RELATION-THEORETIC REVISED FUZZY BANACH CONTRACTION PRINCIPLE AND REVISED FUZZY ELDESTEIN CONTRACTION THEOREM

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Abstract

In this paper we extend and generalize the classical Banach's contraction principle and Edelstein's contraction theorem on a Revised fuzzy metric spaces endowed with a binary relation.

Keywords: *Contraction mappings, complete revised fuzzy metric spaces, compact revised fuzzy metric spaces, binary relation, fixed point.*

1. Introduction

The concept of fuzzy set was initiated by Zadeh [18]. Thereafter, it was developed extensively by many authors which also include interesting applications of this theory in diverse areas. George and Veeramani [9] modified the concept of fuzzy metric space introduced by Kramosil and Michalek [11]. The origin of fixed point theory in fuzzy metric spaces is often traced back to the paper of Grabiec [10] wherein he extended classical fixed point theorems of Banach and Edelstein to complete and compact fuzzy metric spaces respectively. Very recently Alam and Imdad [2] presented new variant of classical Banach contraction principle on a complete metric space endowed with a binary relation which, under universal relation reduces to Banach contraction principle. Many authors proved fixed point theorems in fuzzy metric space including [3]-[6]. Then Muraliraj.A and Thangathamizh.R [13] first introduce the existence of fixed point sets in the fuzzy metric space, which were revised in 2021 based on t-conorm. Muraliraj.A and Thangathamizh.R [14] later prove the Banach and Edelstein contractions in the revised fuzzy metric space. In 2021, Muraliraj.A and Thangathamizh.R [15] introduce the concept of Revised fuzzy modular metric.

The aim of this paper is to extend and generalize the, Banach contraction principle to a complete Revised fuzzy metric space and Edelstein contraction theorem to a compact Revised fuzzy metric space endowed with a binary relation to the contractive conditions which are relatively weaker than usual contraction as it is required to hold only on those elements which are related under the underlying relation rather than the whole space. We extend and generalize the results of Alam and Imdad [2] and Grabiec [12].

2. Preliminaries

To initiate the concept of Revised fuzzy metric space, which was introduced by Alexander Sostak [1] in 2018 is recalled here.

Definition 2.1: [17]

A binary operation $\oplus: [0,1] \times [0,1] \to [0,1]$ is a t-conorm if it satisfies the following conditions:

a) $\oplus$ is associative andcommutative,

b) $\oplus$ is continuous,

c) $a \oplus 0 = a$ for all $a \in [0,1]$,

d) $a \oplus b \leq c \oplus d$ whenever $a \leq c$ and $b \leq d$ for all $a, b, c, d \in [0,1]$.

Examples 2.2: [17]

i.Lukasievicz t-conorm: $a \oplus b = max\{a , b\}$

ii.Product t-conorm: $a \oplus b = a + b - ab$

iii.Minimum t-conorm: $a \oplus b = min(a + b, 1)$

Definition 2.3: [1]

A Revised fuzzy metric space is an ordered triple $(X, \mu, \oplus)$ such that X is a nonempty set, $\oplus$ is a continuous t-conorm and μ is a fuzzy set on $X \times X \times (0, \infty) \to [0,1]$ satisfies the following conditions: $\forall x, y, z \in X \; and \; s, t > 0$

(RGV1) $\mu(x, y, t) < 1 , \forall t > 0$

(RGV2) $\mu(x, y, t) = 0$ if and only if $x = y, t > 0$

(RGV3) $\mu(x, y, t) = \mu(y, x, t)$

(RGV4) $\mu(x, z, t + s) \leq \mu(x, y, t) \oplus \mu(y, z, s)$

(RGV5) $\mu(x, y, -): (0, \infty) \to [0,1)$ is continuous.

Then μ is called a Revised fuzzy metric on X.

