ANALYSIS OF HEAT TRANSFER IN SISKO NANOFLUID OVER NON LINEAR SHEET WITH SUCTION AND INJECTION

Jiya[1], M; Mohammed[2], M. J.; Shaba*[3], A. I ; Saba, A[4] and Yisa[5], E.

[1,2 3,4,]*Department of Mathematics,*
Federal University of Technology, Minna,

[5]*Department of General Studies,*
Niger State College of Agriculture, Mokwa
Corresponding Author Email: proabelshaba@yahoo.com ; e.yisa75@gmail.com

ABSTRACT

This work focus on steady two-dimensional boundary layer flow and heat transfer of sisko nanofluid over non-linear sheet with section and injection. The governing mathematical equation incorporate the influence of Brownian number, Prandtl number, power law and thermophoresis parameter.. Suitable similarity transformation is used to reduce the equation to non-linear ordinary differential equation and solved using Adomian decomposition method (ADM). The result obtained shows thermophoresis number, power law and Prandtl parameter has a positive effect in temperature while thermophoresis number, Brownian number and Prandtl number has no effect on nano concentration profile.

Keywords: *Heat, Linear, Fluid, Differential, Injection, Prandtl.*

1. INTRODUCTION

In several industrialized duties, issues identifying with heat exchange with Nano-fluids have gotten enormous consideration due to its colossal applications in engineering and technological fields. Nanofluid refers to a fluid consisting of nano-meter – sized particles referred to as nanoparticles. Those fluids are built colloidal suspensions of nanoparticles in a based flui**d.** Khan (2010). Flow in addition to heat movement in Nanofluid towards a nonlinear stretching sheet were taken into account by Rana and Bhargava (2012). Wong and De Leon (2010) studied the uses of Nano-fluids.. The method Adomian decomposition investigation meant for the boundary layer arrangement of convective heat exchange with low gradient over a flat plate were studied by Jiya and Oyubu (2012). The stagnation point flow in an extending/contracting slip in a Nanofluid was researched by Bachok, Ishak and Pop (2011). Makinde and Aziz (2011) verified the boundary sheet movement on fluid in Nanofluid along a stretching plate through convective boundary conditions. The non-uniform heat incorporation impacts upon heat exchange of non-Newtonian power law were studied by mahmood and meghed (2012). Flow as well as heat exchange above a contracting sheet that is unsteady with suction in Nanofluid were examined by Roni, Ahmed and Pop (2012). Hydrodynamics edge level micropolar flows along a stretching plane in non darcian middling with penetrability were studied by Aiyesimi, Yusuf and Jiya (2013). Aziz and Khan (2012) considered an innate convective limit layer movement of Nanofluid upon a convective warmed perpendicular shield. Radiation property in temperature plus mass movements in

MHD stagnation - point flow along a porous plane plate through warmth convective boundary conditions were studied by Hamad, Uddin and Ismail (2012).

Although there are many studies on fluid and heat exchange of nano-fluid over non-linear sheet with suction and injection but none has considered sisko nano-fluid with Adomian decomposition method. The purpose of this research paper is to obtain solution to the two dimensional boundary layer flow and transfer of heat in a sisko Nano-fluid over nonlinear stretching part with suction/ injection using Adomian Decomposition Method (ADM).

2. ADOMIAN DECOMPOSITION METHOD

Adomian decomposition method (ADM) is adopted in this work as one of the analytic methods. Adomian decomposition method is used to obtain an obstructed form of numerical approximations of differential equations involving linear and nonlinear terms.

The method is applicable in solving problems like integro-differential, algebraic, differential relay, integral and partial differential equations in the field of science and engineering. An American mathematician, G. Adomian (Adomian 1923- 96) invented the system of Adomian decomposition method. In his to pursuit for a resolutions of certain equations inform of sequences, Adomian polynomials were used recursively to calculate the terms in series from the problem arising from non-linear operators (Adomian, 1994, Adomian 1988). The Adomian decomposition method is however more technical than any other classical methods because it evades agitation in stability to find resolutions of known nonlinear equation as well as, offers a precise estimate of the solution. The technique does not necessitate discretization of the resolution and this makes it a main advantage over traditional numerical methods. Great techniques of equations involving linear or nonlinear equations are needed as compare to other numerical methods. The solution is found within short time and reduces the memory of the computer as well as errors free. We start using the (deterministic) form$f(u) = g(t)$ for considering and illustrating the method of Adomian Decomposition. The operator f having term of linear and nonlinear where the term of the linear is represented as Lu and L is the linear operator. Instead, the linear term is written as $L\,u + Ru$ and L is chosen as the derivative of highest-order. The linear operator is R served as the remainder (considering stochastic expression term)

