E-SUPER EDGE MAGIC LABELING ON SOME CLASSES OF GRAPHS

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Abstract

A (p,q) graph G with p vertices and q edges, a bijection $f: V(G) \cup E(G) \rightarrow \{1,2, \dots . p+q\}$ is called edge magic labeling of G if $f(u)+f(uv)+f(v)=k$, a constant for any edge uv of G.G is known as E-super edge magic if $f(E(G))=\{1,2, \dots q\}$.Herein, we explore some classes of E-super edge-magic graphs.

Keywords*: Edge magic labeling,E-super edge magic labeling, E-super edge magic graphs.*

INTRODUCTION

In this whole paper we deal with only a non-trivial simple undirected graphs.

Let G be a graph with vertex set $V(G)$ and the edge set $E(G)$ such that the order of $G = |V(G)| = p$ and the size of $G = |E(G)| = q$. Assigning of integers to vertices(edges) into a set of numbers is known as Graph Labeling.

Different kinds of labelings have been examined by several experts and an eminent survey of graph labelings can be found in[3]. In 1963, Sedl$\grave{a}\check{c}$ek[5] identified the concept of magic labeling in graphs. A graph G is *magic* if the edges of G can be labeled by a set of numbers$\{1,2, \dots q\}$ so that the sum of labels of all the edges incident with any vertex is the same. In 1966, Stewart also worked on the concept of magic labeling[6].

In 1970, Kotzig and Rosa [4] defined a magic labeling of a graph G as a bijection $f: V(G) \cup E(G) \rightarrow \{1,2, \dots p+q\}$ such that for all edges $uv, f(u)+f(uv)+f(v)=k$,is constant. Enomoto[2] and Wallis[7] call an edge magic total labeling as super edge magic if set of vertex label is $\{1,2, .. p\}$. R.M.Figueroa-Centeno et al[1] discussed the concept of super edge-magic labelings among other classes of labelings.

In 2018,U.VijayaNarayanan and P.Parthiban [8] discussed about Some classes of Super edge magic graphs.

By using the definition of super edge magic labeling, we define a new labeling called E-super edge magic labeling. A (p,q) graph G with p vertices and q edges, a bijection $f: V(G) \cup E(G) \rightarrow \{1,2,,,,p+q\}$ is called edge magic labeling of G if $f(u)+f(uv)+f(v)=k$, a constant for any edge uv of G.G is said to be E-super edge magic if $f\big(E(G)\big)=\{1,2,\dots q\}$.Here we observe that $p=q$.

In this paper we find some graphs that admits E-super edge magic labeling.

E-SUPER EDGE MAGIC GRAPHS

Let C_y be a cycle with vertices $v_1, v_2, \dots . v_y$,where $y \geq 3$ is odd. The Graph $C_y(1)$ is obtained from the cycle C_y by attaching a path of length 1 at the vertex v_1.

THEOREM 1

The graph $C_y(1)$ is E-Super edge magic.

Proof:

We label the vertices of $C_y(1)$ as follows:

Let the vertices of the graph $v_1, v_2, \dots . v_p$.

Define $f: V\big(C_y(1)\big) \rightarrow \{p+1, p+2, .., p+q\}$ by

$$f(v_i)=\begin{cases} \frac{(2p+1)+i}{2}, & if\ i\ is\ odd \\ \frac{3p+i}{2}, & if\ i\ is\ even \end{cases}$$

And $f\left(E\big(C_y(1)\big)\right)=\{1,2,\dots q\}$

Obviously $\{\mathrm{f(u)}+\mathrm{f(v)}: \mathrm{uv} \in \mathrm{E(G)}\}$ contains q-consecutive integers and

$f(u)+f(uv)+f(v)=k$, a constant.

Hence f admits E-super edge magic labeling.

EXAMPLE 1

Consider the graph $C_5(1)$. The vertices of the graph are $v_1, v_2, \dots . v_6$.

Define $f: V(C_5(1)) \rightarrow \{7,8, .., 12\}$ by

$$f(v_i) = \begin{cases} \frac{13+i}{2}, & if\ i\ is\ odd \\ \frac{18+i}{2}, & if\ i\ is\ even \end{cases}$$

And $f(E(C_5(1))) = \{1,2, \dots 6\}$

Obviously $\{f(u) + f(v): uv \in E(G)\}$ contains q-consecutive integers and $f(u) + f(uv) + f(v) = 23$.

Hence G is E-super edge magic.