Example 2.4: [1]

Let (X, d) be a metric space. Define $a \oplus b = max\{a, b\}$ for all $a, b \in [0,1]$, and define $\mu : X \times X \times (0, \infty) \to [0,1]$ as

$$\mu(x, y, t) = \frac{d(x, y)}{t + d(x, y)}$$

$\forall x, y, z \in X$ and $t > 0$. Then $(X, \mu, \oplus)$ is a Revised fuzzy metric space.

Definition 2.5: [1]

Let $(X, \mu, \oplus)$ be a Revised fuzzy metric space, for $t > 0$ the open ball $B(x, r, t)$ with a centre $x \in X$ and a radius $0 < r < 1$ is defined by

$$B(x, r, t) = \{y \in X : \mu(x, y, t) < r\}.$$

A subset $A \subset X$ is called open if for each $x \in A$, there exist $t > 0$ and $0 < r < 1$ such that $B(x, r, t) \subset A$. Let τ denote the family of all open subsets of X. Then τ is topology on X, called the topology induced by the Revised fuzzy metric μ. We call this fuzzy metric induced by the metric d as the standard Revised fuzzy metric.

Definition 2.6: [13]

Let $(X, \mu, \oplus)$ be a Revised fuzzy metric space,

1. A sequence $\{x_n\}$ in X is said to be convergent to a point $x \in X$ if

$$lim_{n\to\infty}\mu(x, y, t) = 0 \; for \; all \; t > 0.$$

2. A sequence $\{x_n\}$ in X is called a Cauchy sequence, if for each $0 < \epsilon < 1$ and $t > 0$, there exists $n_0 \in \mathbb{N}$ such that $\mu(x_n, x_m, t) < \epsilon$ for each $n, m \geq n_0$
3. A Revised fuzzy metric space in which every Cauchy sequence is convergent is said to be complete.

4. A Revised fuzzy metric space in which every sequence has a convergent subsequence is said to be compact.

Lemma 2.7: [13]
Let $(X, \mu, \oplus)$ be a Revised fuzzy metric space. For all $u, v \in X, \mu(u, v, -)$ is non-increasing function.

Definition 2.8.

Let $(X, \mu, \oplus)$ be a Revised fuzzy metric space. μ is said to be continuous on $X^2 \times (0, \infty)$ if

$$\lim_{n\to\infty} \mu(x_n, y_n, t_n) = \mu(x, y, t),$$

whenever $\{(x_n, y_n, t_n)\}$ is a sequence in $X^2 \times (0, \infty)$ which converges to a point $(x, y, t) \in X^2 \times (0, \infty)$; i.e.,

$$\lim_{n \to \infty} \mu(x_n, x, t) = \lim_{n \to \infty} \mu(y_n, y, t) = 0 \text{ and } \lim_{n \to \infty} \mu(x, y, t_n) = \mu(x, y, t).$$

Definition 2.9.

Let $\lim_{n} x_n = x$ and $\lim_{n} y_n = x$. Then

(2.9.1) $\lim_{n \to \infty} \mu(x_n, y_n, t) \leq \mu(x, y, t)$ for all $t > 0$;

(2.9.2) If $\mu(x, y, -)$ is continuous, then $\lim_{n \to \infty} \mu(x_n, y_n, t) \leq \mu(x, y, t)$ for all $t > 0$.

Definition 2.10. [12]

Let X be a nonempty set. A subset R of X^2 is called a binary relation on X. Notice that for each pair $x, y \in X$, one of the following conditions hold:

(i) $(x, y) \in R$; which amounts to saying that "x is R-related to y" or "x relates to y under R". Sometimes, we write xRy instead of $(x, y) \in R$;

(ii) $(x, y) \notin R$; which means that "x is not R-related to y" or "x does not relate to y under R".

Trivally, X^2 and φ being subsets of X^2 are binary relations on X, which are respectively called the universal relation (or full relation) and empty relation.

Throughout this paper, R stands for a nonempty binary relation, but for the sake of simplicity, we write only "binary relation" instead of "nonempty binary relation."

