COMPARATIVE STUDY OF FUZZY MATHEMATICS AS FUZZY SETS AND FUZZY LOGICS

Sonal Dixit [1], Kirti Verma [2]

[1] *Associate Professor,*
Department of Mathematics,
SJHS Gujarati Innovative College of Commerce and Science,Indore (M. P.) India.

[2] *Associate Professor,*
Department of Engineering Mathematics,
Lakshmi Narain College of Technology, Bhopal (M.P.), India.
Corresponding Author Email: kirtivrm3@gmail.com

Abstract:

Fuzzy mathematics is considered to be an important aspect in the field of mathematics that interprets the uncertainties and deals with the unreliable information and vagueness of data. In this chapter we shall discuss the concept of fuzzy mathematics as fuzzy set and fuzzy logics and the beginning of fuzzy set theory and the fuzzy logics with their applications in the real life. As fuzzy mathematics and fuzzy logics are becoming increasingly significant as it is applied in almost every field of developments, engineering design and models and in new technologies also. We also discuss some recent models of fuzzy logics given by T. Patro[8] and C.K. Muthumaniraj [2] Fuzzy logics are playing very important role in many areas. I tried to make this chapter as a strong base for researchers and students so that it can prove strong base for further researches for comparative study of fuzzy set theory and fuzzy logics.

Keywords: *Fuzzy metric space, Fuzzy set theory, Fuzzy logics, Real life application of fuzzy logics*

INTRODUCTION

Fuzzy mathematics is that section of mathematics which includes set theory of fuzzy and logic concepts of fuzzy. First time in 1965 Zadeh work on it. The brain wave of fuzzy, first time proliferate by Zadeh in 1965 seminar paper known as 'Fuzzy sets'. He workedon contor's binary theory and extends their result by adapting the concept of degree of belonging and relationship. After his work used almost all the field of contemporary mathematics and discover the new domain of mathematics in formof new disciplines like fuzzy topology, fuzzy arithmetic, fuzzy algebraic structure, fuzzy geometry, fuzzy differential calculus, fuzzy relation calculus, fuzzy databaseand fuzzy decision making. Initial this concept of fuzzyfication was introduced simply by changing contor's theory using extension of zadeh.

In 1980's they introduce the triangular norms and co-norms for extension of fuzzyffication characteristics. In this chapter i want to give the attention on field a fuzzyand the discoveries and growth of domain of contemporary mathematics.

In recent times the fuzzy logic has been proved as a very powerful tool to solve the uncertainties in many fields in the processes of developments of technologies. In 2016 T. Prato [8] has explained how model based on concept of fuzzy logic is helpful in management actions during climate change. Also in 2019 C.K. Muthumaniyaraja and M. Chinnamuthu [2] give the information how fuzzy logical designs are working for spatial planning. Similar as in 2018 Modestus and Angella have lightened the application of fuzzy logical model for production, distribution and exploration operation of Petroleum.

Basically fuzzy mathematics is a combo of fuzzy set theory and fuzzy logics. In this chapter I will try to cover that part of fuzzy mathematics which can prove the strong base for further studies and discoveries in this field. The fuzzy subset can be defined as a subdivision (subset) of a set 'A' in a interval I [0, 1] such that $F : X \rightarrow I$

where I is the set [0, 1] and fuzzy set F is a function is known as a membership mapping is a abstraction of a characteristic mapping or a indicator mapping of a subset specify on interval I [0, 1].

GROWTH OF FUZZIFICATION

As mentioned above growth of a fuzzyffication can be studied in following part:

⇒ The discoveries and work in the course of sixties decades and seventies decades.

⇒ The growth in decades of eighties.

⇒ The Fuzzyffication and standardization in decades nineties.

Generally the fuzzyffication of these concepts is the growth of these brain waves from characteristic mapping to membership mapping

A straightforward fuzzyffication in general is based on the (min) andminimum maximum (max) for example.

Let P and Q have being two fuzzy subsets out from X. the operation of union $\cup$ and inter traction $\cap$ are specified by

$(P \cap Q)(X) = min\,(P(X), Q(X)).$

And

$(P \cup Q)(X) = \max\,(P(X), Q(X)).$

Rather than using min or max one can use t-norm for example min (P, Q) can be replace by PQ.