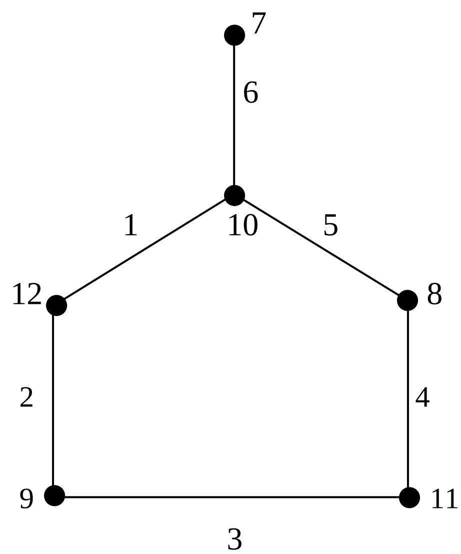

Fig 1: E-Super Edge magic labeling of $C_5(1)$ with $k = 23$.

Let C_y be a cycle with vertices $v_1, v_2, \dots . v_y$ where $y \geq 3$ is odd. The graph $C_y(n,n)$ is obtained from the cycle C_y by attaching a path of length $n - 1$ at the vertices $v_{\left(\frac{y+1}{2}\right)}$ and $v_{\left(\frac{y+3}{2}\right)}$

THEOREM 2

The graph $C_y(n,n)$ is E-super edge magic, for all $n \geq 2$.

PROOF:

We label the vertices of the graph $C_y(n,n)$ as $v_1, v_2, \dots . v_p$.

Define an onto map

$f: V(C_y(n,n)) \to \{p + 1, p + 2, \dots, p + q\}$

$$f(v_i) = \begin{cases} \frac{(2p+1)+i}{2}, & if\ i\ is\ odd \\ \frac{(3p+1)+i}{2}, & if\ i\ is\ even \end{cases}$$

And $f\left(E\left(C_y(n,n)\right)\right) = \{1,2, \dots q\}$.

Obviously $\{\mathrm{f(u)} + \mathrm{f(v)}: \mathrm{uv} \in \mathrm{E(G)}\}$ contains q-consecutive integers and

$f(u) + f(uv) + f(v) = k$, a constant.

Hence f is E-super edge magic labeling.

EXAMPLE 2

Consider the graph $C_5(3,3)$. The vertices of the graph are $v_1, v_2, \ldots . v_9$.

Define $f: V(C_5(3,3)) \rightarrow \{10,11,..,18\}$ by

$$f(v_i) = \begin{cases} \frac{19+i}{2}, & if\ i\ is\ odd \\ \frac{28+i}{2}, & if\ i\ is\ even \end{cases}$$

And $f\big(E(C_5(3,3))\big) = \{1,2, \ldots 9\}$

Obviously $\{\mathrm{f(u)} + \mathrm{f(v)}: \mathrm{uv} \in \mathrm{E(G)}\}$ contains q-consecutive integers and $f(u) + f(uv) + f(v) = 33$.

Hence G is E-super edge magic.

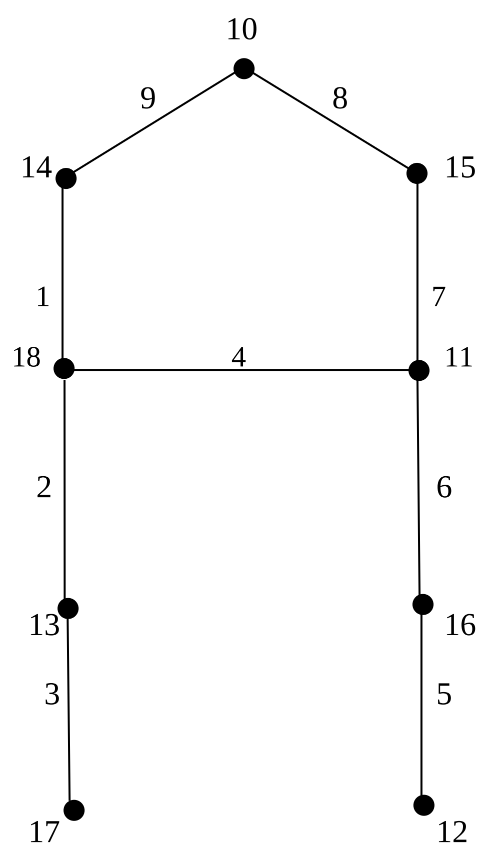

Fig 2: E-Super Edge magic labeling of $C_5(3,3)$ with $k = 33$

Let C_y be a cycle with vertices $v_1, v_2, \dots . v_y$ where $y \geq 3$ is odd. The graph $C_y(1, n, n)$ is obtained from the cycle C_y by attaching a path of length 1 at the vertex v_1 , and a path of length $n - 1$ at the vertices $v_{\left(\frac{y+1}{2}\right)}$ and $v_{\left(\frac{y+3}{2}\right)}$.

THEOREM 3

The graph $C_y(1, n, n)$ is E-super edge magic, for all $n \geq 2$.

PROOF

We label the vertices of the graph $C_y(1, n, n)$ as $v_1, v_2, \dots . v_p$.

