Perturbation Analysis of Magneto hydrodynamic Flow of Third Grade Fluid in an Inclined Cylindrical Pipe

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Abstract

In this paper, analysis of magnetohydrodynamic third grade fluid in an inclined cylindrical pipe was considered. The flow was assumed to be induced by the axial pressure gradient. The resulting governing equations of flow were solved using the regular perturbation method. Results show that the Brinkmann and the Eckert numbers have the tendency to slow the movement of the fluid in the centre line of the cylinder, as a result, the velocity increases near the wall associated with the constant flow rate at each section of the cylinder. Results further show that increase in the Grashhof parameter slightly reduced the temperature of the system, with the profiles converging to zero, satisfying the boundary condition and the walls of the cylinder isothermally stable.

Keywords: *Magnetohydrodynamic, third grade, cylindrical pipe.*

1. INTRODUCTION

In the analysis of flow of magnetohydrodynamic third grade fluid in an inclined cylindrical pipe, many investigators have considered the flow of third grade fluid in different forms. Some of the foremostFosdick and Rajagopal (1980), who examined the thermodynamics and stability of fluids of third grade and showed restrictions on the stress constitutive equation was concern with the relation between thermodynamics and stability for a class of non-Newtonian incompressible fluids of the differential type, and further introduced the additional thermodynamical restriction that the Helmholtz free energy density be at a minimum value when the fluid is locally at rest. They gave detailed attention to the special case of fluids of grade 3 and arrived at fundamental inequalities which restricts its temperature dependent. They discovered that these inequalities requires that a body of such a fluid be stable in the sense that its total kinetic energy must tend to zero in time, no matter what its previous mechanical and thermal fields, provided it is both mechanically isolated and immersed in a thermally passive environment at constant temperature from some finite time onward.

Rajagopal *et al* (1986) examined the flow past an infinite porous flat plates with suction. They discovered that the non-Newtonian fluid mechanics affords an excellent opportunity for studying many of the mathematical methods which have been developed to analyze non-linear problems in mechanics. They established an existence theorem using the shooting and investigated the problem using perturbation analysis and numerical method and concluded that since perturbation method did not converge suitably, it was not the appropriate tool for the problem. Obi *et al* (2017) considered the flow of an incompressible MHD third grade fluid in the annulus of a rotating concentric cylinders with isothermal walls and Joule heating. The flow was assumed to be induced by the axial pressure gradient. They found that at an angular velocity $\omega = 1.0$, the velocity of the fluid tends to be at equilibrium, but with $\omega < 1.0$, the velocity was drastically reduced and when $\omega > 1.0$, it was greatly enhanced. Masoudi and Christie (1995) analysed the effect of temperature-dependent viscosity for the three separate cases treated by Szeri and Rajagopal (1998).

Okedayo *et al* (2019) computationally analyzed the reactive MHD third grade fluid in an electrically conductive cylindrical channel with axial magnetic field. The weighted residual collocation method was used. Results revealed the influence of various thermo-physical parameters and that the critical values of the Frank-Kamenetskii and the third grade parameters exist for which the solution seizes to be unique. Aiyesimi *et al* (2014) dealt with the effects of magnetic field on the MHD flow of third grade fluid through inclined channel with Ohmic heating. They analysed the Couette flow, Poiseuille and Couette-Poiseuille flow and solved the resulting non-linear differential equation by employing regular perturbation technique. In Yurusoy *et al* (2008) on the entropy due to fluid friction and heat transfer in the annular pipe for fluid between concentric circular cylinders with vogels viscosity model. Makinde (2005) examined the hydrodynamic ally and thermally developed Reynolds viscosity liquid film along an inclined heated plate: an exploitation of Hermite-pade approximation technique. Ayub, et al (2003) examined the exact flow of third grade fluid past a porous plate using homotopy analysis method. Yurusoy and Pakdemirli (2002) considered approximate analytical solutions for the flow of a third grade fluid in a pipe. Makinde (2007) studied thermal stability of a reactive third grade fluid in a cylindrical pipe: An exploitation of Hermite Pade approximation technique.

Baldoni *et al* (1993) studied the helical flow of a third grade fluid between eccentric cylinders, using a domain perturbation approach. They discovered a secondary flow contrary to the initial analysis. The consistency of the slow flow approximation has been tested considering the behaviour of the fluid at intermediate and high Reynolds number. According to Rajagopal and Mollica (1999) on secondary deformation due to axial shearing of annular region between two eccentrically placed cylinders followed the works of Fosdick and kao (1978) and extended a conjecture of Ericksen's (1956) for non linear fluid, to nonlinear elastic solids and showed that unless the material modulli of an isotropic elastic material satisfied certain special relations, axial shearing of cylinders would be necessarily accompanied by secondary deformation if the cross-section were not a circle or the annular region between concentric circles.