In case of binary operation in group theory a most important abstraction property used in fuzzyffication is closure property. For example
The closure axiom for a non empty set X for a fuzzy subset A is describe by $\forall x, y \in X$

$$A(x * y) \geq min\ (A(x), A\ (y)).$$

Now presuppose A have being a subset of group $(G, *)$ then A is a fuzzy subgroup of G if and only if $\forall x, y \in X$ in G.
Than $A(x, y^{-1}) \geq min\ (A(x), A\ (y^{-1}))$

We can show this result for transitive property also. As for fuzzyffication transitive axiom. Suppose R is the fuzzy mapping defined for X such that R is a fuzzy subsetof $X \times X$. Again R is a fuzzy transitive assuming that $x, y, z \in X\ as$

$$R(x, z) \geq min(R(x, y), R(y, z)).$$

Similar as we can give example of many more properties.
The concept of fuzzy subgroup and subgroupoids were given by A. Rosenfield in 1971.Similarly many other concepts of different fields of mathematics also translated to the fuzzy mathematics for example, fuzzy geometry, fuzzy graphs, fuzzy topology, fuzzy field theory, fuzzy ordering etc.

To study the growth of fuzzyffication one should have to read the corner stone papers of the fuzzy mathematics. Now we will discuss some basic concept of fuzzy set with their generalization. As Fuzzy mathematics is basically + V e study of basics line fuzzy set theory, fuzzy function and theory logics on basis on basis of this i try to compose the following parts to study theory in details.

FUZZY SET THEORY

The brain wave of fuzzy set theory was proliferating by Zadeh and Dieter Halaua in 1965. According to them fuzzy sets are those sets having such kind of components which have the degrees of membership. Fuzzy sets can also be known as uncertain sets or thisis mapping of sets of real number (x_i) onto membership value (μ_i) in the interval [0, 1]. The fuzzy sets are represented by a set of pairs μ_i / x_i and the element canbe represented

By ($\mu_1/ x_1, \mu_2/ x_2, \mu_3/ x . . . \mu_n/ x_n$) in the ordered pair of (x, y) the fuzzy sets are given by (y/x) which meanings the membership value of y at x. In the way of explanations the definition of fuzzy set we can give the above definition as below

The fuzzy set is a pair up (X, Y) where X is the set of nonempty and Y is a membership mapping points that X can also be represented by µ and is known as universal disclosure and for each x X the value Y (x) is called the group membership mapping of x in (X, Y). The function y = µA is called the membership mapping of fuzzy sets A = (x, y).

For finite set $X = (x_1, x_2, x_3, .. , x_n)$ and the fuzzy set (x, y) is often denote by $(y_1/x_1, y_2/x_2, y_3/x_3, . . . y_n/x_n)$

If $y(x) = 0$ the x is no more include within the fuzzy set

If $y(x) = 1$ the x is thoroughly include within the fuzzy set

If $0 < y(x) < 1$ then x is a partial member of fuzzy set

TYPES OF FUZZY SET

Convex fuzzy set:-

A fuzzy set μ have being convex, assuming that $x, y \in supp\mu$ and $\lambda \in [0, 1]$ In view

$\mu (\lambda x + (1 - \lambda)y) \geq \lambda\mu(x) + (1 - \lambda)\mu(y)$

Crisp set related to a fuzzy set

Let if the universal set is then the crisp set of all the fuzzy set specified By $SF (\cup)$ For any crisp sets $A = (\cup , y)$ for any α in any interval [0, 1] i.e.

$\alpha \in$ [0, 1] the different crisp sets are defined as below.