Define an onto map

$f: V\big(C_y(1, n, n)\big) \to \{p + 1, p + 2, .., p + q\}$

$$f(v_i) = \begin{cases} \frac{(2p+1)+i}{2}, if\ i\ is\ odd \\ \frac{(3p)+i}{2}, if\ i\ is\ even \end{cases}$$

And $f\left(E\big(C_y(1, n, n)\big)\right) = \{1,2, \dots q\}$

Obviously $\{\mathrm{f(u) + f(v): uv \in E(G)}\}$ contains q-consecutive integers and $f(u) + f(uv) + f(v) = k$, a constant.

Hence f is E-super edge magic labeling.

EXAMPLE 3

Consider the graph $C_5(1,4,4)$. The vertices of the graph are $v_1, v_2, \dots . v_{12}$.

Define $f: V(C_5(1,4,4)) \to \{13,14, .., 24\}$ by

$$f(v_i) = \begin{cases} \frac{25+i}{2}, \ if\ i\ is\ odd \\ \frac{36+i}{2}, if\ i\ is\ even \end{cases}$$

And $f\big(E(C_5(1,4,4))\big) = \{1,2, \dots 12\}$

Obviously $\{\mathrm{f(u) + f(v): uv \in E(G)}\}$ contains q-consecutive integers and $f(u) + f(uv) + f(v) = 44$.

Hence G is E-super edge magic.

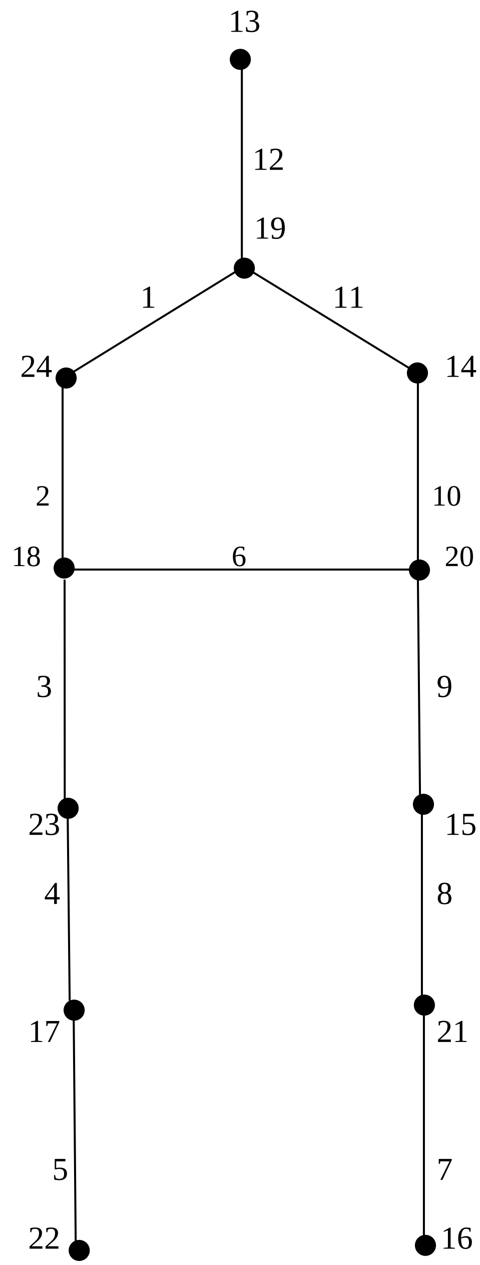

Fig 3: E-Super Edge magic labeling of $C_5(1,4,4)$ with $k = 44$

CONCLUSION

In this paper, we have discussed some classes of graphs that admit E-Super Edge Magic Labeling. In future, we can prove different classes of graphs which satisfy E-Super Edge Magic Labeling.

REFERENCES:

1. R.M.Figueroa-Centeno,r.Ichishima,F.A.Muntaner-Batle,The place of super edge-magic labelings among other classes of labelings,Discrete Math.231(2001)153-168.

2. H.Enomoto,A.S.Llado,T.Nakamigawa,G.Ringel,Super edge magic graphs,SUT J.Math 34(1998) 105-109.

3. J.A.Gallian, A Dynamic survey of graph labeling, Electron J.Cobin.17(2014)#DS6.

4. A.Kotzig,A.Rosa,Magic valuation of finite graphs,Can.Math.Bull.13(1970) 451-461.

5. J.Sedlacek, On magic graphs,Math.Slov.26(1976) 329-335.

6.B.M.Stewart, Magic graphs,Can.J.Math.18(1966) 1031-1059.

7. W.D.Wallis, Magic Graphs,Birkhauser,Boston,2001.

8. U.VijayaNarayanan,P.Parthiban, Some classes of super edge magic graphs,Journal of Analysis and Computation,Vol XI(I),2018.