Yurusoy (2004) examined the flow of a third grade fluid between concentric circular cylinders with heat transfer. He assumed the temperature of the pipe to be higher than that of the fluid, he also considered Reynold's mode viscosity and perturbation technique was used for approximate analytical solu9tions. Yurusoy *et al* (2008) on the entropy due to fluid friction and heat transfer in the annular pipe for fluid between concentric circular cylinders with vogels viscosity model. Akyildiz *et al* (2004) examined the exact solution of nonlinear differential equations arising in third grade fluid flows. They considered a rotating cylinder (unbounded domain case) and between rotating cylinders (bounded domain case). The exact solutions were compared with the numerical ones and it was observed that the difference between the exact and the numerical solutions is about 1% for small R (the non-dimensional distance between the cylinders) and is about 3% when R=100. This difference increases with an increasing R.

2. MATHEMATICAL FORMULATION:

Consider the flow of third grade fluid through an isothermally heated cylinder. Pressure-gradient was assumed to have induced the fluid motion. For magnetohydrodynamically developed flow, both the velocity and the temperature fields depend on r only. Following [2 and 11] and neglecting the reacting viscous fluid assumption, the governing equations for the momentum and energy balance can be represented as:

$$\frac{\mu}{r}\frac{d}{dr}\left(r\frac{du}{dr}\right)+\frac{\beta}{r}\frac{d}{dr}\left(r\left(\frac{du}{dr}\right)^{3}\right)+\rho g\beta\left(T-T_0\right)\sin\phi-\sigma B_0^2u \;=-\frac{dp}{dr} \qquad (1)$$

$$\frac{k}{r}\frac{d}{dr}\left(r\frac{dT}{dr}\right)+\left(\frac{du}{dr}\right)^{2}\left(\mu+\beta_3\left(\frac{du}{dr}\right)^{2}\right)+\sigma B_0u^2=0 \qquad (2)$$

subject to the boundary condition

$$\frac{du}{dr}(0)=\frac{dT}{dr}(0)=0,\; u(a)=0,\; T(a)=T_0 \qquad (3)$$

where T is the temperature of the cylinder, u is the fluid velocity, T_0 is the plate temperature, k is thermal conductivity, μ is the coefficient of dynamic viscosity of the fluid μ is the coefficient of dynamic viscosity of the fluid σ is the coefficient of dynamic viscosity of the fluid, ρ is the density of the fluid, g is the acceleration due to gravity, u is the characteristic fluid velocity, B_0 is the magnetic field effect, p is the pressure of the system, ϕ is the angle of inclination and β_3 is the third grade parameter.

Introduce the following non-dimensional variables (4) and substituting same into equation (1), and dropping the bars for simplicity gives (5)

$$\theta=\frac{T-T_0}{T_0},\; \bar{r}=\frac{r}{a},\; \bar{u}=\frac{u}{u_0},\; and\;\; \sigma=\frac{\beta_3u^2}{a^2\delta} \qquad (4)$$

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{du}{dr}\right)+\frac{\gamma}{r}\frac{d}{dr}\left(r\left(\frac{du}{dr}\right)^3\right)+\eta\theta-\zeta u=p \qquad (5)$$

where

$\eta=\frac{a^2}{\mu u_0}\rho g\beta\left(T_0\theta+T_0-T_0\right)\sin\phi$ is the modified Grashof number

$\gamma=\frac{\beta u_0^2}{a^2\mu}$ is the third grade parameter

$\xi=\frac{\beta_3 u^2}{\mu u_0\delta}B_0^2 u_0$ is the Hartman number and

$P=\frac{a^2}{\mu u_0}\frac{dp}{dz}$ is the modified pressure gradient.

