |
| 16 | 106 | 3.7500 | 3.9751 | 6.0037 |
| 17 | 113 | 3.7500 | 3.9754 | 6.0101 |
| 18 | 120 | 3.7500 | 3.9756 | 6.0161 |
| 19 | 127 | 3.7500 | 3.9758 | 6.0217 |
| 20 | 134 | 3.7500 | 3.9760 | 6.0269 |
| 21 | 141 | 3.7500 | 3.9762 | 6.0319 |

**Table 3.4: Quantifying the effects of increasing $\alpha_1$ $and$ $\alpha_2$ together by 104% on biodiversity gain of a normal Agriculture using ODE15s numerical method**

| Example | LGS TIME (WEEKLY) | $x_1(old)$ | $x_1(new)$ | EBG(%) |
|---|---|---|---|---|
| 1 | 1 | 1.0000 | 1.0000 | 0 |
| 2 | 8 | 3.1976 | 3.5091 | 9.7467 |
| 3 | 15 | 3.6802 | 3.9985 | 8.6408 |
| 4 | 22 | 3.7410 | 4.0411 | 8.0196 |
| 5 | 29 | 3.7472 | 4.0430 | 7.8925 |
| 6 | 36 | 3.7484 | 4.0425 | 7.8454 |
| 7 | 43 | 3.7490 | 4.0420 | 7.8139 |
| 8 | 50 | 3.7494 | 4.0414 | 7.7892 |
| 9 | 57 | 3.7496 | 4.0409 | 7.7687 |
| 10 | 64 | 3.7497 | 4.0404 | 7.7511 |
| 11 | 71 | 3.7498 | 4.0399 | 7.7355 |
| 12 | 78 | 3.7499 | 4.0394 | 7.7216 |
| 13 | 85 | 3.7499 | 4.0390 | 7.7088 |
| 14 | 92 | 3.7500 | 4.0386 | 7.6971 |
| 15 | 99 | 3.7500 | 4.0382 | 7.6863 |
| 16 | 106 | 3.7500 | 4.0378 | 7.6762 |
| 17 | 113 | 3.7500 | 4.0375 | 7.6669 |
| 18 | 120 | 3.7500 | 4.0372 | 7.6582 |
| 19 | 127 | 3.7500 | 4.0369 | 7.6502 |
| 20 | 134 | 3.7500 | 4.0366 | 7.6428 |
| 21 | 141 | 3.7500 | 4.0363 | 7.6359 |

**Table 4.1: Quantifying the effects of increasing $\alpha_1$ $and$ $\alpha_2$ together by 101% on biodiversity gain of a normal Agriculture using ODE23tb numerical method**

| Example | LGS TIME (WEEKLY) | $x_1(old)$ | $x_1(new)$ | EBG(%) |
|---|---|---|---|---|
| 1 | 1 | 1.0000 | 1.0000 | 0 |
| 2 | 8 | 3.1975 | 3.2737 | 2.3851 |
| 3 | 15 | 3.6805 | 3.7599 | 2.1584 |
| 4 | 22 | 3.7410 | 3.8163 | 2.0120 |
| 5 | 29 | 3.7472 | 3.8218 | 1.9918 |
| 6 | 36 | 3.7484 | 3.8232 | 1.9935 |
| 7 | 43 | 3.7490 | 3.8239 | 1.9980 |
| 8 | 50 | 3.7494 | 3.8245 | 2.0028 |
| 9 | 57 | 3.7496 | 3.8248 | 2.0071 |
| 10 | 64 | 3.7497 | 3.8251 | 2.0108 |
| 11 | 71 | 3.7498 | 3.8253 | 2.0138 |
| 12 | 78 | 3.7499 | 3.8255 | 2.0163 |
| 13 | 85 | 3.7499 | 3.8256 | 2.0182 |
| 14 | 92 | 3.7500 | 3.8257 | 2.0197 |
| 15 | 99 | 3.7500 | 3.8258 | 2.0209 |
| 16 | 106 | 3.7500 | 3.8258 | 2.0218 |
| 17 | 113 | 3.7500 | 3.8258 | 2.0225 |
| 18 | 120 | 3.7500 | 3.8259 | 2.0231 |
| 19 | 127 | 3.7500 | 3.8259 | 2.0235 |
| 20 | 134 | 3.7500 | 3.8259 | 2.0238 |
| 21 | 141 | 3.7500 | 3.8259 | 2.0240 |