$\Rightarrow \alpha-$ level set $\Rightarrow$ For any $\alpha \in [0, 1]$ a $\alpha-$ level set is given by

$\alpha^{>}\alpha = A = \{x \in u/y(x) \geq \alpha\}$

$\Rightarrow$ *Strong* $\alpha-$ level set $\Rightarrow$ for $\alpha \in [0, 1]$ strong $\alpha-$level set is given by

$\alpha A^{>\alpha} = A^{t} = \{x \in v/y(x) > \alpha\}$

Support set

For $\alpha \in [0, 1]$ a support set is given by $S(A) = Supp(A) = A^{>0}$

and $S(A) = \{x \in v \quad / \quad y(x) \geq 0\}$

Core set $\Rightarrow$ For $\alpha \in [0, 1]$ a core set is given by

$C(A) = Core(A) = A^{=1}$

$A^{=1} = \{x \in v \ / \ y(x) = 1\}$

Some basic concepts

* A fuzzy set is empty strictly on the assumption

$\forall x \in U: \ \mu_A(x) = y(x) = 0$

Where $y = \mu_A$ is a membership function

* Fuzzy sets are equal on assumption that

$\forall x \in U: \ \mu_A(x) = \mu_B(x)$

*If a fuzzy set called A is subset of another fuzzy set called B then first fuzzy set A is called include in second fuzzy set B strictly on assumption that $\forall x \in U: \ \mu_A(x) \leq \mu_B(x)$

* *The Height of fuzzy set is specifying* by Hgt $(A) = Sup\ (\mu_A\ (A)) = Sup\ \{\mu A(x)/x \in U\}$

Here $\mu A(U)$ is non empty and bounded above here sup can be replaced by maximum

Normalized fuzzy set

The fuzzy sets are termed as normalized if and only if $Hgt(A) = 1$

Width of a fuzzy set

If A is a fuzzy set real of number for bounded support then the width have being establish by width $(A) = sup\ (supp\ (A)) - inf\,(supp\ (A))$

If support (A) i.e. supp (A) is a closed set then the width of the fuzzy set is given by

$$Width\ (A) = max\ (supp\ (A)) - min\ (supp\ (A))$$

Disjoint fuzzy set

According to the definition of crisp sets if the supports of fuzzy sets are disjointthen fuzzy sets are called the disjoint fuzzy sets. According to the operations intersection and union the disjoint fuzzy set are given by the statement that fuzzy set A and fuzzy set B are disjoint if and only if

$\forall x \in U : \mu_A(x) = 0 \ \ or \ \ \mu_B(x) = 0$ Equivalent as $\forall x \in U : min(\mu_A(x), \mu_B(x) = 0.$

L - Fuzzy sets

From time to time more developed approach fuzzy sets were with the membership function having values in structure L in which L can be a lattice i.e. μ can be a abstract structure of partially ordered set everywhere in two sets have unique supremum and unique infimum, to characterise these function over unit time interval these were called as L- Fuzzy sets.

INTUTIONISTIC FUZZY SETS

The development of fuzzy set proposed by Atanassov and Baruah.

Based on two function μ_A and γ_A where$\mu_A(x)$ is the degree of membership of X and where $\gamma_A(x)$ is the degree of non-membership of $X \rightarrow$ $\forall \in$with function $\mu_A, \gamma_A : v\,[0, 1]$ such that $\mu_A(x) + \gamma_A(x) = 1$.This extension of fuzzy set is known by intuitionistic fuzzy set. The intuitionistic fuzzy sets can be further extended in two models

⇒Neutrosophic fuzzy set

⇒Pythagorean fuzzy set

Neutrosophic fuzzy set

In 1998 samarandhache introduce the neutrosophic fuzzy sets. These neutrosophic fuzzy sets are combo of two functions one for degree of membership $\mu_A(x)$ and other for other non-membership function $\mu_A A$, along with these functions neutrosophic fuzzy sets have one more function called intermediate function $i_A(x)$which indicate the degree of undecidedness. This function works in a condition of undecidedness in the membership function or non-membershipand hence neutrosophic fuzzy sets where they are having three function as follows :

$\mu_A(x)$ is for the degree of membership

$\gamma_A(x)$ is for the degree of non-membership.

$\imath_A(x)$ is for the degree of intermediate values of x.

Pythagoean fuzzy sets

The development in intuitionistic fuzzy set is pythagorean fuzzy set.

As the IFSs based upon the condition that $\mu_A(x) + \gamma_A(x) \leq 1$.