3. METHOD OF SOLUTION

Defining the perturbation series as follows:

$$u=u_0+\beta u_1+\beta^2 u_2+0\left(\beta^3\right),\ \theta=\theta_0+\beta\theta_1+\beta^2 u_2+0\left(\beta^3\right),$$
$$\eta=\beta G,\quad \xi=\beta M\quad and\quad \gamma=\beta N \qquad (6)$$

substituting equation (6) into equation (5) yields equations (7-9), subject to condition (10)

$$\beta^0:\frac{1}{r}\frac{d}{dr}\left(r\frac{du_0}{d}\right)=p \qquad (7)$$

$$\beta:\frac{1}{r}\frac{d}{dr}\left(r\frac{du_1}{dr}\right)+\frac{\gamma}{r}\frac{d}{dr}\left(r\frac{du_0}{dr}\right)^3+G\theta_0+Mu_0=0 \qquad (8)$$

$$\beta^2:\left(\frac{1}{r}\frac{d}{dr}\left(r\beta^2\frac{du_2}{dr}\right)+\left(9r\left(\frac{du_0}{dr}\right)^2\left(\frac{du_1}{dr}\right)+\left(3r^2\frac{du_0}{dr}\right)^2\left(\frac{d}{dr}\left(\frac{du_1}{dr}\right)\right)\right.\right.$$
$$\left.\left.+6r^2\left(\frac{du_0}{dr}\right)\left(\frac{du_1}{dr}\right)\left(\frac{d}{dr}\left(\frac{du_0}{dr}\right)\right)\right)-G\theta_1-Mu_1\right)=0 \qquad (9)$$

$$u_0(1) = u_1(1) = u_3(1) = 0 \tag{10}$$

Solving equations (7-9) using equation (10), yields

$$u(r) = \frac{p}{4}(r^2 - 1) - \frac{1}{2304}Gr^6E_c - \frac{1}{48}r^6p^3 + \frac{1}{64}Mr^4p + \frac{1}{256}Gr^2E_c - \frac{1}{16}r^2Mp - \frac{1}{288}GE_c$$

$$+\frac{1}{48}p^3 + \frac{3}{64}Mp - \frac{1}{18432}GP^4r^8Br - \frac{1}{18432}GP^2r^8E_cM + \frac{1}{192}Mr^2P^3 + \frac{1}{2304}M^2r^6P$$

$$-\frac{19}{3072}P^3r^8M + \frac{3}{320}P^5r^{10} - \frac{1}{147456}Mr^8GE_c - \frac{1}{1024}P^2r^6GE_c + \frac{11}{98304} + G^2Pr^2E_c^{\,2} + \frac{1}{4096}Mr^4GE_c$$

$$-\frac{1}{73728}G^2\,\mathrm{Pr}^6\,E_c^{\,2} - \frac{1}{2048}GP^4r^2E_c + \frac{1}{1152}GP^4r^2Br + \frac{1}{51200}GP^4r^{10}E_c + \frac{1}{2457600}G^2\,\mathrm{Pr}^{10}\,E_c^{\,2}$$

$$+\frac{1}{5120}P^2r^{10}GE_c - \frac{15}{1024}MP^3 - \frac{19}{2304} - M^2P - \frac{3}{320}P^5 - \frac{5}{18432}GP^2E_cM + \frac{13}{49152}MGE_c$$

$$-\frac{91}{921600}G^2PE_c^{\,2} + \frac{3}{6400}GP^4E_c - \frac{5}{6144}GP^4B_r + \frac{1}{1280}P^2GE_c - \frac{1}{1152}Mr^2GE_c$$

$$+\frac{1}{2304}GP^2r^6E_cM - \frac{1}{1024}GP^2r^4E_cM + \frac{1}{1152}GP^2r^2E_cM + \frac{1}{64}P^3r^6M - \frac{1}{256}M^2r^4P$$

$$+\frac{3}{256}M^2r^2P \tag{11}$$

4. HEAT TRANSFER ANALYSIS

Using equation (4) in equation (2) and dropping the bars for simplicity yields

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{d\theta}{dr}\right) + \beta\left[\lambda\left(\frac{du}{dr}\right)^2 + 2B_r\left(\frac{du}{dr}\right)^4\right] + E_c\beta\tau u^2 = 0 \tag{12}$$

where

$B_r = \dfrac{u_0^2}{kT_0}$ is the Brinkman number

Substituting Equation (6) into equation (12), yields

$$\beta^0 : \frac{1}{r}\frac{d}{dr}\left(r\frac{d\theta_0}{dr}\right) + B_r\left(\frac{du_0}{dr}\right) = 0 \tag{13}$$

$$\beta : \frac{1}{r}\frac{d}{dr}\left(r\frac{d\theta_1}{dr}\right) + 2B_r\left(\frac{du_0}{dr}\right)^4$$
$$+2B_r\left(\frac{du_0}{dr}\right)\left(\frac{du_1}{dr}\right) + E_c M u_0^2 = 0 \tag{14}$$