**Table 4.2: Quantifying the effects of increasing $\alpha_1$ $and$ $\alpha_2$ together by 102% on biodiversity gain of a normal Agriculture using ODE23tb numerical method**

| Example | LGS TIME (WEEKLY) | $x_1(old)$ | $x_1(new)$ | EBG(%) |
|---|---|---|---|---|
| 1 | 1 | 1.0000 | 1.0000 | 0 |
| 2 | 8 | 3.1975 | 3.3511 | 4.8041 |
| 3 | 15 | 3.6805 | 3.8395 | 4.3201 |
| 4 | 22 | 3.7410 | 3.8916 | 4.0237 |
| 5 | 29 | 3.7472 | 3.8963 | 3.9800 |
| 6 | 36 | 3.7484 | 3.8976 | 3.9797 |
| 7 | 43 | 3.7490 | 3.8985 | 3.9866 |
| 8 | 50 | 3.7494 | 3.8992 | 3.9951 |
| 9 | 57 | 3.7496 | 3.8997 | 4.0039 |
| 10 | 64 | 3.7497 | 3.9002 | 4.0123 |
| 11 | 71 | 3.7498 | 3.9006 | 4.0199 |
| 12 | 78 | 3.7499 | 3.9009 | 4.0267 |
| 13 | 85 | 3.7499 | 3.9012 | 4.0327 |
| 14 | 92 | 3.7500 | 3.9014 | 4.0379 |
| 15 | 99 | 3.7500 | 3.9016 | 4.0423 |
| 16 | 106 | 3.7500 | 3.9017 | 4.0461 |
| 17 | 113 | 3.7500 | 3.9018 | 4.0494 |
| 18 | 120 | 3.7500 | 3.9019 | 4.0521 |
| 19 | 127 | 3.7500 | 3.9020 | 4.0544 |
| 20 | 134 | 3.7500 | 3.9021 | 4.0564 |
| 21 | 141 | 3.7500 | 3.9022 | 4.0581 |

**Table 4.3: Quantifying the effects of increasing $\alpha_1$ *and* $\alpha_2$ together by 103% on biodiversity gain of a normal Agriculture using ODE23 numerical method**

| Example | LGS TIME (WEEKLY) | $x_1(old)$ | $x_1(new)$ | EBG(%) |
|---|---|---|---|---|
| 1 | 1 | 1.0000 | 1.0000 | 0 |
| 2 | 8 | 3.1975 | 3.4295 | 7.2580 |
| 3 | 15 | 3.6805 | 3.9190 | 6.4816 |
| 4 | 22 | 3.7410 | 3.9666 | 6.0289 |
| 5 | 29 | 3.7472 | 3.9703 | 5.9526 |
| 6 | 36 | 3.7484 | 3.9711 | 5.9401 |
| 7 | 43 | 3.7490 | 3.9717 | 5.9398 |
| 8 | 50 | 3.7494 | 3.9722 | 5.9440 |
| 9 | 57 | 3.7496 | 3.9727 | 5.9504 |
| 10 | 64 | 3.7497 | 3.9731 | 5.9581 |
| 11 | 71 | 3.7498 | 3.9736 | 5.9661 |
| 12 | 78 | 3.7499 | 3.9739 | 5.9742 |
| 13 | 85 | 3.7499 | 3.9743 | 5.9821 |
| 14 | 92 | 3.7500 | 3.9746 | 5.9897 |
| 15 | 99 | 3.7500 | 3.9749 | 5.9969 |
| 16 | 106 | 3.7500 | 3.9751 | 6.0037 |
| 17 | 113 | 3.7500 | 3.9754 | 6.0101 |
| 18 | 120 | 3.7500 | 3.9756 | 6.0161 |
| 19 | 127 | 3.7500 | 3.9758 | 6.0217 |
| 20 | 134 | 3.7500 | 3.9760 | 6.0269 |
| 21 | 141 | 3.7500 | 3.9762 | 6.0319 |