Later on yager explain this on the basis of pythagorean theorem i.e. $\mu_A(x)^2, \gamma_A(x)^2 \leq 1$

The IFSs is based on this constraints is called pythagorean fuzzy set which aresuitable for real life operation.

This development in fuzzy sets can be understand using the following chart

The brain wave of Fuzzy set generated by zadeh in 1965 with membership functionμ(x)

Intiutionistic fuzzy set indused by Atanassov in 1986 with condition $\mu_A(x) + \gamma_A(x) \leq 1$ asumming that $\mu_A(x)$ is membership function & $\gamma_A(x)$ is non membership function

Neutrosophic Fuzzy set by Samarandache in 1988 with $\mu(x)$= membership mapping ,$\gamma(x)$ = non membership mapping i(x) = intermediate function

Pythagorean fuzzy sets by yager 2013,2014 with condition $\mu(x)$= membership mapping $\gamma(x)$ = non membership mapping & precondition $\mu(x)^2 + \gamma(x)^2 \leq 1$

Fuzzy operations

There is some uncertainty in the algebraic operation of fuzzy sets. But the compliment of fuzzy set has one single value.

Intersection of Fuzzy sets

The algebraic operation of intersection of fuzzy set A and B with *t-norm* and corresponding δ-norm given by $\mu_{(A\cap B)}(x) = t(\mu_A(x), \mu_B(x)) \quad \forall x \in U.$

Union of Fuzzy sets

The union of fuzzy sets A and B with corresponding t-norm and s -norm isgiven by

$$\mu_{(A\cup B)}(x) = s(\mu_A(x), \mu_B(x)), \forall x \in v.$$

Compliments of Fuzzy set

Compliments of fuzzy set (A^c) is given by the $\mu_{A^c}(x) = 1 - \mu_A(x)$.

Difference about fuzzy set A and B is denoted by A/B or $A - B$ and is given by

$\mu_{A/B}(x) = t(\mu_A(x) \cap (\mu_B(x))) \forall x \in U$ or $\mu_{A-B}(x) = \mu_A(x) - t(\mu_A(x), \mu_B(x)) \forall x \in U$.

Symmetric difference about Fuzzy sets

The symmetric difference about fuzzy set is specified by help of membership function

$\mu_{A\Delta B}(x) = |\mu_A(x) - \mu_B(x)| \forall x \in V$

or

$\mu_{A\Delta B}(x) = max\,(min\,(\mu_A(x), 1 - \mu_B(x)), min\,(\mu_B(x), 1 - \mu_A(x)))$.

Now we come to the second important part of the fuzzy mathematics i.e. fuzzy logics also plays a very most important part in the process of fuzzification.

Fuzzy logics

In fuzzy mathematics second most important part is fuzzy logic which is a form of propositional calculus which have more than 2 truth values and the truth values of variables may be any real number between 0 and 1.

Though infinite valued logic was studied since the 1920s but the term was introduced in 1965 by Lotfi Zadeh.

The brain wave of fuzzy logic was established on the basis of the observation that people take decision depend on the information which are not accurate or defective, unreliable and non-numerical. The mathematical model of fuzzy show the un- reliable information and uncertainty in mathematical form. These mathematical models can work on the data and information which is unreliable and uncertain. And therefore fuzzy logic have a many applications in applied field also like control theory to artificial intelligence.

As on the basis of degrees of truth the probabilities and fuzzy logic are seem to besame because both degrees of truth and probabilities between 0 and 1 but we can say that fuzzy logics are mathematical layout of uncertainty whereas probabilitiesis based upon mathematical model of unawareness or unintelligence.

Operators of fuzzy logic

As the boolean algebra deals with the truth value be true or false which are de- noted by 1 and 0 Fuzzy logic deals with the membership values.

Fuzzy logic work with the replacement of basic operator like AND, OR and NOTby MIN, MAX and (1,X). this replaced operator are known as Zadeh operator.

Fuzzy rules

The fuzzy logic system used same Fuzzy rule to interpret and output results based on input information. In fuzzy rule the modes pones and modus tollens are sig-nificant rules of interpretation. For example: If then implication for modes ponens rules is as

Proposition: If X is P

Implication: If X is P. Then Y is Q.