$$\beta^2 : \frac{1}{r}\frac{d}{dr}\left(r\frac{d\theta_2}{dr}\right) + 8\left(\frac{du_0}{dr}\right)^3 B_r\frac{du_1}{dr} + 2\lambda\frac{du_0}{dr}\frac{du_1}{dr} + 2E_c M u_0 u_1 = 0 \tag{15}$$

$$\theta_0(1) = \theta_1(1) = \theta_2(1) = 0 \tag{16}$$

Solving equations (13-15) with the condition (16) gives

$$\theta(r) = p^2 B_r\left(\frac{1}{80} - \frac{r^5}{80}\right) + \frac{1}{200}B_r p^4 r^5 + B_r p\frac{r^2}{4}\left(\frac{r^2 p}{8}\right)$$
$$\left(-\frac{1}{13824}Gr^6 E_c - \frac{1}{288}r^6 p^3 + \frac{1}{256}Mr^4 p + \frac{1}{512}Gr^2 E_c - \frac{1}{32}Mr^2 p\right)$$
$$+E_c M\frac{r^2}{4}\left(\frac{1}{36}pr^3 - \frac{1}{4}rp\right) - \frac{1}{200}B_r p^4 r^5 - B_r p\frac{r^2}{4}\left(\frac{r^2 p}{8}\right)\left(+\frac{1}{13824}Gr^6 E_c + \frac{1}{288}r^6 p^3 - \frac{1}{256}Mr^4 p\right.$$
$$\left.-\frac{1}{512}Gr^2 E_c + \frac{1}{32}Mr^2 p\right) - E_c M\frac{r^2}{4}\left(\frac{1}{36}pr^3 + \frac{1}{4}rp\right) + \left(-\frac{1}{800}B_r P^6 + \frac{137}{42467328}G^2 E_c^3\right.$$
$$+\frac{1}{288}B_r P^4 r^6 M + \frac{3}{51200}E_c M P^4 - \frac{3}{513}E_c P^2 r^4 M^2$$

$$+\frac{3}{513}E_cP^2r^2M^2-\frac{85}{49152}E_cM^2P^2+\frac{1}{513}E_cP^2r^6M^2-\frac{13}{6144}E_cP^4r^8M+\frac{1}{384}E_cP^4r^2M$$

$$\frac{1}{55296}r^{12}E_c^2P^3G-\frac{1}{737280}r^{12}E_c^2P^5G-\frac{1}{35389440}r^{12}E_c^3P^2G^2+\frac{29}{38400}r^{10}E_cMP^4$$

$$+\frac{1}{8192}r^8E_c^2P^3G-\frac{1}{1024}r^8B_rP^4M-\frac{11}{49512}r^8E_cM^2P^2+\frac{1}{786432}r^8E_c^3P^2G^2-\frac{1}{768}r^4E_cMP^4$$

$$+\frac{1}{16384}r^4E_c^2P^5G+\frac{11}{1843200}r^{10}E_c^2MPG+\frac{1}{230400}r^{10}E_c^2P^3GM+\frac{1}{230400}r^{10}E_cP^5GB_r$$

$$+\frac{1}{38400}r^{10}B_rP^3GE_c-\frac{1}{4608}r^6B_rP^3GE_c-\frac{1}{9216}r^4E_cP^5GB_r-\frac{1}{9216}Gr^6E_c^2MP-\frac{7}{9216}r^{12}E_cP^6$$

$$-\frac{1}{21233664}r^{12}G^2E_c^3+\frac{1}{800}r^{10}B_rP^6+\frac{1}{1572864}r^8G^2E_c^3-\frac{1}{262144}r^4G^2E_c^3+\frac{1}{9600}E_cP^5GB_r$$

$$+\frac{67}{1843200}E_c^2P^5GM+\frac{11}{57600}B_rP^3GE_c+\frac{7}{9216}E_cP^6+\frac{41}{460800}E_c^2MPG-\frac{11}{184320}E_c^2P^5G$$

$$+\frac{67}{1843200}E_c^2P^5GM+\frac{11}{57600}B_rP^3GE_c+\frac{7}{9216}E_cP^6+\frac{41}{460800}E_c^2MPG-\frac{11}{184320}E_c^2P^5G$$

$$-\frac{23}{9216}B_rP^4M+\frac{451}{35389440}E_c^3P^2G^2-\frac{23}{221184}E_c^2P^3G-\frac{1}{24576}E_c^2P^3r^8GM-\frac{1}{73728}E_c^2Pr^8MG$$

$$+\frac{17}{36864}E_c^2\Pr^4MG-\frac{1}{2304}E_c^2\Pr^2MG+\frac{1}{9216}E_c^2P^3r^6GM-\frac{1}{9216}E_c^2P^3r^4GM \qquad (17)$$

5. RESULTS AND ANALYSIS

Specifying the various values of the thermo-physical parameters, the following results were generated to see the effects of various dimensionless numbers on the velocity field and the energy balance.

