A BRIEF SURVEY ON DYNAMIC NETWORK FLOWS IN CONTINUOUS-TIME MODEL

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Abstract

In dynamic network flows, when some commodities move from one place to another, time taken by them to traverse the path are taken into account, unlike the static flows. Time being continuous entity, network flow problems are modeled in its continuous setting to get more realistic solution to the optimization problems. Rather, it can also be discretized to get more computational results. In this paper, insight is given to the models formulated by the researchers in dynamic network flows along with some practical applications regarding the triple optimization problem in continuous time setting mainly focusing on cost minimization problems, since the quickest path problem and the maximum flow problem are considered as two important special cases of the minimum cost flow problem. Different solution strategies and their efficiencies carried out by them are also observed. Further possible modification in the existing formulations and algorithms for better solution of problems are suggested as research endeavor.

Keywords: *Network optimization, dynamic flows, continuous-time model, discretization*

INTRODUCTION

Network flow problems are widely faced problems in natural as well as social sciences and their various applied disciplines. In network flow problems, to solve the three fundamental problems of optimization: minimum cost flow problem, the quickest path problem, and the maximum flow problem, sufficient formulations and algorithms are developed. Further modifications on those formulations and algorithms have also been done by many researchers based on different situations, necessity, and assumptions. Among many books, the book by Ahuja et al.[1] is one of the books to get knowledge about these interest and concern.

In traditional network flows models, the transit time on arc was ignored and only the flows were assumed to change over time. But in many real world problems, traversal time plays a vital role. In many real-world network problems, due to fluctuation in seasonal demands and supplies, arc costs and capacities, storage costs and capacities, flow values on arc etc. vary with respect to time. Such type of flows are called *dynamic network flows.* Fleischer and Skutella [17] used the term *network flows over time* instead of *dynamic network flows.* Unlike the static flow models, the following three features are considered in dynamic flows: *flow value on an arc varies over time, transit time of flow along an arc is finite, and waiting is allowed at the nodes.*

Ford and Fulkerson [19, 20] introduced the concept of dynamic networks flows. They studied to send *maximum dynamic flow* from a source to a sink (target) in discrete time setting, and solved in minimum cost by taking transit times as arc costs.

Among various types of network flow problems, the dynamic maximum flow problem, the quickest flow problem, and the dynamic minimum cost flow problem are discussed here. In these problems, the parameters related to arcs and nodes are taken constant or variable and time is taken either in discrete steps or in continuous setting according to the nature of the problem.

The temporal dimension of dynamic network flows give a more real modeling tool for real world problems such as communication networks, production systems, financial flows, transportation, road traffic control and logistics etc. When time is modeled in continuous setting, the problems become more complex and less computational, that's why enough literature are not found so far.

In discrete-time setting, time-expanded network are used to develop more efficient algorithms. The discrete-time model is computationally easier but not realistic to world problems and hence continuous-time model is carried out.

Fleischer and Tardos [18] studied the relation between the discrete and continuous-time dynamic flows models. Some results and algorithms for the discrete-time model even in case of non integral time were transformed to the continuous-time model by them. These results do not remain true when network parameters fluctuate over time. There exists a one-to-one correspondence between these models, if the network parameters are independent of time and the transit times and the time horizon are integral.

The organization of the paper is as follows. Section 2 gives literature review related to dynamic network flows regarding to various aspects of flow and storage mainly focusing on continuous-time model. The notations used in the formulation of flow models are introduced in Section 3. Starting from the triple optimization problem models, some models related to them are introduced along with scaling and discretization of time and bounding of the problem. Section 3 is closed by introducing an example with further splitting of time intervals. The comparison of efficiency and deficiency of uniform discretization, descent and adaptive discretization algorithms are discussed in Section 4 and is followed by conclusion in Section 5.

LITERATURE REVIEW

Philpott [33] introduced the formulation of maximum flow problem in continuous-time setting. It was further studied by Anderson et al.[4] taking variable arc and storage capacities omitting the transit time. They derive a max-flow min- cut theorem and also introduced the concept of source-sink generalized cut over time. The flows vary with time and are taken as Lebesgue measurable and storages is allowed at the nodes. Later, Philpott [32] extended their work by taking arbitrary transit time.

