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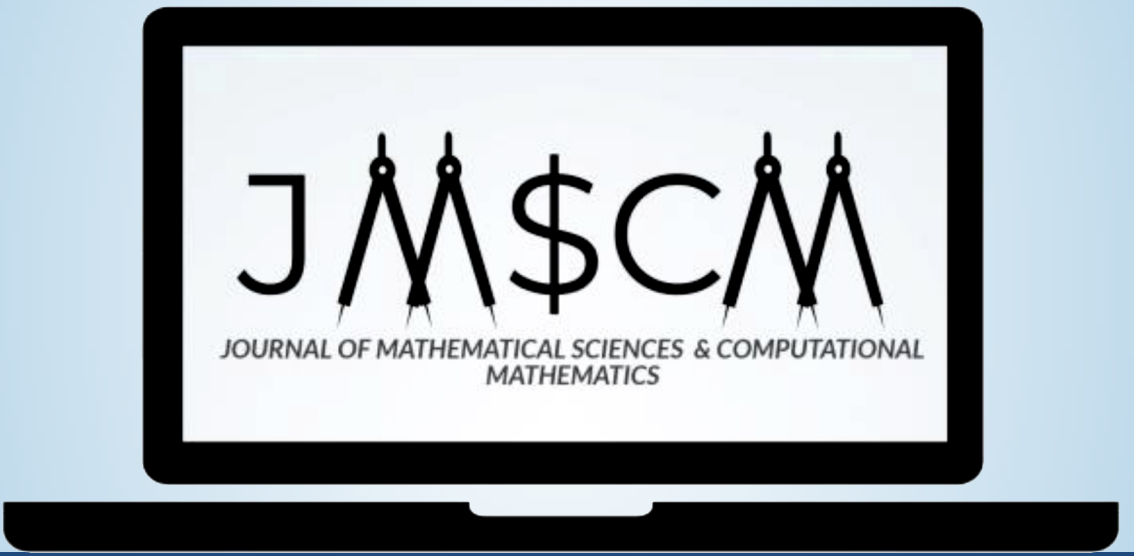

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Editorial and Administrative Information

From Managing Editor's Desk

"Mathematics is a more powerful instrument of knowledge than any other that has been bequeathed to us by human agency" - 'Descartes'

The Journal of Mathematical Sciences & Computational Mathematics (JMSCM) is a peer reviewed, international journal which promptly publishes original research papers, reviews and technical notes in the field of Pure and Applied Mathematics and Computing. It focuses on theories and applications on mathematical and computational methods with their developments and applications in Engineering, Technology, Finance, Fluid and Solid Mechanics, Life Sciences and Statistics.

The spectrum of the Journal is wide and available to exchange and enrich the learning experience among different strata from all corners of the society. The contributions will demonstrate interaction between various disciplines such as mathematical sciences, engineering sciences, numerical and computational sciences, physical and life sciences, management sciences and technological innovations. The Journal aims to become a recognized forum attracting motivated researchers from both the academic and industrial communities. It is devoted towards advancement of knowledge and innovative ideas in the present world.

It is the fourth issue of third volume and we expect to bring it to the zenith through the cooperation of my friends and colleagues.

We sincerely believe that the quarterly issues of the Journal will receive patronage from our esteemed readers and contributors throughout the globe.

Valuable suggestions from any corner for further improvement of the Journal are most welcome and highly appreciated.

Our thanks are due to the SMART SOCIETY, USA in bringing out the Journal in an orderly manner.

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USING THE CONCEPT OF FUZZY LOGIC AND CONTROLLER SYSTEM IN INTERPOLATION AND FUNCTIONS APPROXIMATION

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Abstract:

The main aim of this contribution is to present an alternative approach for interpolation and function approximation from a given data. Instead of the traditional interpolation methods we consider and propose a numerical procedure for interpolation using the concept of fuzzy logic and membership functions. The method can be used for interpolating data resulting from physical experiments, engineering, medicine, applied sciences etc. Fuzzification will be applied to the given data according to Mamdani technique and membership functions will be chosen to satisfy the interpolation mathematical condition. The defuzzification process will be implemented to get the crisp values of the interpolation. The procedure will be implemented on the mathematical code MATLAB and its Simulink fuzzy logic features. Finally the applicability and efficiency of the procedure is illustrated by numerical examples.

2010 AMS Subject Classification: 68T37, 65N22, 26A33, 41A55.

Keywords: *Fuzzy sets, Fuzzy logic, Fuzzy Controller System (FCS), Crisp sets Crisp logic, Membership functions, interpolation.*

1. Introduction

Interpolation is at root a simple mathematical concept. It is considered to be a mathematical and statistical method by which related known data are used to estimate an unknown values. Interpolation is achieved by using other established data that are located in the domain of the unknown value.