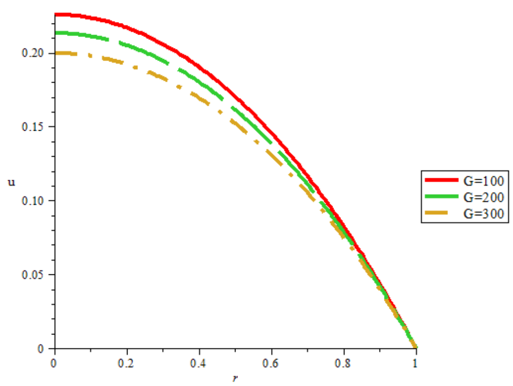

Figure 1: Velocity Profile For Various Values the Grashhof Number

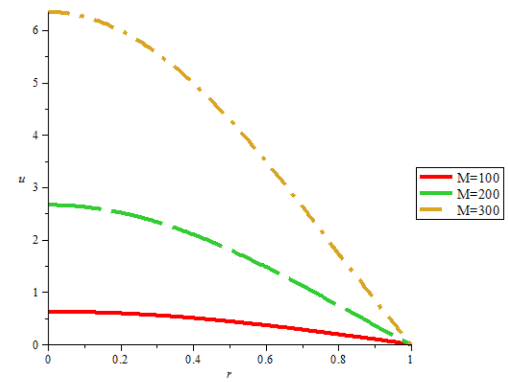

Figure 2: Velocity Profile For Various Values Of the Magnetic Parameters

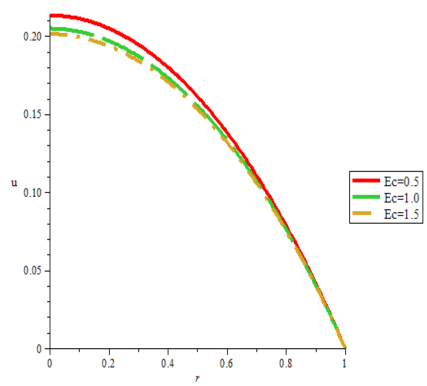

Figure 3: Velocity Profile for Various Values of the Eckert Num

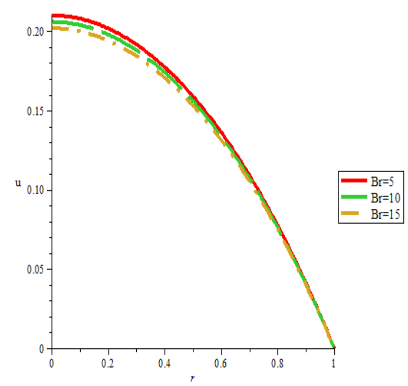

Figure 4: Velocity Profile for Various Values of The Brinkman Number

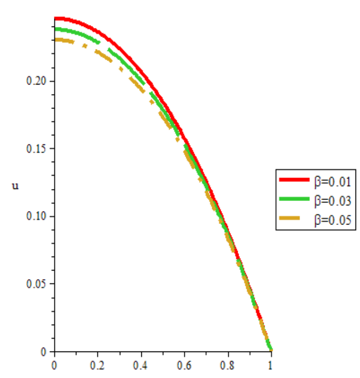

Figure 5: Velocity Profile for Various Values Of the Third Grade Parameters

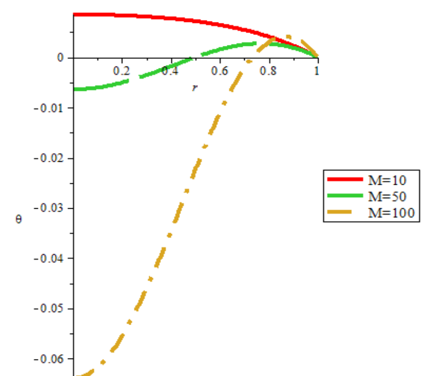

Figure 6: Temperature Profile For Various Values of the Magnetic Parameter

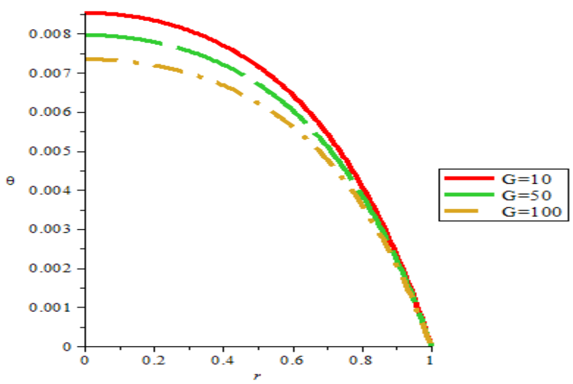

Figure 7: Temperature Profile for Various Values of the Grashhof Number

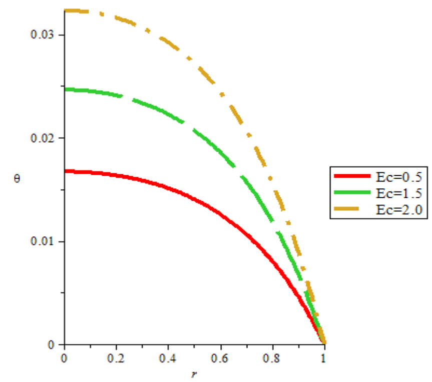

Figure 8: Temperature Profile for Various Values of the Eckert Number

In figure 1, the Grashhof number was varied between 100 and 300, while the other parameters were held constant that is $p=-1,\ M=5,\ E_c=0.5,\ B_r=1\ and\ \beta=0.1$, results show that the increase in the Grashhof number reduces the velocity of the fluid. Results further revealed that the velocity profiles converges to zero, when $r=1$, satisfying the boundary condition. Figure 2 shows that the magnetic field was varied between 100 and 300 with the rest of the parameters kept constant that is $p=-1,\ G=100,\ E_c=0.5,\ B_r=1\ and\ \beta=0.1$, and that the magnetic field reduces the velocity of the fluid flow. Result further revealed that the velocity profiles converged to zero at $r=1$, satisfying the boundary condition. In figure 3 the Eckert number was varied between 0.5 and 1.5, when the other parameters were kept constant, that is $p=-1,\ G=100,\ M=5,\ B_r=1\ and\ \beta=0.1$ .Results show that the application of the Eckert number in figure 4 tend to slow down the velocity of the fluid flow and that the velocity profiles converged to zero at the point $r=1$, satisfying the boundary condition. The Brinkman number was varied between 5 and 15, when the other parameters were kept constant that