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Discretization of Laplacian Operator in Polar Coordinates System on 9-Point Stencil with Mixed PDE's Derivative Approximation Using Finite Difference Method

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Abstract:

In this research paper,we have discretized the nine point stencils inclduding four neighbourer point of symmetric and asymmetric lines from the centre of proposed molecule for the laplacian operator by implementing polar gird system.It is the modifiied five point scheme.For the discretization of laplacian operator and concerning the explicit PDES and mixed derivative the finite onward time central space differencing has been conceded Polar mesh system is very important to source out the track preference ascending to computational and mathematical models.In ordinary differntial equation ODES the laplacian operator labeled so many physical,fluid dynamics and chemical models just as gratvitational potentials and electric,diffusion equation of heat,steady flow indcluding fluid flow with its types ,circulation of wave and physics of quantumn and so many other examples.

Keywords: *Finite differences method 65F05*.

Introduction:

Isotropic discrete operator of lapacian equation is very important and major and important requirement are considerating,operating and estimating to resove the modern mathematical and computational simulations.The precesion of Finite Difference techanique is associated to the addition of stencils and assilmilation to upper direction terms in the planned order.The modern planned a Finite Difference formula,is being descreibed as a isotropic,which indicates that the process of descritization for laplacian in PCS is conceded over the techanique of explicit of finite differecing scheme.The perference of PCS is considered to keep away from anistropy within inside the compuatatuional consequences,PDEs is descritized on applicable alternative of polar mesh structure and BVC.The stability,representation and steadiness exploration will be passed out for planned structure.

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Laplacian Operator and its Scope.

In this computational world, for the derivation of discretization of some fundamental differentiation of numerical results the laplacian operator keeps important role, this laplacain operator is represented as $\nabla^2 u$ here u is numerical axioms of operator which is non identical in field space. The indicated term characterizes the CCS Cartesian coordinate system and keeps huge number of physical assets and usage in CCS such mathematical simulation are transformation of heat in shapes of conduction, convection and radiation as well. In addition diffusion of wave and fixed length are commonly simulated by PDES and the PDES comprises the operators such as Gradient, divergence curl and laplacian. [1]. for simulation of physical computational and mathematical models associated problems laplacian plays important rule. In addition it contribute in physics such as quantum waves and classical mechanics which is simulated on designed models such as mathematical or computational models [2].This is the operator which is being used in wide range in engineering field and fluid mechanics specially electricity, fluid flow, and steady heat conduction [3].The discrete laplacian operator is functioned for some various types of geometric models such as surface renaissance, shape depiction, shape utterance and gird insertion [4]. Laplacian or Poisson equation of linear self-governing underwrites a significant discussion of modeling which is based on boundary value condition (BVC) including this operator is being used to resolve in many classical and absolute gird structure or system. In ODES the boundary value condition is being practiced for numerical models.in case the given perimeter section is ingenuous than logical sections are valuable to slog out on given rectilinear problems for precise field that consensus to parallelepipeds in various systems [2]. Latest computational, mathematical and numerical structures is applied to express and used on these fundamental mathematical models problems for resolving higher order numerical approximation. For such numerical approximations there are three major method which is being used for resolving the PDES with reliable outcomes, which are finite difference method FDM, Finite Element method FEM and Finite volume method FVM [5].FDM is not only applicable in computational fluid dynamics but also comprehensively applied in several computational and numerical models such as steady state flow and including approximation of gradient function, image processing which are commonly considered as a numerical approximation of first differentiation [6].This method has wide range of usage and able to implement very quickly as compare to Fourier Transform. It followed BVC problems with nonlinear examples which is widely applied in mesh system of finite difference method [7] and also applied on linear 2nd order Partial differential equation. The scientific outcomes on computational and numerical girds by discretization of quantity dominion disturbed to FDM. Furth more it comprises the system of linear differential equation is exactly and perfectly resolved with the help of discretization of 1st and 2nd order equations. By his huge consequences the linear differential equations creates a stare for sparse matrix which is being classified by some well-known iterative methods such as Gauss-Jacobi Method, Gauss Seidal Method Gauss elimination optimization and LU decomposition techniques etc [2]. The PDES is anxious with the theory of variety and it received the scientific and numerical results of classical and discretization operator to be evaluated. Quantum theory and it is basic need of the discretization of operator of differential is widely applied in lesser order of FDM and the correctness of this technique is unswervingly associated with the length of stencils [8,9].

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METHODOLOGY AND DERIVATION OF MOLECULE.