In the mathematical field of numerical analysis, interpolation is a type of estimation, it's a method of introducing and constructing new data points based on the range of a discrete set of known or given data points. [6, 9] It's widely used to replace complicated and non-smooth functions by polynomials or splines.

There is a rich history behind interpolation and function approximation. It really began with the early studies of astronomy when the motion of heavenly bodies was determined from periodic observations. The names of many famous mathematicians are associated with interpolation: Gauss, Newton, Bessel, and others.[9]

Interpolation is widely needed and used in engineering, applied sciences and medical sciences, where often has a number of available data points, obtained by sampling or experimentation, which represent the values of a function for a limited number of values of the independent variable. It is often required to interpolate; that is, estimate the value of that function for an intermediate value of the independent variable.

There are different types of interpolation. The n^{th} degree polynomial interpolation has been used in many interpolation methods for uniform and non-uniform data points, i.e. Newton-Gregory, Lagrange, Divided difference methods, least square method … etc.. Linear interpolation uses a linear function for each sub-interval $[x_i, x_{i+1}]$. Spline interpolation is another type that uses low-degree polynomials in each of the sub-intervals, and chooses the polynomial pieces such that they fit smoothly together. The resulting function is called a spline. For example, the natural cubic spline is piecewise cubic and twice continuously differentiable. Furthermore, its second derivative is zero at the end points. B-splines are often used to numerically integrate and differentiate functions that are defined only through a set of data points. [6, 9]

Saman [1] used the concept of fuzzy logic and system controllers together with the membership functions to give more reasonable values for the loss precisions in solving a system of equations. He considered the condition number and the rate of singularity of the system of equation as an input variables and the rate of accuracy of computed solution as the output variable. Fuzzy logic allows us also to model a non-specific mathematical and scientific language notions, especially, so called vogue expressions (small rate, very small determinant, large condition number, nearly singular matrix … etc.).

The aim of this paper is to overview the main concepts of fuzzy control based on logic and its applications. Moreover, to provide a technique using fuzzy control developed by Zadeh [18-21] and Mamdani [11-13] to help in choosing and evaluating the technique that best suits the given data set: the fuzzy membership functions and the logic rules will be chosen carefully to improve the quality of interpolation compared to traditional approximation methods, and to satisfy the interpolation condition at the given data. Data sets in the examples will be chosen to be connected to known functions to facilitate computing the actual error. Examples will be given with the given data as input or behaviour parameters , and the interpolating values as output or action parameters. The input crisp data will be fuzzified, and then defuzzified according to given rules.

2. Basic Concept of Fuzzy Logic

Fuzzy logic is considered as the concept of linguistic variable, it was supposed to be the elite tool for computing with wards variables where computers couldn't process linguistic variables. It considers linguistic variables, that is, variables whose values are not numbers but words or sentences in a natural or artificial language[15]. The term fuzzy logic was introduced with the 1965 proposal of fuzzy set theory by Lotfi

Zadeh who is considered to be the father of fuzzy logic. [18-20] Fuzzy logic had, however, been studied since the 1920s, as infinite-valued logic notably by Łukasiewicz and Tarski.[1, 10, 13, 21] It provided mathematicians with an appropriate tool for modelling vagueness phenomenon and shed new light into non-digital variables and control theory for engineers.[1-3, 8, 10, 13]

Fuzzy set and crisp set are the part of the distinct set theories, where the fuzzy set implements and allow the whole interval [0 1] to be the range of their characteristic functions which we call membership functions $\mu_A(x)$, and employ infinite-valued logic. The fuzzy membership functions $\mu_A(x)$ enabled us to overcome the difficulty of having very different control actions for a small a change in the inputs. while crisp set employs bi-valued logic and normally gives different actions for a very close similar inputs. [13, 18]

$$\mu_A(x) = \begin{cases} 1, & x \in A \\ 0, & x \notin A \end{cases}$$

Where A is a classical (crisp) set.

In logic, fuzzy logic is a form of many-valued logic in which the truth value of variables may be any real number between 0 and 1 both inclusive $0 \leq \mu_A(x) \leq 1$. It is employed to handle the concept of partial truth, where the truth value may range between completely true and completely false. By contrast, in crisp logic, the truth values of variables may only be the integer values 0 or 1.[13, 20]

Since computers can't process wards, fuzzy logic works as a bridge between words and numbers. It works in combination with a tool named membership functions $\mu_A(x)$ in a way that mimics Crisp logic.

membership functions $\mu_A(x)$ quantifies the degree of belongingness of the element x to the fuzzy set A. The type and the number of membership functions control the demanded degree of accuracy.