**Table 4.4: Quantifying the effects of increasing $\alpha_1$ $and$ $\alpha_2$ together by 104% on biodiversity gain of a normal Agriculture using ODE23tb numerical method**

| Example | LGS TIME (WEEKLY) | $x_1(old)$ | $x_1(new)$ | EBG(%) |
|---|---|---|---|---|
| 1 | 1 | 1.0000 | 1.0000 | 0 |
| 2 | 8 | 3.1975 | 3.5091 | 9.7467 |
| 3 | 15 | 3.6805 | 3.9985 | 8.6408 |
| 4 | 22 | 3.7410 | 4.0411 | 8.0196 |
| 5 | 29 | 3.7472 | 4.0430 | 7.8925 |
| 6 | 36 | 3.7484 | 4.0425 | 7.8454 |
| 7 | 43 | 3.7490 | 4.0420 | 7.8139 |
| 8 | 50 | 3.7494 | 4.0414 | 7.7892 |
| 9 | 57 | 3.7496 | 4.0409 | 7.7687 |
| 10 | 64 | 3.7497 | 4.0404 | 7.7511 |
| 11 | 71 | 3.7498 | 4.0399 | 7.7355 |
| 12 | 78 | 3.7499 | 4.0394 | 7.7216 |
| 13 | 85 | 3.7499 | 4.0390 | 7.7088 |
| 14 | 92 | 3.7500 | 4.0386 | 7.6971 |
| 15 | 99 | 3.7500 | 4.0382 | 7.6863 |
| 16 | 106 | 3.7500 | 4.0378 | 7.6762 |
| 17 | 113 | 3.7500 | 4.0375 | 7.6669 |
| 18 | 120 | 3.7500 | 4.0372 | 7.6582 |
| 19 | 127 | 3.7500 | 4.0369 | 7.6502 |
| 20 | 134 | 3.7500 | 4.0366 | 7.6428 |
| 21 | 141 | 3.7500 | 4.0363 | 7.6359 |

## 6. RESULTS & DISCUSSION

In this paper, we have applied a mathematical model to discuss and predict the growth of normal agricultural assets by comparing four differential methods of ordinary differential equation (ODE45, ODE23, ODE15s, and ODE23tb). From the result obtained, we have seen that the growth of normal agricultural assets depends on time and the normal agricultural assets changes deterministically as the length of growing season changes where all the model parameter values are fixed, however when the model value $\alpha_1$ *and* $\alpha_2$ are increase 101%-104%, the normal agricultural variable also changes. From the numerical prediction using four method of ODE which are similar, ODE45 appears to be more efficient than the others. Applying these four methods, we have seen that the pattern of growth of these two interacting normal agricultural data have finite instance of biodiversity.

## REFERENCES

[1] I. Agyemang, H.I. Freedman, J.W. Macki, An ecospheric recovery model for agriculture industry interactions, Diff. Eqns. Dyn. Syst. 15 (2007) 185-208.

[2] I. Agyemang, H.I. Freedman, An environmental model for the interaction of industry with two competing agricultural resource, Elsevier. 49(2009) 1618-16

[3] H. Amman, Ordinary Differential Equations: Introduction to Non-Linear Analysis, Walter de Gruyter, Berlin, 1990.

[4] A.G Eleki, R.E Akpodee and E.N Ekakaa, The differential effects of a scenario of a non-additive environmental perturbation on biodiversity ecospheric assets: Kalabari kingdom of ecospheric assets, African Publication and research international, 2019 1-15

[5] A.G Eleki and E.N Ekakaa, Computational Modelling of ecosperic assets: alternative numerical approach, African Scholar publications and research international, 15(2), 2019, 160-165

[6] L.P. Apedaile, H.I. Freedman, S.G.M. Schilizzi, M. Solomonovich, Equilibria and dynamics in an economic predator prey model of agriculture, Math. Comput. Model. 19 (1994) 1 -15.

[7] G.J. Butler, H.I. Freedman, P.E. Waltman, Uniformly persistent systems, Proc. Amer. Math. Soc. 96 (1986) 425 430.