The Laplacian operator under consideration is given below in Cartesian coordinate system.

$$\nabla^2\omega = \frac{\partial^2\omega}{\partial x^2} + \frac{\partial^2\omega}{\partial y^2} \qquad (1)$$

Following is the polar form of Laplacian operator in the continuous equation

$$\nabla^2\omega = \frac{\partial^2\omega}{\partial r^2} + \frac{1}{r}\frac{\partial\omega}{\partial r} + \frac{1}{r^2}\frac{\partial^2\omega}{\partial\theta^2}$$

I. Pictorial form of Molecule

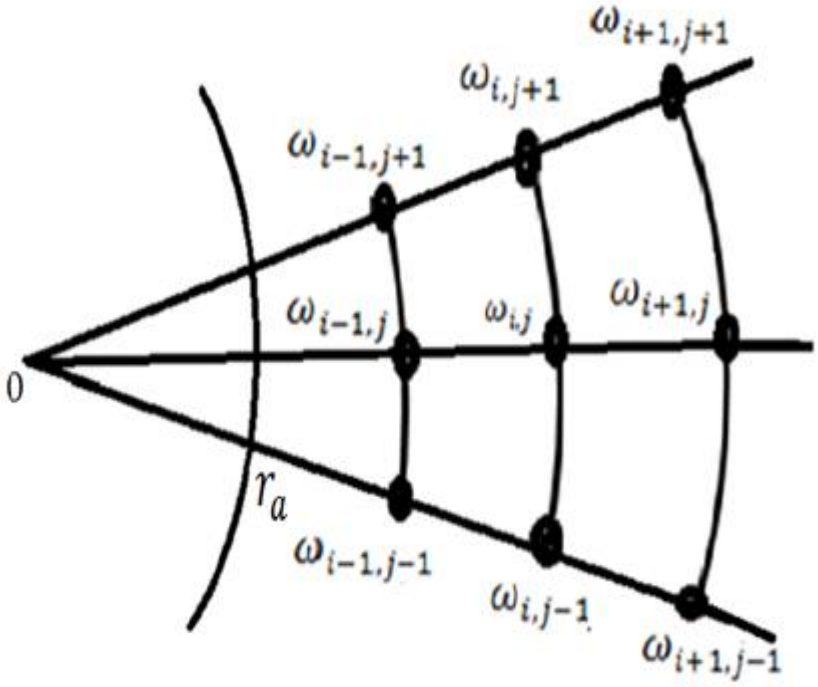

Figure 1 Proposed Molecule

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II. Derivation of molecule.

$$\frac{\partial^2\omega}{\partial r^2}=\frac{1}{4(\Delta r)^2}\left[\omega_{i+1,j+1}+\omega_{i-1,j+1}+\omega_{i+1,j-1}+\omega_{i-1,j-1}+2\omega_{i+1,j}+2\omega_{i-1,j}-8\omega_{i,j}\right]\dots\dots\dots\dots(1)$$

$$\frac{\partial\omega}{\partial r}=\frac{1}{8(\Delta x)}\left[\omega_{i+1,j+1}+\omega_{i+1,j-1}-\omega_{i-1,j+1}-\omega_{i-1,j-1}+2\omega_{i+1,j}-2\omega_{i-1,j}\right]\dots\dots\dots(2)$$

$$\frac{\partial^2\omega}{\partial\theta^2}=\frac{1}{4(\Delta\theta)^2}\left[\omega_{i+1,j+1}+\omega_{i-1,j+1}+\omega_{i+1,j-1}+\omega_{i-1,j-1}+2\omega_{i,j-1}+2\omega_{i,j+1}-8\omega_{i,j}\right]\dots\dots\dots\quad(3)$$

Put the above derivative in to polar form equation no (4)

$$\nabla^2\omega=\frac{\partial^2\omega}{\partial r^2}+\frac{1}{r}\frac{\partial\omega}{\partial r}+\frac{1}{r^2}\frac{\partial^2\omega}{\partial\theta^2}\dots\dots\dots\dots(4)$$

$\nabla^2 w$ Is equal to

$$\frac{2r^2(\Delta r)^2(\Delta\theta)^2)}{(2r^2(\Delta\theta)^2+2(\Delta r)^2)}\begin{bmatrix}\frac{1}{4(\Delta r)^2}\begin{bmatrix}\omega_{i+1,j+1}+\omega_{i-1,j+1}+\omega_{i+1,j-1}\\+\omega_{i-1,j-1}+2\omega_{i+1,j}+2\omega_{i-1,j}\end{bmatrix}+\\ \left[\frac{1}{8r(\Delta r)}\begin{bmatrix}\omega_{i+1,j+1}+\omega_{i+1,j-1}-\omega_{i-1,j+1}\\-\omega_{i-1,j-1}+2\omega_{i+1,j}-2\omega_{i-1,j}\end{bmatrix}\right]+\\ \frac{1}{4r^2(\Delta\theta)^2}\begin{bmatrix}\omega_{i+1,j+1}+\omega_{i-1,j+1}+\omega_{i+1,j-1}\\+\omega_{i-1,j-1}+2\omega_{i,j-1}+2\omega_{i,j+1}\end{bmatrix}\end{bmatrix}-\omega_{i,j}\dots(5)$$

Which the required polar meshed nine-point formula is for scheme I using Laplacain operator.