The nonlinear expression is indicated as Lu.

$$Lu + Ru + Nu = g \qquad (2.1)$$

Thus,

$$L^{-1}Lu = L^{-1}g - L^{-1}Ru - L^{-1}Nu \qquad (2.2)$$

The problems value were defined as L^{-1} for $L = \frac{\partial^n}{\partial t^n}$ as the n-fold define operator from 0 to t that served as integration. For the operator $L = \frac{\partial^2}{\partial t^2}$, for instance we have,

$$L^{-1}Lu = u - u(0) - tu'(0) \qquad (2.3)$$

$$u = u(0) + L^{-1}g - L^{-1}Ru - L^{-1}Nu \quad (2.4)$$

Taking into consideration problem of boundary value for the same equation, operator L^{-1} is represented as an indefinite integral and $u = A + Bt$ for the initial expressions which are two and A, B are evaluated from the set Condition. The first three terms in the supposed decomposition represented by u_0

$$U = \sum_{n=0}^{\infty} u_n \quad (2.5)$$

Nu is assumed analytical solution.

$$Nu = \sum_{n=0}^{\infty} A_n \quad (2.6)$$

Therefore, A_n are in particular created (Adomian polynomials used for the specific nonlinearity).

These depend on the u_0 to u_n sections which figure quickly a convergent sequences. The A_n indicated as:

$$A_0 = f(u_0) \quad (2.7)$$

$$A_1 = u_1\left(\frac{d}{du_0}\right)f(u_0) \quad (2.8)$$

$$A_2 = u_2\left(\frac{d}{du_0}\right)f(u_0) + \frac{{u_1}^2}{2}\frac{d^2}{{u_0}^2}f(u_0) \quad (2.9)$$

It is capable to be discovered early stage of expression (for $n \geq 1$)

$A_n = \sum_{n=1}^{\infty} C(v, n) f^v u_0$ In the linear situation when

$G(u) = u$ the A_n reduce to u_n. Then $A_0 = A(u_0, u_1, u_2 \ldots\ldots u_n)$

Several means are considered to generate the polynomials. The successive recursive formulations were used

$$A_n = \frac{1}{n!}\left[\frac{d^n}{d^\lambda}\left[N\left(\sum_{n=0}^{\infty} \lambda^i u_i\right)\right]\right]_\lambda \quad n = 0,1,2,3,4 \ldots\ldots \quad (2.10)$$

The solution is a created solution through the form of conventional solution and it is more accurate and stand the test to the technique of streamlining the physical problems since it does not help in linearizing or postulation of unstable non-linearity,.

3. PROBLEM FORMULATION

Consider the laminar, two- dimensional, steady flow and heat transfer in Sisko Nano-fluid in the region $y > 0$ determined with a sheet stretching with power-law velocity $U = cx^s$ were considered and a non-negative actual number represented as c and the sheet stretching rate represented as $(s > 0)$. The horizontal axis called the x-axis is to be corresponded to the stretching sheet and the vertical axis (y-axis) correspondent to the sheet of the plane. A method of heat transfer by convention that provides heat transfer coefficient h_f is employed in cooling down or heating up the sheet surface (to be regulated) in using warm temperature of the fluid, T_w. The equivalent nano particle bulk fraction of the exterior of the stretching sheet was presumed to be C_w whereas T_∞ and C_∞ were the temperature of the ambient as well as volume fractions of nano particle separately. Based on these postulations combined with estimation of the boundary layer. The convective boundary-layer flow of force is directed below:

Continuity Equation:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{3.1}$$

Momentum Equation:

$$u\frac{\partial u}{\partial x} + \text{v}\frac{\partial u}{\partial x} = \frac{a}{\rho}\frac{\partial^2 u}{\partial y^2} - \frac{b}{\rho}\frac{\partial}{\partial y}\left(-\frac{\partial u}{\partial y}\right)^n \tag{3.2}$$