Dynamic network flow in continuous time (DNF-CT) to find total minimum cost of flow and storage was studied by Anderson [2]. He also introduced the concepts of arc linked, arc connected, arc cycle, improper node-time pair and characterization of the extreme point solutions. Later, Anderson and Philpott [6] surveyed network flows results related to continuous-time and DNF-CT. They introduced a weak duality result and show strong duality holds satisfying some complementary slackness conditions.

Continuous-time network flow problem (CNFP) was first introduced by Philpott [31]. DNF-CT is known as CNFP, if transit times on the arcs and storage capacities and costs at the nodes are not considered. Anderson and Philpott [5] studied further about CNFP. They develop a continuous time simplex algorithm for CNFP assuming flows cost piecewise linear and other functions (supply and demand rates and arc capacities) piecewise constant. The convergence of this algorithm to an optimal solution may not be guaranteed. Following the same assumptions as Anderson and Philpott did in [5], Philpott and Craddock [34] developed an adaptive discretization algorithm for solving CNFP and applied satisfactorily.

A typical problem which occurs in both communication and transportation networks in which one seeks to direct an accumulation of traffic at the nodes of the network to a destination node in minimum time was first addressed by Segall [42]. Segall purposed a continuous time model for which optimal routing strategies are easier to compute.

For a problem of continuous time dynamics, Orda and Rom [29] give an algorithm and convergence result. Philpott and Mees [35] studied on quickest path problems with holding and starting costs and extended the study to provide a practically useful terminating algorithm. Lovetskii and Melamed [26] describe several problems and techniques in both discrete and continuous time. Powell et al [36] focus on modeling issues.

To turn an arbitrary feasible solution into an extreme point solution without degrading the value of objective function, purification algorithm for *separated continuous linear program* (SCLP) was developed by Anderson and Pullan [7]. In SCLP, storage and processing constraints are separated as: integral and instantaneous constraints. Extensive study on SCLP has been carried out by Pullan through his series of papers.

MATHEMATICAL FORMULATION OF THE PROBLEM

Consider a directed network N = (V, A) with set of vertices V and set of arcs A, such that A ⊤ V × V i.e; vertices are the crossing-over of arcs. Let f_{ij} : A × [0,T] → $R_{\geq}0$ be a continuous-time dynamic flow in arc (i, j) over the time horizon [0,T], where f_{ij} (t) is the rate of flow entering the arc (i, j) in time t. Also, u_{ij} (t), w_{ij} (t), and τ_{ij} (t) respectively be an upper bound on the flow rate, cost of unit flow and traversal time in arc (i, j) at time t.

Let s_i: V × [0,T] → $R_{\geq}0$ be a continuous-time dynamic storage at vertex i over [0,T]. Also, v_i (t), x_i (t), and m_i (t) respectively be the storage capacity, per time unit storage cost per unit flow, and supply/demand rate at vertex i at time t.

The formulation for DNF-CT for minimum cost first introduced by Anderson [2] is as follows:

DNF-CT :

$$\min \quad \int_0^T w(t)f(t)dt + \int_0^T x(t)s(t)dt \qquad (1)$$

$$\int_0^t \sum_{j:(i,j)\in A} f_{ij}\,(\mathrm{t})dt - \int_0^t \sum_{j:(j,i)\in A} \sum_{t_1:t_1+\tau_{ji(t_1)}=t} f_{ji}\,(\mathrm{t}_1)\mathrm{dt} + s_i(\mathrm{t}) = s_i(0) + \widetilde{m_i}\,(t) \qquad (2)$$

$$\widetilde{m_i}\,(t) = \int_0^t m_i\,(\mathrm{t})\,dt \qquad (3)$$

$$0 \leq f(t) \leq \mathrm{u(t)},\ 0 \leq s(t) \leq \mathrm{v(t)},\ \mathrm{i} \in V,\ \mathrm{t} \in [0,\mathrm{T}] \qquad (4)$$

Maximum flow problem (MFP) and *shortest path problem* are considered as special cases of *minimum cost problem*. Here we consider *shortest path problem* as *minimum time problem* (MTP). Their formulation in continuous-time version are given as follows:

MFP :

$$\max \quad \int_0^T \sum \,[f_{ji}\left(t-\tau_{ji}\right) - f_{i\,j}(t)]dt \qquad (5)$$

$$\text{s.t.} \quad \int_0^t \sum f_{ij}\,(\mathrm{t})\,dt - \int_0^t \sum \, f_{ji}\left(t-\tau_{ji}\right) + s_i(\mathrm{t}) = s_i(0) \qquad (6)$$

$$0 \leq f_{ij} \leq u_{ij}\,,\ 0 \leq s_i \leq v_i \qquad (7)$$

MTP :

$$\min\ \mathrm{T} \qquad (8)$$

$$\text{s.t.} \quad \int_0^t \sum f_{ij}\,(\mathrm{t})\,dt - \int_0^t \sum \, f_{ji}\left(t-\tau_{ji}\right) + s_i(\mathrm{t}) = s_i(0) \qquad (9)$$

$$0 \leq f_{ij} \leq u_{ij}\,,\ 0 \leq s_i \leq v_i\,,\ \mathrm{s(t)} = 0,\ \mathrm{t} \geq \mathrm{T},\ f_{ij}\,(t) = 0,\ \mathrm{t} \geq \mathrm{T} - \tau_{ij} \qquad (10)$$

Some related models. DNF-CT is reduced to *continuous-time network flow problem* (CNFP), if transit times on the arcs and storage capacities and costs at the nodes are ignored. Other models related to DNF-CT are *separated continuous linear program* (SCLP) and SCLP with traversal time

(SCLP-TT). CNFP is a special type of SCLP. Bellman [9] introduces *continuous linear problem* (CLP) to model some economic processes called *bottleneck processes*.

CNFP : (Philpott '85)

$$\min \int_0^T w_{i,j}(t) f_{i,j}(t) dt \tag{11}$$

$$\text{s. t.} \int_0^t \sum_{j:(i,j)\in A} f_{ij}(\mathrm{t})\ dt - \int_0^t \sum_{j:(j,i)\in A} f_{j,i}(t) dt + s_i(\mathrm{t}) = s_i(0) + \widetilde{m_i}(t) \tag{12}$$

$$\widetilde{m_i}(t) = \int_0^t m_i(\mathrm{t})\, dt \tag{13}$$

$$0 \le f_{i,j}(t) \le u_{i,j}(t), \quad 0 \le s_i(t),\ \mathrm{i} \in V,\ \mathrm{t} \in [0,\mathrm{T}] \tag{14}$$

SCLP : (Anderson'78)

$$\min \int_0^T w(t) f(t) dt \tag{15}$$

$$\text{(storage)} \qquad \int_0^t K f(t) dt + \mathrm{s(t)} = \mathrm{u(t)} \tag{16}$$

$$\text{(processing)} \qquad \mathrm{H\ f(t)} \le \mathrm{v(t)} \tag{17}$$

$$0 \le f(\mathrm{t}), \mathrm{s(t)}, \mathrm{t} \in [0,\mathrm{T}] \tag{18}$$

SCLP-TT: (Pullan'97)

$$\min \int_0^T w(t) f(t) dt \tag{19}$$

$$\int_0^t (K f(t))_i\ \mathrm{d}t + \sum_{j=1}^{n_1} \int_0^t \eta_{ij} f_j\left(t - \tau_{ij}\right) dt + s_i(t) = u_i(t) \tag{20}$$

$$\mathrm{H\ f(t)} \le \mathrm{v(t)} \tag{21}$$

$$0 \le f(\mathrm{t}), \mathrm{s(t)} \tag{22}$$

$$\mathrm{t} \in [0,\mathrm{T}] \tag{23}$$

CLP : (Bellman- bottleneck process)

$$\min \int_0^T w(t) f(t) dt \tag{24}$$

$$\text{s. t.} \qquad B(t) f(\mathrm{t}) + \int_0^{\mathrm{t}} \mathrm{K}(\mathrm{t}_1, \mathrm{t})\, \mathrm{f}(\mathrm{t}_1) \mathrm{dt}_1 \le \mathrm{v(t)} \tag{25}$$

$$0 \le f(\mathrm{t}),\ \mathrm{t} \in [0,\mathrm{T}] \tag{26}$$

In above formulations, the dimensions of the functions are taken such that the operations are defined.