is $p=-1,\ M=5,\ E_c=0.5,\ G=100$ and $\beta=0.1$. Results show that the Brinkman number have the tendency to slow the movement of the fluid in the centre line of the cylinder, and as a result of this, velocity increases near the walls associated with the constant flow rate at each section of the cylinder. The velocity profiles converged appropriately. In figure 5, the third grade parameter was varied between 0.01 and 0.05 while the other parameters were kept constant that is $p=-1,\ G=100,\ M=5,\ B_r=1\ and\ E_c=0.5$. Results revealed that the velocity profiles converged appropriately to zero at the point $r=1$ and that the third grade parameter has tendency of reducing the velocity of the fluid flow. The magnetic field parameter was varied between 10 and 100, while the other parameters were kept constant that is $p=-1,\ G=10,\ E_c=0.5,\ B_r=1\ and\ \beta=0.1$. Results show in figure 6, that increase in the magnetic field greatly reduced the temperature of the system. Results further show that although there appeared to be a negative profiles, yet all the temperature profiles converged to zero at the point $r=1$, satisfying the boundary condition and the walls of the cylinder at a constant temperature. The Grashhof number was varied between 10 and 100 while the other parameters were held constant in figure 7, that is, $p=-1,\ E_c=0.5,\ M=5,\ B_r=1\ and\ \beta=0.1$, within the inclined cylindrical pipe. Results show that increase in the Grashoff number slightly reduced the temperature of the system, with the profiles converging to zero, satisfying the boundary condition and the walls of the cylinder isothermally stable. In figure 8, the Eckert number was varied between 0.5 and 2.0, when $p=-1,\ G=10,\ M=10,\ B_r=1\ and\ \beta=0.1$ were kept constant.

6. CONCLUSION

In this present paper on the analysis of magnetohydrodynamic flow of third grade fluid in an inclined cylindrical pipe, the coupled nonlinear equation is solved using perturbation technique. Results show that solutions exist for values of the thermo-physical parameters.

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