

MATHEMATICAL ANALYSIS OF A MODEL FOR THE DYNAMICS OF BROILER PRODUCTION IN NIGERIA

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Abstract

In this paper, we propose a system of first order differential equations divided into four classes to describe broiler production in Nigeria. The model incorporates previous training, years of experience, capital and stocking capacity as major variables determining the dynamics of the physical phenomenon. The model equations are stated with underlying assumptions. Using basic mathematical techniques, positivity, boundedness, existence and uniqueness, of solution of the model are explored. The results showed that there exists a unique, positive and bounded solution of the model. This suggests the applicability of the model to study the determinants of broiler production in Nigeria and gives useful insights to assist economists and researchers in their empirical studies.

Keywords: *broiler, mathematical model, positivity, boundedness, existence and uniqueness of solution*

1. Introduction

In the Nigeria domestic poultry industry, the significance of broiler production cannot be over-emphasized as it contributes a great turn over cycle and requires far less amount of capital (Aminu et al., 2010). In addition, it plays a very important role in protein supply in Nigeria (Omolayo, 2008). According to Food and Agriculture Organization (FAO), broiler has the most astounding protein content of 21-50%, when compared with other animal protein obtained from cattle, goat and sheep (FAO, 2008) Beyond direct food production, poultry industry generally is a great contributor to Nigeria Gross Domestic Production (GDP), employment, poverty alleviation as well as income generation among the teeming population (Philip et al., 2009).

In order to meet the growing concerns of inadequate protein that Nigeria like most developing countries is confronted with, strategies that will encourage higher production is required.

Surprisingly, mathematical modeling is central to solving many physical problems arising in the field of science, engineering and finance (Burges and Wood, 1980, Madubueze et al., 2017). As a solution to the problem, the role of mathematical models is inevitable. These models can be used to assess the critical impacts of capital, years of experience, previous training and stocking capacity as key parameters in determining broiler production and profitability of the business.

The present paper considers all these variables, proposes a model incorporating the parameters and performs mathematical analysis to suggest crucial data that should be collected to make general forecasts, estimations and predictions which would in turn assists policy makers and people involved in poultry enterprise.

2. Mathematical Formulation

We propose the following set of 1st order differential equations to describe the dynamics of broiler production in Nigeria.

$$\frac{dP}{dt} = \gamma\alpha_m + \gamma\alpha_s - \alpha_m P - \alpha_s P \quad (1)$$

$$\frac{dQ}{dt} = \gamma\Omega_t + \gamma\Omega_{ut} - \Omega_t Q - \Omega_{ut} Q \quad (2)$$

$$\frac{dR}{dt} = \gamma\ell_c + \gamma\ell_n - \ell_n R - \ell_c R \quad (3)$$

$$\frac{dW}{dt} = \gamma\theta_l + \gamma\theta_h - \theta_h W - \theta_l W \quad (4)$$

With the initial conditions which are non-negative in the positive invariant region λ such that

$$\{P_0, Q_0, R_0, W_0\} \in \lambda.$$

The descriptions of the variables and parameters of the model are given in Table 1.

Variable parameter	Description
P	years of experience
Q	previous training
R	capital
W	stocking capacity
γ	growth rate
α_m	many years of experience
α_s	small years of experience
Ω_t	training rate
Ω_{ut}	untrained
ℓ_c	capital rate
ℓ_n	no capital
θ_h	high capacity rate
θ_l	low capacity rate

Table 1: Model Variables and Parameters

3. Mathematical Analysis

3.1 Positivity of Solution

Theorem 1

Given that the initial conditions P_0, Q_0, R_0, W_0 are non-negative and exist in the positive invariant region λ , then the solution set $\{P(t), Q(t), R(t), W(t)\}$ are all positive for all time $t \geq 0$.

Proof

Consider equation (1)

$$\frac{dP}{dt} = \gamma\alpha_m + \gamma\alpha_s - \alpha_m P - \alpha_s P$$

which can be rewritten as

$$\frac{dP}{dt} \geq -(\alpha_m + \alpha_s)P \quad (5)$$

Integrating by separable variables and applying the initial conditions gives

$$P(t) \geq P_0 e^{-(\alpha_m + \alpha_s)t} \geq 0 \quad (6)$$

In a similar fashion, it can be shown that respective solution $Q(t)$, $R(t)$, $W(t)$ of the model equations (3) - (4) remains positive in the invariant region λ . Hence, we conclude that the solution set $\{P(t), Q(t), R(t), W(t)\}$ of our model is positive in λ for all $t \geq 0$.