Heat Equation

$$u\frac{\partial T}{\partial x} + \text{v}\frac{\partial T}{\partial y} = \propto \frac{\partial^2 T}{\partial y^2} + \tau\left[D_B \frac{\partial C}{\partial y}\frac{\partial T}{\partial y} + \frac{D_T}{T_\infty}\left(\frac{\partial T}{\partial y}\right)\right] \tag{3.3}$$

Concentration Equation

$$u\frac{\partial C}{\partial x} + \text{v}\frac{\partial C}{\partial y} = D_B\frac{\partial^2 C}{\partial y^2} + \frac{D_T}{T_\infty}\frac{\partial^2 T}{\partial y^2} \tag{3.4}$$

Subject to the boundary conditions:

$$\text{u} = \text{U} = \text{c}x^s,\ v_w = \text{S},\ K\frac{\partial T}{\partial y} = -h_w[T_w - T_\infty],\ \text{C}=C_w\ at\ y = 0, \tag{3.5}$$

$$u \to 0, v \to 0, T \to \infty, C \to C_w\ as\ y \to \infty \tag{3.6}$$

Applying suitable similarity transformation and stream function to equation (3.1) to (3.6) the equation is reduced to the following local solution

$$f''' - \frac{AR}{AR - n(f'')^{n-1}} {L_1}^{-1}[ff' - (f')^2] = 0 \tag{3.7}$$

$$\theta'' + P_r f\theta' + N_b\theta'\vartheta' + N_t\theta'^2 = 0 \tag{3.8}$$

$$\vartheta'' + Lef\vartheta' + \frac{N_t}{N_b}\theta' = 0 \tag{3.9}$$

With required conditions:

$$f(0) = 0, \quad f'(0) = 1, \quad \theta'(0) = -Bi[1 - \theta(0)], \quad \vartheta(0) = 1, \tag{3.10}$$

$$f'(1) = 0, \quad \theta(1) = 0, \quad \vartheta(1) = 0. \tag{3.11}$$

SOLUTIONS

Applying ADM and Apply inverse operator on equation (3.7) to (3.11) we have

$$f(\eta) = \frac{AR}{AR - nL_1^{-1}(f'')^{n-1}} L_1{}^{-1}[ff' - (f')^2 + S + \eta + \frac{\eta^2}{2} \propto \quad (3.12)$$

$$\theta(\eta) = -L_2^{-1}[P_r f\theta' + N_b\theta'\vartheta' + N_t\theta'^2] + \beta + \eta Bi(1 - \beta\ ) \quad (3.13)$$

$$\vartheta(\eta) = -L_2^{-1}\left[Lef\vartheta' + \frac{N_t}{N_b}\theta'\right] + 1 + \gamma \quad (3.14)$$

where

$$L_1^{-1} = \iiint (.)\partial\eta\partial\eta\partial\eta \text{ and } L_2^{-1} = \iint (.)\partial\eta\partial\eta \quad (3.15)$$

Series polynomial is used to decompose the depending variable in equation (3.12) to (3.15) as a function of nonlinear terms, so we have

$$\mathrm{P}(\mathrm{F}(\eta)) = \sum_{m=0}^{\infty} A_m(\eta) - \sum_{m=0}^{\infty} B_m(\eta) \quad (3.16)$$

$$G\big(\theta(\ \eta)\big) = \sum_{n=0}^{\infty} C_n(\eta) + \sum_{n=0}^{\infty} D_n(\eta) + \sum_{m=0}^{\infty} E_n(\eta) \quad (3.17)$$

$$G\big(\vartheta(\ \eta)\big) = \sum_{n=0}^{\infty} K_n(\eta) \quad (3.18)$$

Adomian polynomials $A_m, B_m, C_n, D_n, E_n\ and\ K_n$ represent the nonlinear terms $\mathrm{P}\big(f(\eta)\big), G\big(\theta(\eta)\big), G\big(\vartheta(\eta)\big)$can be gotten from:

$$A_m = \sum_{V=0}^{m} f'_{m-V} f'_V \quad (3.19)$$

$$B_m \sum_{V=0}^{m} f_{m-V} f''_V \quad (3.20)$$

$$C_n = \sum_{V=0}^{n} f_{n-V}\, \theta'_V \quad (3.21)$$

$$D_n = \sum_{V=0}^{n} \theta'_{n-V}\, \vartheta'_V \quad (3.22)$$

$$E_n = \sum_{V=0}^{n} \theta'_{n-V}\, \theta'_V \quad (3.23)$$

$$K_n = \sum_{V=0}^{n} f_{n-V}\vartheta'_V \quad (3.24)$$

For determination of other components of$f(\eta), \theta(\eta), \vartheta(\eta)$we have that,

$$\sum_{m+1}^{\infty} f(\eta) = \frac{AR}{AR - nL_1{}^{-1}\sum_{m=0}^{\infty}(f'')^{n-1}} L_1{}^{-1} \sum_{m=0}^{\infty}[A_m - B_m] \quad (3.25)$$

$$\sum_{m+1}^{\infty} \theta(\eta) = \sum_{n=0}^{\infty}[P_r C_n + N_b D_n + N_t E_n] \quad (3.26)$$

$$\sum_{m+1}^{\infty} \vartheta(\eta) = -L_2^{-1} \sum_{m=0}^{\infty}\left[LeK_n + \frac{N_t}{N_b}\theta'\right] \quad (3.27)$$

Maple 18 software is used to compute the integrals, where

$$f_0(\eta) = S + \eta + \frac{\eta^2}{2} \propto \quad (3.28)$$

$$\theta_0(\eta) = \beta + \eta Bi(1-\beta) \tag{3.29}$$

$$\vartheta_0(\eta) = 1 + \eta\gamma \tag{3.30}$$

4. RESULTS AND DISCUSSION

Maple 18 software was use to simulation the solution of Adomian Decomposition method of sisko nano-fluid over nonlinear sheet with suction and injection. The graphs of velocity $f'(\eta)$, temperature $\theta(\eta)$ and Nano Concentration profiles $\vartheta(\eta)$ were plotted against Prandtl Number, thermophoresis number, power law and Brownian number. The graphs shows, the influence of the physical parameters such as material constant of sisko Nano-fluid (A)and Prandtl number(P_r) on the velocity (a), temperature(b)and nano concentration profiles (c)

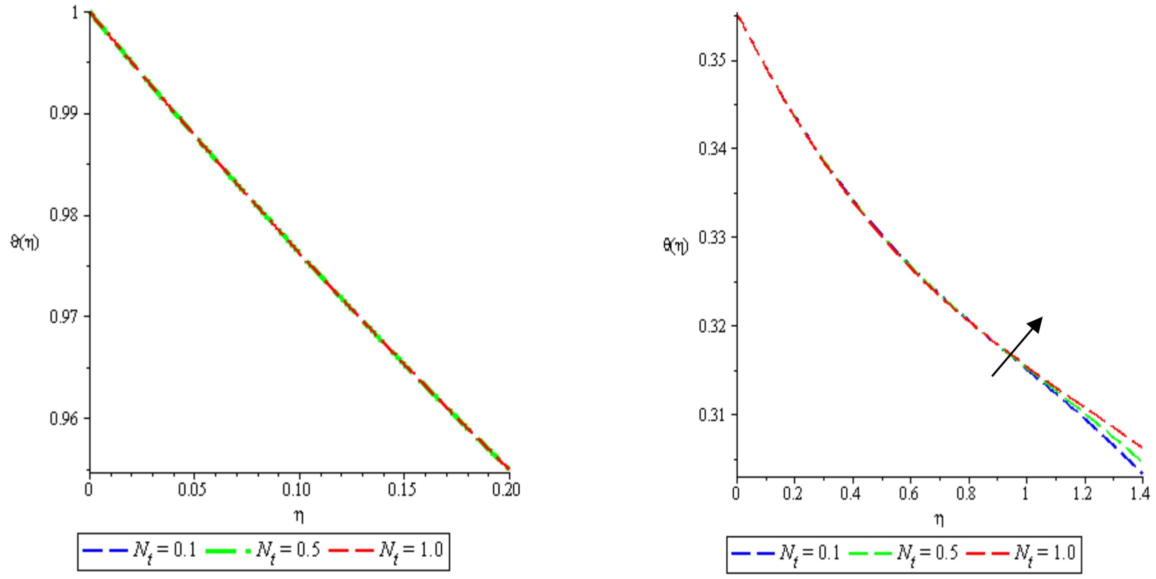

Figure 1 Effect of Thermophoresis number on (a) Concentration Profile (b) Temperature profile