III. Example and Results.

$$w_{(r,\theta)}=r^3 sin\theta$$

For the discretization scheme following notation are adopted

$$r_i=r_a+i(\Delta r)\qquad i=0,1,2,3\dots\dots\dots\dots n$$

$$\theta_j=j(\Delta\theta)\qquad i=0,1,2,3\dots\dots\dots\dots\dots\dots n$$

Some important value of proposed scheme.

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Table No 1: Results of planned molecule is given below

r	Radian	Exact Value	Approximate value	Error
3.1	0.017453293	0.00907486	0.009089024	1.42E-05
3.1	0.034906585	0.01814971	0.018178048	-2.83E-05
3.1	0.052359878	0.05235988	0.027267067	-4.25E-05
3.1	0.06981317	0.03629941	0.036356088	-5.67E-05
3.1	0.087266463	0.04537427	0.045445103	-7.08E-05
3.2	0.017453293	0.0099817	0.009996327	-1.46E-05
3.2	0.034906585	0.01996341	0.019992653	-2.92E-05
3.2	0.052359878	0.02994511	0.029988976	-4.39E-05
3.2	0.06981317	0.03992681	0.039985303	-5.85E-05
3.2	0.087266463	0.0499085	0.04998162	-7.31E-05
3.3	0.017453293	0.01094704	0.010962118	-1.51E-05
3.3	0.034906585	0.02189441	0.021924233	-3.02E-05
3.3	0.052359878	0.03284111	0.032886349	-4.52E-05
3.3	0.06981317	0.04378814	0.043848459	-6.03E-05
3.3	0.087266463	0.05473516	0.054810569	-7.54E-05
3.4	0.017453293	0.01197268	0.011988222	-1.55E-05
3.4	0.034906585	0.02394537	0.023976443	-3.11E-05
3.4	0.052359878	0.03591805	0.03596466	-4.61E-05
3.4	0.06981317	0.04789073	0.047952872	-6.21E-05
3.4	0.087266463	0.0598634	0.059941087	-7.77E-05
3.5	0.017453293	0.01306043	0.013076467	-1.60E-05
3.5	0.034906585	0.02612094	0.026152935	-3.20E-05
3.5	0.052359878	0.03918141	0.039229401	-4..79E-05
3.5	0.06981317	0.05224188	0.052305859	-6.40E-05
3.5	0.087266463	0.06530233	0.065382317	-8.00E-05

Figure 2 shows the results with error of planned scheme.

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Table for the values of graph showing stability

Change in Δr	*Change in Error*
0.000001	-5.7220459E-06
0.00001	0
0.0001	-3.8146973E-06
0.001	-1.1440918E-05
0.01	-4.7492981E-04
0.1	-4.65087891E-02

Figure 3 shows the stability analysis of error.