Result: Y is B

According as crisp logic X is can be only true or false where as in fuzzy rules the preposition X is P can be partially true as a result Y is a can also be partially true and the result can be represent in truth value 0 and 1.

Fuzzy logic and probability theory show different kinds of vagueness. The prob- ability theory uses the according as crisp logic X is P can be only true or false where as in false rules the preposition X is P can be partially true as a result Y can also be partially true and the result can be represent in truth value 0 and 1.

Comparison of probability

Fuzzy logic and probability theory show different kinds of vagueness. The probability theory uses the prepositions that are either true or false and fuzzy logic deals with the membership values. The measurement of uncertainties done by fuzzy logic is completely different from the done by the probability. This is actu-ally a very deep subject to study to study the difference because both work on theuncertainty and unreliable information on others. The vagueness or uncertainties measure by measure by using fuzzy logics are not forced to follow, the rules of probability. In case of probability measurement can be lies within the range 0 and 1. Whereas uncertainty of fuzzy is not certain. Probability than works with the occurrence of events whereas the fuzzy logic ruled by the other than this fuzzy logics have numerous application in vacuum cleaner, washing, machines, in breaking system in facial pattern recognition, transmission system, control of subway system in helicopters in weather forecasting system etc. more over it is very much useful for people doing research and development in different field.

Application of fuzzy logics

Fuzzy logics and the fuzzy system is very simple are easy to understands and because of these reasons is very widely used in many fields of engineering andin medical also. This is one of most fortunate tool in technologies and development in the engineering field also it works on the engineering design also. Mathematicalmodels of pure math cannot work completely fuzzy mathematics fill the gap of pure mathematicalmodel. Following are some application of Fuzzy logics in real life:

- An important real life application of fuzzy logic is traffic light controller. This model is based on the behavior under the different traffic conditions.
- Fuzzy logic is also important in uncertainties in computing problems it gives a powerful means of problem solving in many areas of computation.
- Fuzzy Logic is commonly used in Spatial Planning to recognize spatial elements to be used as elements of sets also.
- Fuzzy logics are used in calculation of climate change and management actions also. This design allows the sets of natural and human systems to get the proper information of management actions with time when there is a condition of uncertainty of climate change.

CONCLUSION

Fuzzy mathematics is playing very most important role in almost every field like agriculture, medical, engineering, computing and many more. There are so many field of fuzzy like fuzzy geometry, fuzzy topology, fuzzy decision making,fuzzy database, fuzzy differential equation it was difficult to work on all at some time in above discussion it clear that the fuzzy set theory and the concept of fuzzy mathematics given by Zadeh is a very useful mathematical model in all the field of development. In this chapter I tried to compile the comparative study of fuzzy math as fuzzy sets and fuzzy logics with discussion of the mathematical model based on fuzzy concept which can work on uncertainties of information.There are so many models of real life application of fuzzy mathematics as fuzzy sets and fuzzy logics which are playing very important role in every field of developments.

References

1. Zadeh, L. A.(1965) , ”Fuzzy sets” information on control.,8, 338-353.

2. C.K. Muthumaniraja,M. Chinnamuthu (2019) in GIS and Geostatistical Technique,Eisevier ISBN 978-0-12-815413-7, 365-371.

3. Goguen I. (1967): L. ”Fuzzy sets” J. Math. anals and app., 18. 149-174.

4. Rosenfed, A. (1971): ”Fuzzy Groups” J. Math. anals and app., 35. 512-517.

5. Backley J.J. Eslami. E. (1997),: Fuzzy plane geometry I. Points and lines, Fuzzy sets andsyst,86, 179-187.

6. Smarandache, Florentin (1998),: Neutrosophy, Neutrosophic probability setand Logic analytic synthesis and synthetic analysis.American research press ISBN 978-1879585638.

7. Yagor Ronald (2013), : ”Pythogorean membership grades in multicriteria decision making”IEEE Transactions on Fuzzy system. 22(4),958-965.

8. T. Prato (2016), in Developments in Enviromental Modelling, 28,1-268

9. Tanassov, K.T, (1986): Intutionistic fuzzy set, fuzzy sets and system V.20. No. 87-96.