Definition 2.11. [2]

Let R be a binary relation defined on a nonempty set X and $x, y \in X$. We say that x and y are R-comparative if either $(x, y) \in R$ or $(y, x) \in R$. We denote it by $[x, y] \in R$.

Definition 2.12. [2]

Let X be a nonempty set and R a binary relation on X.

(i) The inverse transpose or duel relation of R, denoted by R^{-1}, is defined by $R^{-1} = \{(x, y) \in X2 : (y, x) \in R\}$.

(ii) The symmetric closure of R, denoted by R^s, is defined to be the set $R \cup R^{-1} (i.e., R^s := R \cup R^{-1})$. Indeed, R^s is the smallest symmetric relation on X containing R.

Proposition 2.13 [3]

For a binary relation R defined on a nonempty set X,

$$(x, y) \in R^s \Leftrightarrow [x, y] \in R.$$

Proof.

The observation is straightforward as

$$\begin{aligned}(x, y) \in R^s &\Leftrightarrow (x, y) \in R \cup R^{-1}\\ &\Leftrightarrow (x, y) \in R \; or \; (x, y) \in R^{-1}\\ &\Leftrightarrow (x, y) \in R \; or \; (y, x) \in R \Leftrightarrow [x, y] \in R \cup R^{-1}\end{aligned}$$

Definition 2.14. [2]

Let X be a nonempty set and R a binary relation on X.A sequence $\{x_n\} \subset X$ is called R-preserving if

$$(x_n, x_{n+1}) \in R \quad \forall n \in N_0.$$

Definition 2.15.

Let $(X, \mu, \oplus)$ be a Revised fuzzy metric space. A binary relation R defined on X is called d-self-closed if whenever $\{x_n\}$ is an R-preserving sequence and $x_n \to x$. then there exists a subsequence $\{x_{n_k}\}$ of $\{x_n\}$ with $[x_{n_k}, x] \in R$ for all $k \in N_0$.

Definition 2.16. [2]

Let X be a nonempty set and T a self mapping on X. A binary relation R defined on X is called T-closed if for any $x, y \in X$,

$$(x.y) \in R \Rightarrow (T\,x, T\,y) \in R.$$

Proposition 2.17. [2]

Let X, T and R be the same as in definition 14. If R is T-closed, then R^s is also T-closed.

Definition 2.18. [2]

Let X be a nonempty set and R a binary relation on X. A subset E of X is called R-directed if for each $x, y \in E$, there exists $z \in X$ such that $(x, z) \in R$ and $(y, z) \in R$.

Definition 2.19. [2]

Let X be a nonempty set and R a binary relation on X. For $x, y \in X$, a path of length k (where k is a natural number) in R from x to y is a finite sequence $\{z_0, z_1, z_2, \ldots, z_k\} \subset X$ satisfying the following conditions:

(i) $z_0 = x$ and $z_k = y$,

(ii) $(z_i, z_{i+1}) \in R$ for each $i(0 \leq i \leq k-1)$.

Notice that a path of length k involves $k + 1$ elements of X, although they are not distinct.

In this paper, we use the following notations:

(i) $F(T)$ = The set of all fixed points of T,

(ii) $X(T\,;\,R) := \{x \in X : (x, Tx) \in R\}$,

(iii) $\Upsilon(x, y, R) :=$ The class of all paths in R from x to y.

3. Main Results

In order to establish our main result, we first prove the following proposition:

Proposition 3.1.

If $(X, \mu, \oplus)$ is a Revised fuzzy metric space, R is a binary relation on X, T is a self-mapping on X, $0 < k < 1$ and $t > 0$, then the following contractivity conditions are equivalent:

(i) $\mu(Tx, Ty, kt) \le \mu(x, y, t)\ \forall\, x, y \in X$ with $(x, y) \in R$,

(ii) $\mu(Tx, Ty, kt) \le \mu(x, y, t)\ \forall\, x, y \in X$ with $[x, y] \in R$.

Proof.