Scaling of time and dimensions of variables. DNF-CT is most closely related to the SCLP-TT studied by Pullan [38] by assuming transit time and time horizon rationals. These are scaled by some common denominator so that they are changed into integers. Then, the dimensions of variables scaled by time horizon, which was increased enormously by previous scaling. These scalings make the problem huge and is difficult to use his algorithm. For irrational and time-varying transit times this transformation can no longer be used.

No restriction in transit time. Unlike the assumption of Pullan, Nasrabadi takes transit time as irrational and not constant. He assumes arc costs (w) and storage capacities (v) piecewise linear and supply/demand rate (m), storage cost(x), arc capacity (u) and transit time (τ) as piecewise constant. So, Pullan and other used scaling technique and Nasrabadi used discretization technique.

Discretization of time and bounding. Although, continuous-time model is realistic, it is more difficult to handle. Discrete-time model is more computational and easier to handle then continuous one but it cannot give optimum solution in general situations. Here, we discretize the time horizon corresponding to two ways: averaging the network properties over each sub-interval and corresponding to dual problem of DNF-CT (DNF-CT*).

To get a lower and upper bound respectively on the optimum solution value of DNF-CT, the discretized problems can be solved by the well-known minimum cost flow algorithms on a time-expanded network. Finer the partitions of time closer will be the solution to the optimum value of DNF-CT. In case of uniform discretization, where sub-intervals of equal lengths are made, information from solutions of discrete approximations is not used to further finer process.

While computing an upper bound on the optimum solution value of DNF-CT, a non-uniform time-expanded network of N, $N_U(P)$ is created, where P is given set of partition. Any static flow in $N_U(P)$ corresponding to a dynamic flow in N is shown by defining dynamic flow and storage in terms of their static values. Also it's converse result is shown by averaging dynamic flow value on any arc in each time interval and by interpreting storage as the amount of flow on the corresponding waiting arcs. Moreover dynamic flow in N and static flow in $N_U(P)$ have equal cost.

Lower bound on the optimum solution value of DNF-CT is computed by formulating the dual of DNF-CT. This dual is based on that given by Pullan [37] as the dual of SCLP.

DNF-CT*:

$$\max \quad -\int_0^T s(0) + \tilde{m}_i(t)\ d\zeta(t) - \int_0^T u(t)\eta(t)\ dt - \int_0^T v(t) d\xi(t) \tag{27}$$

$$\text{s. t.} \quad \zeta_i(t) - \sum_{t_1:\, t_1 - \tau_{i,j}(t) = t} \zeta_j(t_1) - \eta_{i,j}(t) \le w_{i,j}(t), \quad (i, j) \in A,\ t \in [0,T] \tag{28}$$

$\zeta(t)$, $\xi(t)$ and $\eta(t)$ are decision variables of DNF-CT*

Generally, weak duality condition holds between the value of DNF-CT and it's dual. Strong duality holds between them under some complementary slackness conditions. Corresponding to DNF-CT* and further half split of given partition another discretized formulation is done to give lower bound of DNF-CT.

The feasibility of DNF-CT and the discretization problem giving upper bound and that of DNF-CT* and lower bound giving discretization problem are shown in various corollaries and lemmas in [22]. Here we summarize the weak duality condition and bounding of value of continuous-time model of DNF as,

V [LB(Q)] ≤ V [DNF-CT*] ≤ V [DNT-CT] ≤ V [UB(P)]

V [LB(Q)] : value of lower bound discretized problem with partition Q

V [UB(P)] : value of upper bound discretized problem with partition P.

Reduction in duality gap. In case of continuous-time linear programming problems, the objective value of the given problem may or may not be equal to the objective function value of the dual problem But in finite-dimensional linear programming problems, it is yes [3].

The difference between the objective function value for DNF-CT and LB(P) is bounded (see lemma [22]) and it's value tends to zero when n tends to infinity in the sequence of partitions $\{P_n\}^{\infty}_{n=1}$ in P (see corollary [22]) which leads to the absence of duality gap between DNF-CT and DNF-CT*.