3.2 Existence and Uniqueness of Solution

Consider the IVP

$$\left. \begin{array}{l} x'_1 = f_1(t, x_1, \dots, x_n), \quad x_1(t_0) = x_{10} \\ x'_2 = f_2(t, x_1, x_2, \dots, x_n), \quad x_2(t_0) = x_{20} \\ \cdot \quad \quad \quad \cdot \quad \quad \quad \cdot \quad \quad \quad \cdot \\ \cdot \quad \quad \quad \cdot \quad \quad \quad \cdot \quad \quad \quad \cdot \\ \cdot \quad \quad \quad \cdot \quad \quad \quad \cdot \quad \quad \quad \cdot \\ x'_n = f_n(t, x_1, x_2, \dots, x_n), \quad x_n(t_0) = x_{n0} \end{array} \right\} \quad (7)$$

We write the vectors

$$x = \begin{pmatrix} x_1 \\ \cdot \\ \cdot \\ \cdot \\ x_n \end{pmatrix} \quad (8)$$

$$f(t, x) = \begin{pmatrix} f_1(t, x_1, \dots, x_n) \\ \cdot \\ \cdot \\ \cdot \\ f_n(t, x_1, x_2, \dots, x_n) \end{pmatrix} \quad (9)$$

$$x_0 = \begin{pmatrix} x_{10} \\ x_{20} \\ \cdot \\ \cdot \\ \cdot \\ x_{n0} \end{pmatrix} \quad (10)$$

Then, we may write equation (7) in the compact form

$$x' = f(t, x), \quad x(t_0) = x_0 \quad (11)$$

Applying equation (11) to the model equations (1) - (4) results in the following differential equations

$$f_1(t, x) = \gamma\alpha_m + \gamma\alpha_s - \alpha_m P - \alpha_s P \quad (12)$$

$$f_2(t, x) = \gamma\Omega_t + \gamma\Omega_{ut} - \Omega_t Q - \Omega_{ut} Q \quad (13)$$

$$f_3(t, x) = \gamma\ell_c + \gamma\ell_n - \ell_n R - \ell_c R \quad (14)$$

$$f_4(t, x) = \gamma\theta_t + \gamma\theta_h - \theta_h W - \theta_t W \quad (15)$$

Then, we state the following theorem necessary for the proof of existence and uniqueness of solution of the model

Theorem 2

Let V denote a positive region such that $|t - t_0| \leq a, \|x - x_0\| \leq b$ (16)

Suppose that $f(t,x)$ satisfies the Lipschitz condition

$$\|f(t, x_1) - f(t, x_2)\| \leq k \|x_1 - x_2\| \quad (17)$$

Whenever the pairs (t, x_1) and (t, x_2) belong to V where k is a non-negative constant. Then there is a constant $\delta > 0$ such that there exists a continuous unique vector solution $x(t)$ of the system in equation (11) in the interval $|t - t_0| \leq \delta$

Remark: It is important to note that the condition (17) is satisfied by the requirement that

$$k = \frac{\partial f_i}{\partial x_j}, i = 1, 2, \dots, n, j = 1, 2, \dots, n \text{ continuous and bounded in } V.$$

Now, if $f(t, x)$ defined by equation (12) - (15) has continuous partial derivatives on a bounded domain R such that

$$1 \leq t \leq R, \quad 0 < R < \infty \quad (18)$$

Then, $f(t, x)$ satisfies a Lipschitz condition in R and we have the following theorem

Theorem 3

Let V denote the region defined in equation (16) such that equation (18) holds, then there exists a unique and bounded solution of the model system (1) - (4) in the region V .

Proof

By Theorem 2, it suffices to show that

$$\left| \frac{\partial f_i}{\partial x_j} \right|, i, j = 1, 2, 3, 4 \text{ are bounded}$$

Now,

$$\left| \frac{\partial f_1}{\partial x_j} \right| = |-\alpha_m - \alpha_s| = |-(\alpha_m + \alpha_s)| < \infty \quad (19)$$

$$\left| \frac{\partial f_1}{\partial Q} \right| = \left| \frac{\partial f_1}{\partial R} \right| = \left| \frac{\partial f_1}{\partial W} \right| = 0 < \infty \quad (20)$$

$$\left| \frac{\partial f_2}{\partial Q} \right| = |-\Omega_t - \Omega_{ut}| = |-(\Omega_t + \Omega_{ut})| < \infty \quad (21)$$

$$\left| \frac{\partial f_2}{\partial p} \right| = \left| \frac{\partial f_2}{\partial R} \right| = \left| \frac{\partial f_2}{\partial W} \right| = 0 < \infty \quad (22)$$

$$\left| \frac{\partial f_3}{\partial R} \right| = |-\ell_n - \ell_c| = |-(\ell_n + \ell_c)| < \infty \quad (23)$$

$$\left| \frac{\partial f_3}{\partial p} \right| = \left| \frac{\partial f_3}{\partial Q} \right| = \left| \frac{\partial f_3}{\partial W} \right| = 0 < \infty \quad (24)$$

$$\left| \frac{\partial f_4}{\partial W} \right| = |-\theta_h - \theta_l| = |-(\theta_h + \theta_l)| < \infty \quad (25)$$

$$\left| \frac{\partial f_4}{\partial p} \right| = \left| \frac{\partial f_4}{\partial Q} \right| = \left| \frac{\partial f_4}{\partial W} \right| = 0 < \infty \quad (26)$$

Clearly, equations (19) - (26) are bounded.

Hence, by Theorem 2 there exists a unique solution of equations (1) - (4) which satisfies equation (17). Theorem 3 is hereby established.

4. Discussion of Results and Conclusion

In this paper, we have analysed a model of broiler production. The solution set of the model was shown to be bounded, positive and unique. The importance of existence and uniqueness of solution is to identify the determinants of broiler production in Nigeria for significant improvement among broiler producers. Based on the findings of this paper, broiler production data on capital, previous training, years of experience and stocking capacity should be collected, fitted into the existing model, simulated over a period of time and make general assessments and forecasts on the potential impacts of the incorporated variables.

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