The implication $(ii) \Rightarrow (i)$ is trivial. Conversely, suppose that (i) holds. Take $x, y \in X$ with $[x, y] \in R$, then (ii) directly follows from (i). Otherwise, if $(y, x) \in R$ then using the symmetry of μ and (i), we obtain

$$\mu(Tx, Ty, kt) = \mu(Ty, Tx, kt) \le \mu(y, x, t) = \mu(x, y, t).$$

This shows that $(i) \Rightarrow (ii)$.

Theorem 3.2. Revised fuzzy Banach contraction principle

Let $(X, \mu, \oplus)$ be a complete Revised fuzzy metric space with $\lim_{t \to \infty} \mu(x, y, t) = 0$ for all $x, y \in X$, R a binary relation on X and T a self mapping on X. Suppose that the following conditions hold:

(a) $X(T\,;\,R)$ is non empty,

(b) R is T-closed,

(c) either T is continuous or R is d-self closed.

(d)

$$\mu(Tx, Ty, kt) \le \mu(x, y, t)\ \forall\, x, y \in X \text{ with } (x, y) \in R$$

where $0 < k < 1$ and $t > 0$.

Then T has a fixed point. Moreover, if

(e) $\Upsilon(x, y, R^s)$ is non empty, for each $x, y \in X$, then T has a unique fixed point.

Proof.

Let $x0$ be an ordinary point of $X(T, R)$. Define an iterative sequence $\{x_n\}$ by

$x_n = T^n(x_0)$ or $x_{n+1} = Tx_n \; \forall\, n \in N_0$. As $(x_0, Tx_0) \in R$, using assumption (b), we obtain

$$(Tx_0, T^2x_0), (T^2x_0, T^3x_0), \ldots, (T^nx_0, T^{n+1}x_0) \in R$$

so that

$$(x_n, x_{n+1}) \in R \; \forall\, n \in N_0 \tag{1}$$

Thus the sequence $\{x_n\}$ is R-preserving. Applying the contractive condition (d) to (1),

we deduce for $n \in N_0$, that

$$\mu(Tx_n, Tx_{n+1}, kt) = \mu(x_{n+1}, x_{n+2}, kt) \leq \mu\left(Tx_0, Tx_1, \frac{t}{k^{n-1}}\right)$$

$$= \mu\left(x_1, x_2, \frac{t}{k^{n-1}}\right) \text{ for all } n \in N_0 \text{ and } t > 0.$$

Thus for any positive integer p, we have

$$\mu(Tx_n, Tx_{n+p}, t) = \mu(x_{n+1}, x_{n+p+1}, t)$$

$$\leq \mu\left(x_{n+1}, x_{n+2}, \frac{t}{p}\right) \oplus \left(\overset{(p)}{\ldots}\right) \oplus \mu\left(x_{n+p}, x_{n+p+1}, \frac{t}{p}\right)$$

$$\leq \mu\left(x_1, x_2, \frac{t}{pk^n}\right) \oplus \ldots . \oplus \mu\left(x_1, x_2, \frac{t}{pk^n}\right)$$

since $\mu(x, y, t) \to 0$ as $t \to \infty$ we get

$$\lim_{\text{n}} \mu(Tx_n, Tx_{n+p}, t) \leq 0 \oplus \ldots . \oplus 0 = 0$$

which implies that the sequence $\{x_n\}$ is Cauchy, hence convergent. so there exists such that

$$\lim_{n \to \infty} x_n = x.$$

Now assume that T is continuous, we have $x \in X$

$$\lim_{n \to \infty} x_{n+1} = \lim_{n \to \infty} Tx_n = T\,x$$

Owing to the uniqueness of limit, we obtain $T\,x = x$, i.e., x is a fixed point of T.