Example 1. *To compute the lower and upper bound on the optimum value of DNF-CT, the formulation is discretized.*

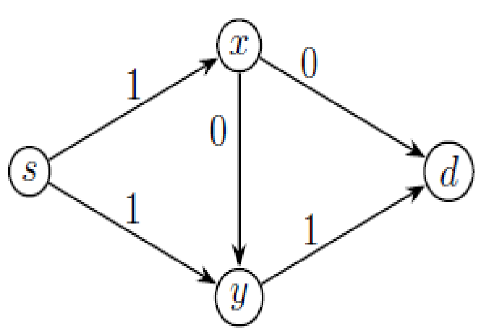

Figure 1: A simple instance example of given network arc with transit time

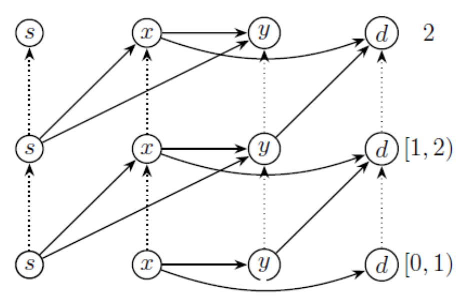

Figure 2: Time-expanded network of figure 1 over T = 2

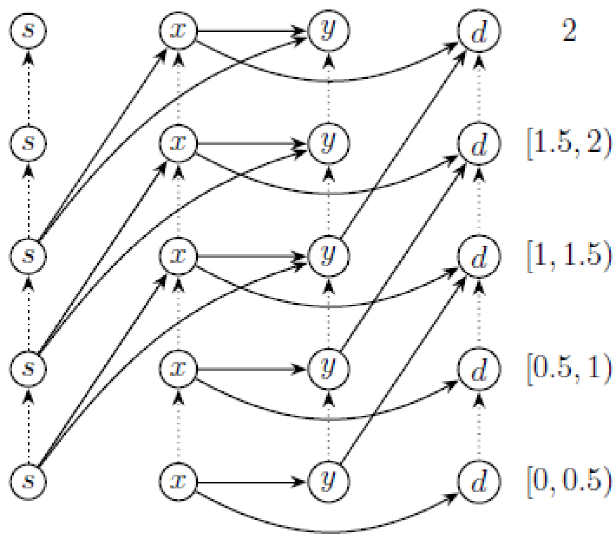

Figure 3: Time-expanded network of figure 1 over T = 2 with further splitting in time intervals

METHOD OF SOLUTION

For a given partition P, the values of LB(P) and UB(P) respectively give the lower and upper bound on the optimum solution value. With finer and finer partition, their values converge to the value of DNF-CT.

Uniform discretization (UD) algorithm. In this algorithm, partition of equal size intervals are taken and breakpoints are successively made doubled to reduce the duality gap so that the value of the discretized problems converge to the optimum value of DNF-CT.

Descent algorithm (DA). In UD algorithm no information from the solutions of LB(P) and UB(P) is used in finer process and the number of breakpoints grows exponentially. To overcome the drawbacks of UD algorithm, DA algorithm was developed by Pullan [37] for SCLP and extended to DNF-CT. This algorithm successively improves the objective function value until an optimum solution is obtained by moving between feasible solutions of DNF-CT. Convergence of DA is discussed by Hashemi and Nasrabadi [22].

Adaptive discretization (AD) algorithm. In DA, breakpoints are increased by a multiple of 2 or 3 at each iteration. This may be impractical. So, AD algorithm based on the concepts of *node time pair* (NTP) was proposed by Philpott and Craddock [34]. This algorithm can also be used in other cases more beneficially.

In finer process, since the redundant breakpoints of the partition increase the size of the sub-problem, so it is better to remove them. Their removable is done so that the convergence of algorithm is guaranteed It means the redundant breakpoints are removed only if, difference of values of discretized problem is reducing and the *adaptive discretization with removable of breakpoints* (ADR) algorithm is proceeded.

CONCLUSION

In this paper, we presented a review of the main literature on dynamic network flows focusing on continuous-time setting. Various flows models such as; maximum flow, minimum cost and minimum time, which are the norms of optimization are presented. Time can be discretized to get more computational results. In these models, the values of parameters and decision variables are taken at every instant of time respecting their corresponding capacity constraints, which is a challenging job. To make the execution of the models more soft and to reduce the complexity of convergence of algorithms, some assumptions such as; aggregate arc capacity and aggregate transit time etc will be taken as our further research endeavor.

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