[8] Michael A. Obe, Godspower C. Abanum, Innocent C. Eli, Variational Iteration Method for first and second ordinary differential equation using first kind chebychev polynomials, IJASRE. 4(2018) 126-130

[9] Godspower C.A, Charles O and Ekakaa E.N, Numerical Simulation of biodiversity Loss: Comparison of numerical methods,IJMTT. 66(2020) 53-64

[10] G.J. Butler, P. Waltman, Persistence in three-dimensional Lotka Volterra systems, Math. Comput. Model. 10 (1988) 13 16.

[11] C.W. Clark, Mathematical Bioeconomics: The Optimal Management of Renewable Resources, Wiley, New York, 1990.

[12] G.C. Daily, Natures Services: Societal Dependence on Natural Ecosystem, Island Press, Washington, DC, 1997.

[13] G.C. Daily, K. Ellison, The New Economy of Nature: The Quest to Make Conservation Profitable, Island Press, Washington, DC, 2002.

[14] G.C. Daily, P.A. Matson, P.M. Vitousek, Ecosystem services supplied by soil, in: Natures Services: Societal Dependence on Natural Ecosystem, Island Press, Washington, DC, 1997.

[15] L. Edelstein-Keshet, Mathematical Models in Biology, Random House, New York, 1988.

[16] H.W. Eves, Foundations and Fundamental Concepts of Mathematics, PWS-Kent, Boston, 1990.

[17] H.I. Freedman, Deterministic Mathematical Models in Population Ecology, Marcel Dekker Inc., New York, 1980.

[18] H.I. Freedman, P. Moson, Persistence definitions and their connections, Proc. Amer. Math. Soc. 109 (1990) 1025 1033.

[19] H.I. Freedman, J.W.-H. So, Global stability and persistence of simple food chains, Math. Biosci. 73 (1985) 89 91.

[20] H.I. Freedman, M. Solomonovich, L.P. Apedaile, A. Hailu, Stability in models of agricultural-industry-environment interactions, in: Advances in Stability Theory, vol. 13, Taylor and Francis, London, 2003, pp. 255 265.

[21] H.I. Freedman, P. Waltman, Persistence in a model of three competitive populations, Math. Biosci. 73 (1985) 89 91.

[22] H.I. Freedman, P. Waltman, Persistence in a model of three interacting predator-prey populations, Math. Biosci. 68 (1984) 213 231.

[23] G. Heal, Nature and the Marketplace: Capturing the Value of Ecosystem Services, Island Press, Washington, DC, 2000.

[24] L. Horrigan, R.S. Lawrence, P. Walker, How sustainable agriculture can address the environmental and human health harms of industrial agriculture, Environ. Health Persp. 110: 445 456.

[25] J. Ikerd, The ecology of sustainability, University of Missouri, MO, USA.

[26] J. Ikerd, Sustaining the profitability of agriculture, Paper presented at extension pre-conference, The economists role in the agricultural sustainability paradigm, San Antonio, TX, 1996.

[27] M. Solomonovich, L.P. Apedaile, H.I. Freedman, A.H. Gebremedihen, S.M.G. Belostotski, Dynamical economic model of sustainable agriculture and the ecosphere, Appl. Math. Comput. 84 (1997) 221 246.

[28] M. Solomonovich, L.P. Apedaile, H.I. Freedman, Predictability and trapping under conditions of globalization of agricultural trade: An application of the CGS approach, Math. Comput. Model. 33 (2001) 495 516.

[29] M. Solomonovich, H.I. Freedman, L.P. Apedaile, S.G.M. Schilizzi, L. Belostotski, Stability and bifurcations in an environmental recovery model of economic agriculture-industry interactions, Natur. Resource Modeling 11 (1998) 35 79.

[30] M.L Jhingan, The economics of development and planning, 39th revised and enlarged edition

[31] M. Donatelli, R.D Magarey, S Bregagho, L. Willocquet, J.P.M Whish and S. Savary, Modelling the impacts of pests and diseases on agricultural systems, Elseavier, agricultural system, 155(2017) 213-224