NON-CANONICAL TO CANONICAL K-ESSENCE AND ACCELERATED EXPANSION OF THE UNIVERSE

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Abstract

The main aim of this paper is to study late time accelerated expansion of the universe with the help of k-essence scalar field. We have chosen non-canonical k-essence lagrangian of the form $V(\phi)F(X)$, and converted to canonical lagrangian with the help of scaling ralation $\sqrt{X}F_X$=Ca^{-3}. The obtained lagrangian has two generalised co-ordinate, logarithm of the scale factor (q=ln$a$) and scalar field (ϕ). From the obtained lagrangian we studied two most important cosmological parameter, Equation of state parameter ($\omega \approx -1$) and deceleration parameter ($q_0 \approx -1$) which are to some extent consistent with the current observations. Both the parameters indicates an accelerated expansion of the universe driven by negative pressure known as dark energy.

Keywords: *k-essence, Dark energy, Hubble parameter, Acceleration of Universe*.

1. Introduction

Observation of Type 1a Supernovae (SNe 1a) by The Supernova Cosmology Project [1-4] and the High-Z-Supernova search team [5-7] indicates an accelerated expansion of the universe. Recent observation with WMAP satellite [8,9] and Planck satellite [10] led to the deep understanding of the mysterious energy known as dark energy as the source of negative pressure that leads to an accelerated expansion of the universe.

Several cosmological model has been established to understand the role of dark energy in the universe, out of which we have chosen k-essence model [11-16] of scalar field ϕ(r,t)with non-canonical kinetic term X as our field of study.

The k-essence scalar field ϕ(r,t) minimally coupled to background spacetime metric $g_{\mu\nu}$ has the action given by [16-23]

$$S_k\left[\phi,g_{\mu\nu}\right] = \int d^4 x\sqrt{-g}L(\phi,X) \qquad (1)$$

where $L(\phi,X)$ is the k-essence Lagrangian and $X=\frac{1}{2}g^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi$ is the canonical kinetic term.

We will work with the flat Friedmann-Lemaitre-Robertson-Walker (FLRW) metric of the form

$$dS^2=dt^2-a^2(t)\sum_{i=1}^{3}\left(dx^i\right)^2 \quad (2)$$

where $a(t)$ is the cosmological scale factor.

The energy -momentum tensor with respect to action (1) is given by

$$T_{\mu\nu}=\frac{2}{\sqrt{-g}}\frac{\delta S_k}{\delta g^{\mu\nu}}=L_X\nabla_\mu\phi\nabla_\nu\phi-g_{\mu\nu}L \quad (3)$$

Where $L_X=\frac{dK}{dX}$ and ∇_μ is the covariant derivative defined with respect to gravitational metric. We will consider energy-momentum tensor of perfect fluid with density and pressure is given by

$$T_{\mu\nu}=(\rho+p)U_\mu U_\nu-pg_{\mu\nu} \quad (4)$$

For our study we take the form of k-essence Lagrangian (or the pressure) with non-canonical kinetic term [11-23] given by

$$L[\phi,X]=p=V(\phi)F(X) \quad (5)$$

where $V(\phi)$ is the scalar field potential and $F(X)$ is the kinetic part.

Comparing (3) and (4) energy density for the k-essence Lagrangian (5) is given by:-

$$\rho=V(\phi)[2XF_X-F(X)] \quad (6)$$

where $F_X=\frac{dF}{dX}$.

2. Scaling relation

Equation of continuity is given by

$$\dot{\rho}+3H(\rho+p)=0 \quad (7)$$

where H is the Hubble parameter defined in terms of scale factor $a(t)$ as $H=\frac{\dot{a}}{a}$.

From (6) we get

$$\dot{\rho}=V(\phi)[F_X+2XF_{XX}]\dot{X}+\dot{\phi}[2XF_X-F(X)]V_\phi \quad (8)$$

where $F_X = \frac{\mathrm{dF}}{\mathrm{dX}}$, $F_{\mathrm{XX}} = \frac{\mathrm{dF}_x}{\mathrm{dX}}$ and $V_\varphi = \frac{\mathrm{dV}}{\mathrm{d}\varphi}$.