Now let us assume that R is d-self-closed. As $\{x_n\}$ is R-preserving sequence and

$$\lim_{n \to \infty} x_n = x$$

there exists a subsequence $\{x_{n_k}\}$ of $\{x_n\}$ with

$$[x_{n_k}, x] \in R \quad \forall k \in N_0.$$

Using (d) and proposition (17), $[x_{n_k}, x] \in R$ and $\lim_{n \to \infty} x_{n_k} = x.$ we obtain

$$\mu\left(x_{n_k+1}, T x, kt\right) = \mu\left(T x_{n_k}, T x, kt\right) \leq \mu(x_{n_k}, x, t) \rightarrow 0 \; as \; k \rightarrow \infty,$$

i.e.,
$$\lim_{n \to \infty} \mu(x_{n_k}, x, t) = 0$$

so that

$$\lim_{n \to \infty} x_{n_k+1} = T x,$$

Again owing to the uniqueness of limit, we obtain

$$T x = x$$

so that *x* is a fixed point of *T*.

To prove uniqueness, let us take $x, y \in F(t)$,

i.e., $$T(x) = x \,\&\, T(y) = y \quad (2)$$

By assumption (e), there exists a path $(say\{z_0, z_1, z_2, \ldots z_r\})$ of some finite length r in R^s from *x* to *y* so that

$$z_0 = x, \; z_r = y, [z_i, z_{i+1}] \in R \; for \; each \; i(0 \leq i \leq r-1) \quad (3)$$

As *R* is *T* -closed, by using proposition (17), we have

$$[T^n z_i, T^n z_{i+1}] \in R \; for \; each \; i(0 \leq i \leq r-1) \text{ and for each } n \in N_0 \quad (4)$$

Now making use of (2), (3), (4), RFm-4, assumption (d) and proposition 17, we obtain

$$\begin{aligned} M(x, y, t) &= M(T x, T y, t) \\ &= M(T z_0, T z_r, t) \\ &= \mu(T^n z_0, T^n z_r, t) \\ &\leq \mu\left(T^n z_0, T^n z_1, \tfrac{t}{p}\right) \oplus \mu\left(T^n z_1, T^n z_2, \tfrac{t}{p}\right) \oplus \ldots \oplus \mu\left(T^n z_{r-1}, T^n z_r, \tfrac{t}{p}\right) \\ &\leq \mu\left(z_0, z_1, \tfrac{t}{p\, r^n}\right) \oplus \mu\left(z_0, z_1, \tfrac{t}{p\, r^n}\right) \oplus \ldots \oplus \mu\left(z_0, z_1, \tfrac{t}{p\, r^n}\right) \\ &\leq 0 \oplus 0 \oplus \ldots . \oplus 0 \end{aligned}$$

$= 0$

so that $x = y$. Hence T has a unique fixed point. _

If R is complete or X is R^s-directed, then the following consequence is worth recording:

Corollary 3.3.

Theorem (3.2) remains true if we replace condition (e) by one of the following conditions (besides retaining the rest of the hypothesis):

(e') R is complete

(e'') X is R^s-directed.

Proof.

If R is complete, then for each $x, y \in X, [x, y] \in R$, which amounts to saying that $\{x, y\}$ is a path of length 1 in R^s from x to y so that $\Upsilon(x, y, R^s)$ is nonempty. Hence Theorem 3.2 gives rise to the conclusion.

Otherwise, if X is R^s-directed, then for each $x, y \in X$, there exists $z \in X$ such that $[x, z] \in R$ and $[y, z] \in R$ so that $\{x, z, y\}$ is a path of length 2 in R^s from x to y. Hence $\Upsilon(x, y, R^s)$ is nonempty, for each $x, y \in X$ and again by Theorem 3.2 the conclusion is immediate.

Theorem 3.4. Revised Fuzzy Edelstein contraction theorem:

Let $(X, \mu, \oplus)$ be a compact Revised fuzzy metric space with $\mu(x, y, -)$ continuous for all $x, y \in X$, R a binary relation on X and T a self mapping on X. Suppose that the following conditions hold:

(a) $X(T, R)$ is non empty,

(b) R is T-closed,

(c) either T is continuous or R is d-self closed.