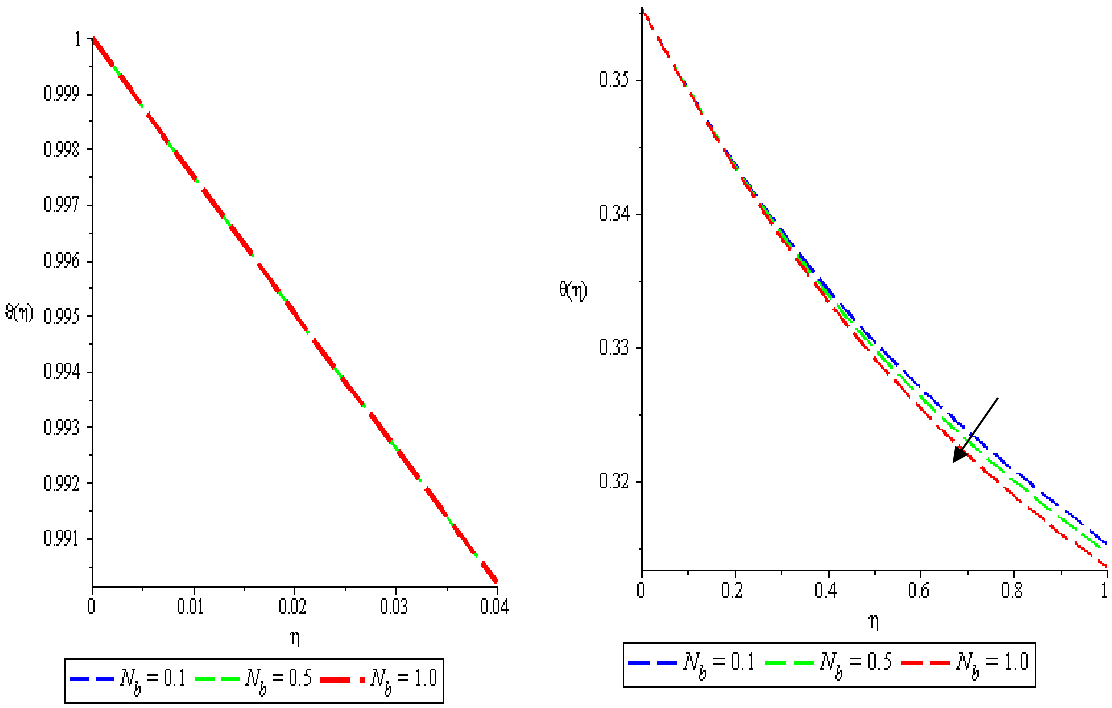

Figure 2 Effect of Brownian number on (a) Concentration (b) Temperature profile

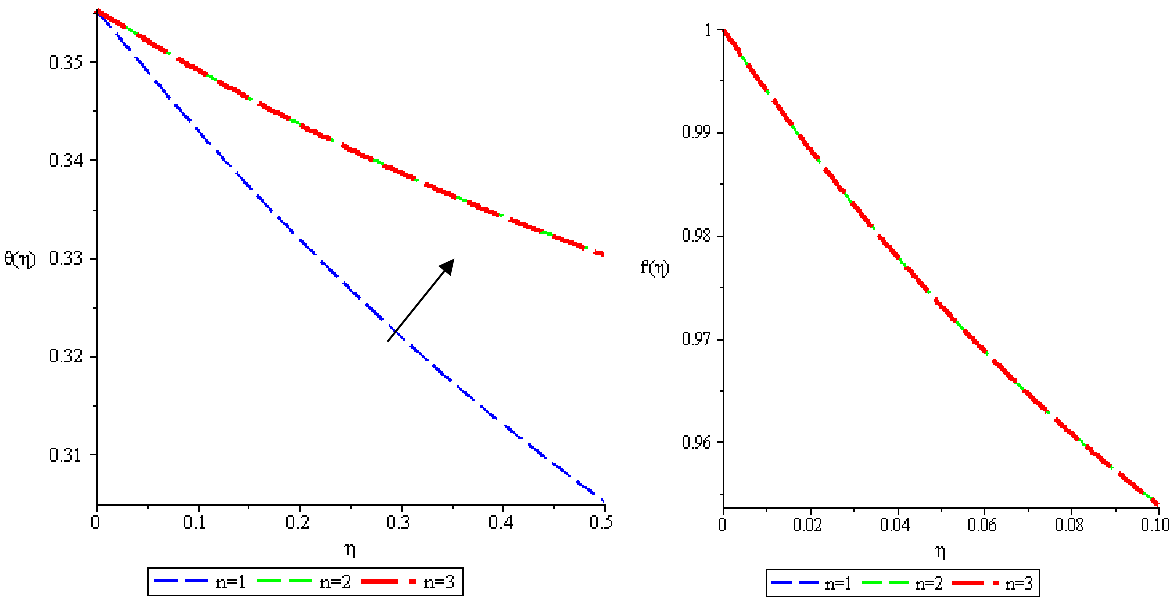

Figure 3 Effect of Power law on (a) Temperature Profile (b) Velocity profile

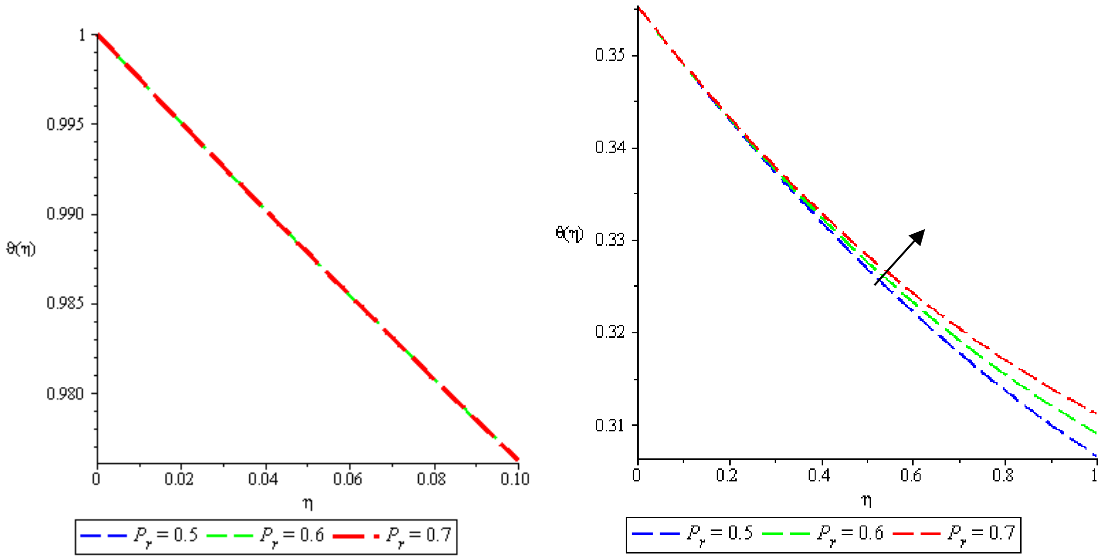

Figure 4 Effect of Prandt number on (a) Concentration profile (b) Temperature profile

Figure 1 shows the effect of thermophoresis on nano concentration and Temperature profile. As the thermophoresis number increase there is raise in temperature and no change in concentration profile

Figure 2 shows the effect of Brownian number on nano concentration and Temperature profile. As the Brownian number increase there is negative decrease in temperature and no change on concentration profile

Figure 3 shows the effect of Power law on Velocity profile and Temperature profile. As the Power law increase there is positive increase in temperature and no change on velocity profile

Figure 4 shows the effect of Prandtl number on nano concentration and Temperature profile. As the Prandtl number increase there is an increase in temperature and no change on concentration profile

5. CONCLUSION

This work focuses on the sisko nano fluid over non-linear sheet with suction and injection. Local similarity solutions were obtained and solution solved using Adomian Decomposition method. The solution was presented which depends on Brownian number, thermophoresis number, Prandtl number and power law on the velocity, temperature and nano concentration profiles. It was found that

1. Brownian number does not have effect on nano concentration profile
2. Increase in power law has positive effect on temperature and no change on nano concentration profile.
3. Increase in thermophoresis increase the temperature profile but no effect on nano concentration.
4. Increase in Prandtl number increase the temperature but no effect on concentration profile

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