Putting (5), (6) and (8) in equation of continuity (7) we get

$$V(\varphi)[F_X + 2\mathrm{XF}_{\mathrm{XX}}]\dot{X} + \dot{\phi}[2\mathrm{XF}_X - F(X)]V_\varphi + 6\mathrm{HV}(\varphi)\mathrm{XF}_X = 0 \quad (9)$$

For a constant potential $V_\phi = 0$so that equation (9) reduces to

$$[F_X + 2\mathrm{XF}_{\mathrm{XX}}]\frac{\mathrm{dX}}{\mathrm{dt}} + 6\mathrm{HXF}_X = 0 \qquad (10)$$

Since H=$\frac{1}{a}\frac{\mathrm{da}}{\mathrm{dt}}$, this reduces to the form

$$\left[\frac{1}{X} + 2\,\frac{F_{\mathrm{XX}}}{F_X}\right]\mathrm{dX} = -\,6\,\frac{\mathrm{da}}{a} \qquad (11)$$

On integration this results in scaling relation of the form

$$\sqrt{X}F_X = \mathrm{Ca}^{-3} \qquad (12)$$

where C is an integration constant. This is the scaling relation [12-14] which relates kinetic term with

the cosmological scale factor of the FLRW metric.

3. K-essence Lagrangian

Considering Friedmann equation

$$H^2 = \frac{8\pi\mathrm{G}}{3}\rho \qquad (13)$$

Comparing (6) and (13) we get

$$H^2 = \frac{8\pi G}{3} V[F(X) - 2XF_X] \qquad (14)$$

Since we have considered constant potential hence we will write $V(\phi)$=V=constant.

From (12) we get $F_X = \frac{Ca^{-3}}{\sqrt{X}}$, substituting in (14) we get

$$F(X) = \frac{3}{8\pi GV} H^2 + 2C\sqrt{X}a^{-3} \qquad (15)$$

Let q=ln$a$, so that Hubble parameter becomes H=$\frac{\dot{a}}{a} = \dot{q}$. Considering present observable

universe to be homogeneous , we will consider scalar field to be function of time only i.e.,

ϕ(r,t)=ϕ(t),so that X=$\frac{1}{2}\dot{\phi}^2$. Equation (15) becomes

$$F(X) = \frac{3}{8\pi GV} \dot{q}^2 + C\sqrt{2}\dot{\phi}e^{-3q} \qquad (16)$$

Putting (16) in (5) we get

$$L = -c_1\dot{q}^2 - c_2\dot{\phi}e^{-3q} \qquad (17)$$

where $c_1 = \frac{3}{8\pi G}$ and c_2=C$\sqrt{2}V$.

This is the k-essence Lagrangian in canonical form with first term as the kinetic term and second term

as a polynomial interaction. It has two generalised co-ordinate logarithm of the scale factor q=ln$a$

and scalar field $\phi(t)$.

Equation of state parameter ω

Equation of state parameter in terms of pressure (p) and density (ρ) is define as

$$\omega=\frac{p}{\rho} \qquad (18)$$

Since the lagrangian density is equivalent to pressure, hence we can write

$$p=-c_1\dot{q}^2-c_2\dot{\phi}e^{-3q} \qquad (19)$$

The energy density is given by the Friedmann equation

$$\rho=\frac{3}{8\pi G}H^2=c_1\dot{q}^2 \qquad (20)$$

Putting (19) and (20) in (18) we get

$$\omega=-1-\frac{c_2e^{-3q}\dot{\phi}}{c_1\dot{q}^2} \qquad (21)$$

Since $\dot{q}$=H, hence q=Ht. Therefore at late time when $t\to\infty$, $q\to\infty$, so that $e^{-3q}\to 0$.

Thus EOS becomes $\omega\approx -1$,which is consistent with the recent observations [1-10,24].

Deceleration parameter q_0

Deceleration parameter $q_0$is defined as

$$q_0=\frac{-a\ddot{a}}{\dot{a}^2} \qquad (21)$$

In terms of EOS (ω) , (21) can be defined as

$$q_0=\frac{1}{2}(1+3\omega) \qquad (22)$$

Since for late time cosmology for $t\to\infty$ we get $\omega\approx -1$,thus (22) shows that

$q_0\approx -1$.Negativity of $q_0$indicates late time acceleration of the universe.

Conclusion

k-essence lagrangian with non-canonical kinetic term is transformed into lagrangian with canonical kinetic term using scaling relation. Two most important results we get from the work of this paper is at late time when $t\to\infty$, Equation of state parameter $\omega\approx -1$as well as deceleration parameter

$$q_0\approx -1.$$

Both the results indicates an accelerated expansion of the universe driven by negative pressure nown as dark energy.

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