(d) $\mu(T x, Ty, t) < \mu(x, y, t)\ \forall\, x, y \in X$ with $(x, y) \in R\ \&\ x \neq y\ \&\ t > 0$.

Then T has a fixed point. Moreover, if

(e) $\Upsilon(x, y, R^s)$ is non empty, for each $x, y \in X$, then T has a unique fixed point.

Proof.

Let x_0 be an ordinary point of $X(T, R)$. Define an iterative sequence $\{x_n\}$ by $x_n = T^n(x_n)$ or $x_{n+1} = Tx_n$, $\forall\, n \in N_0$. As $(x_0, Tx_0) \in R$, using assumption (b), we obtain

$$(T\, x_0, T^2\, x_0), (T^2\, x_0, T^3\, x_0), \ldots (T^n\, x_0, T^{n+1}\, x_0) \in R$$

so that

$$(x_n, x_{n+1}) \in R \,\forall\, n \in N_0$$

Thus the sequence $\{x_n)$ is R-preserving.

Now since X is compact, $\{x_n\}$ has a convergent subsequence $\{x_{n_i}\}$

Let $\lim_i x_{n_i} = x$. Now assume that T is continuous, we have

$$\lim_i \, x_{n_i} = T\,x \tag{5}$$

$$\lim_i \, x_{n_i} = T^2\,x \tag{6}$$

As $(x_n, T\,x_{n_i}) \in R$. Now observe that

$$\begin{aligned} \mu(x_n, T\,x_{n+1}, t) &< \mu(T\,x_{n_i}, T^2 x_{n_i}, t) < \cdots < \mu(x_{n_i}, T\,x_{n_i}, t) \\ &< \mu(T\,x_{n_i}, T^2 x_{n_i}, t) < \cdots < \mu(x_{n_{i+1}}, T\,x_{n_{i+1}}, t) \\ &< \mu\left(T\,x_{n_{i+1}}, T^2 x_{n_{i+1}}, t\right) < \cdots < 0, \forall\, t > 0. \end{aligned}$$

Thus $\{\mu(x_{n_i}, T\,x_{n_i}, t)\}$ and $\{\mu\left(T\,x_{n_{i+1}}, T^2 x_{n_{i+1}}, t\right)\}$ $(t > 0)$ are convergent to a common limit. So by (5),(6) and (2.9.2) we get

$$\begin{aligned} \mu(x, T\,x, t) &= \mu(\lim x_{n_i}, T \lim x_{n_i}, t) \\ &= \lim \mu(x_{n_i}, T\,xni, t) \\ &= \lim \mu\,(T\,x_{n_i}, T^2 x_{n_i}, t) \\ &= \mu\,(\lim T x_{n_i}, \lim T^2 x_{n_i}, t) \\ &= \mu(T\,x, T^2 x, t) \end{aligned}$$

for all $t > 0$. Now suppose $x \neq T\,x$. Then by (d), we have

$$\mu(x, T\,x, t) < \mu(T\,x. T^2 x, t), t > 0$$

which is a contradiction. Hence $x = T\,x$, i.e., x is a fixed point of T.

Now assume that R is d-self-closed. As $\{x_n\}$ is an R-preserving sequence and

$\lim_{n\to\infty}\{x_n\} = x$, there exists a sequence $\{x_{n_i}\}$ of $\{x_n\}$ with $[x_{n_i}, x] \in R \,\forall\, i \in N$

using (d), proposition 3.1, $[x_{n_i}, x] \in R$ and $\lim_{i\to\infty} x_i = x$, we obtain

$$\mu(x_{n_{i+1}}, T\,x, t) = \mu(T x_{n_i}, T x, t) > \mu(x_{n_i}, x, t) \to 0 \; as \; i \to \infty$$

Thus $x_{n_{i+1}} \rightarrow T\, x$

Owing to the uniqueness of limit, we obtain $T\, x = x$. so that x is a fixed point of T. Uniqueness follows from (d).

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