Stability analysis by graphically

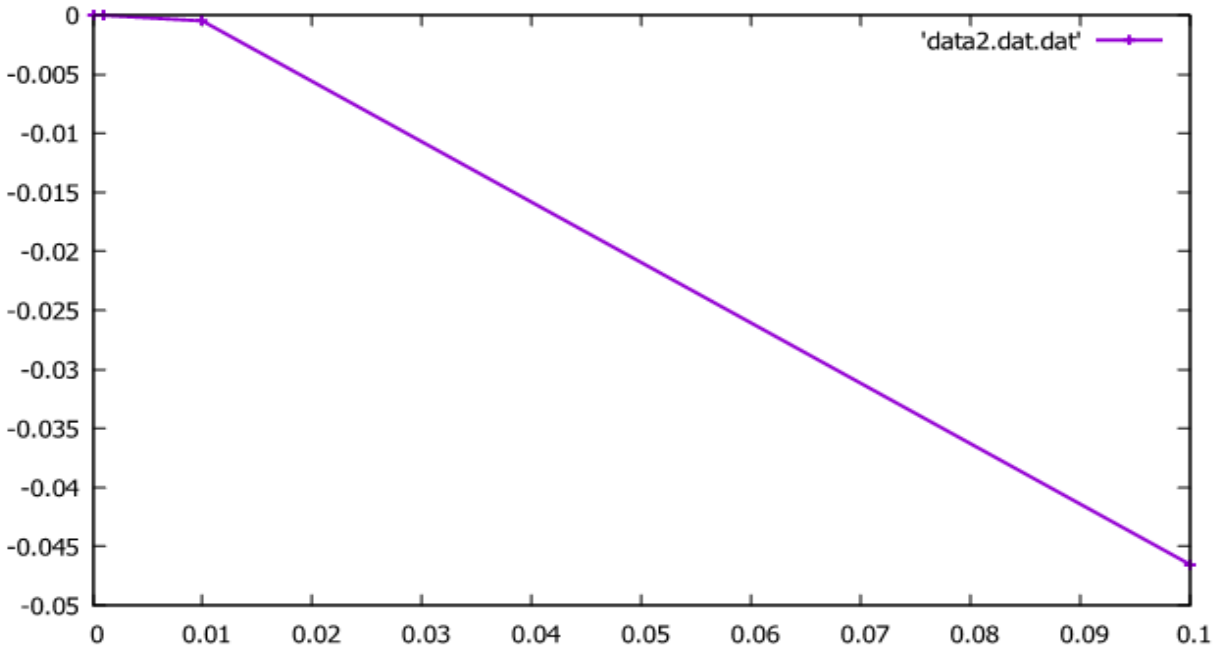

Figure 4 shows the stability of the scheme by graphically

Stability analysis of proposed scheme.

The stability analysis is found out in planned proposed molecule for the discretization of PCS. The approximation outcomes has been acquired for reducing Δr against error.There are some different values have been taken for Δr, which can be seen in above figure. The plotted figure shows that the error reduces linearly and hastily among reduced value of Δr 0.1 towards 0.00998302.As reducing the value from 0.00998302 the error converge very quickly to zero.

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DISCUSSION

Here nine point discretization laplacian equation in polar coordinate system has be developed and acquired the outcomes with the help of FORTRAN code of computational language. The developed outcomes of discretization of planned scheme in polar coordinate are present in Table 1.The estimation results of proposed scheme indicates the accuracy. In these results of isotropic discrete laplacian operator, accuracy remains more than six decimal point.

FUTURE WORK

I. There is need of time to work on the literature of partial differential equation as well as on mixed derivative approximation by using FDM, to evaluate the discretization in polar coordinate system on thirteen, seventeen and twenty five stencils.

II. It is the need of time to work on the literature of mixed partial differential equation by applying the FDM to discretize the laplacian operator in cylindrical and spherical coordinates system on thirteen, seventeen and twenty five stencils.

REFERENCES

[1] A. Bruno-alfonso and H. A. Navarro, "Alternate treatments of jacobian singularities in polar coordinates within," vol. 8, no. 3, pp. 163–171, 2012.

[2] A. S. Reimer and A. F. Cheviakov, "A Matlab-based finite-difference solver for the Poisson problem with mixed Dirichlet-Neumann boundary conditions," *Comput. Phys. Commun.*, vol. 184, no. 3, pp. 783–798, 2013.

[3] A. K. Mitra, "Finite Difference Method for the Solution of Laplace Equation."

[4] E. Abstract and Y. Wang, "Discrete Laplace Operator on Meshed Surfaces."

[5] M. Esmaeilzadeh, "Scholarship at UWindsor A Cartesian Cut - Stencil Method for the Finite Difference Solution of PDEs in Complex Domains," 2016.

[6] S. L. M. El-hachem and J. T. M. Reggio, "High Order Spatial Generalization of 2D and 3D Isotropic Discrete Gradient Operators with Fast Evaluation on GPUs," pp. 545–573, 2014.

7] M. Lai, Z. Li, and X. Lin, "Fast solvers for 3D Poisson equations involving interfaces in a finite or the infinite domain," vol. 191, pp. 106–125, 2006.

[8] E. Letters, T. Madras, and M. Sciences, "Lattice differential operators for computational physics," no.

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January 2017, 2013.

[9] V. Thomã, “From ÿnite di erences to ÿnite elements A short history of numerical analysis of partial di erential equations,” vol. 128, pp. 1–54, 2001.

[10] A. Kumar, “Isotropic finite-differences,” *J. Comput. Phys.*, vol. 201, no. 1, pp. 109–118, 2004.

[11] C. Shin and H. Sohn, “A frequency-space 2-D scalar wave extrapolator using extended 25-point finite-difference operator,” *Geophysics*, vol. 63, no. 1, pp. 289–296, 1998.

[12] F. Costa, D. De Oliveira, E. Ogasawara, A. A. B. Lima, and M. Mattoso, “Resource.