There is a significant difference between the classical crisp sets and fuzzy sets, the classical sets are considered as a subset of the fuzzy sets. The classical sets are a yes or no sets and defined as:

$$A = \{ x | \, x \varepsilon \, X \; and \;\; x \; is \;\; \Delta \}$$

Where X is the universe of discourse and x has some property Δ.

While in the fuzzy set, every element x should be combined with a membership $\mu_A(x)$, determines it's degree of membership

$$A_{fuzzy} = \{ (x, \mu_A(x) | \, x \epsilon \, X, \mu_A(x) \epsilon [0,1] \}$$

To this end, replacements for basic operators and, or must be available. There are several ways to this. A common replacement is called the Zadeh operators [16], where

the $\cap(x,y)$ and $\cup(x,y)$ in crisp logic are replaced by $min(x,y)$and $max(x,y)$ respectively.

Let A and B be two fuzzy sets in D with membership functions $\mu_A(x)$ and $\mu_B(x)$ respectively. Then the union and intersection sets operations are defined in terms of their membership functions $\mu_A(x)$as following,

Definition 1 [10]: The membership function of $A \cup B$ is denoted by $\mu_{A\cup B}(x)$, and defined pointwise for all $x \in D$ by,

$\mu_{A\cup B}(x)= \max\{\mu_A(x), \mu_B(x)\}$

Definition 2 [10]: The membership function of $A \cap B$ is denoted by $\mu_{A\cap B}(x)$, and defined pointwise for all $x \in D$ by,

$\mu_{A\cup B}(x)= \min\{\mu_A(x), \mu_B(x)\}$

Fuzzy logic is considered to be a basic control system that relies on the degrees of state of the input and the output depends on the state of the input and rate of change of this state. [5] In other words, a fuzzy logic system works on the principle of assigning a particular output (action) depending on the probability of the state of the input. An interesting property is that the behaviour of a fuzzy system is not described using algorithms and formulas, but rather as a set of rules that may be expressed in natural language.

3. Fuzzy Control System (FCS)

In 1974, fuzzy logic was applied and implemented practically for the first time by Mamdani and his colleagues [13] in steam engine control theory. fuzzy control based on fuzzy logic and fuzzy sets has been considered as one of the most active and valuable topics for research in the application of fuzzy logic and fuzzy set theory. Fuzzy control is a logical system which is much closer in spirit to human thinking and natural language than traditional logic systems.

Mamdani and his colleagues [11- 13] have developed the theory of fuzzy control systems based on the theory of fuzzy sets motivated by the pioneer research of Zadeh's [18-20] on the linguistic approach and system analysis. Mamdani's work opened the way for industrial and practical applications of fuzzy control in water quality control, automatic train operations systems, automatic transmission system, nuclear reactors, city planning and many other applications in engineering and medicine.

Fuzzy logic controllers have many advantages over the conventional controllers [7]: they are cheaper to develop, they cover a wider range of operating conditions, and they are more readily customizable in natural language terms.[13, 19]

Unlike the other logic controllers, fuzzy controller also prevents different actions for almost same behaviours. [19]

After 1990's, Fuzzy sets and logic were made more practically useful [10, 21]. Several models based on the fuzzy sets, the fuzzy transform has been proposed and widely used as a numerical tools for solving differential equations and image processing [1, 2, 3] . Fuzzy logic allows scientists to model vogue human language notions, especially, so called linguistic expressions (small, very big, more or less … etc.).

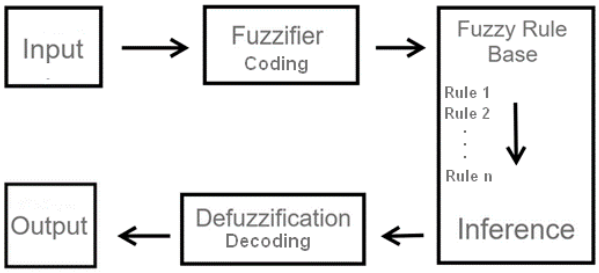

Fig. 1 Fuzzy controller system (FCS)

Numerical Steps:

Given the set of experimental data or values that satisfy a function or a mathematical equation,
Define the inputs and output variables,
Define the descriptors of the input and output variables.
Define membership functions for each of the input and output variables, such that the given data can be satisfied or nearly satisfied i.e.

$$\mu_A(x_i) = 1, i = 0,1,2, \dots, n$$

x_i are the given input data

Form the logic rules based on the mathematical problem to satisfy or nearly satisfy the given data (i.e. consistent with the interpolation condition) ,
Evaluate the Rules,
Defuzzification,
Increase the number of membership functions or their definition to get better results,
Results

4. Numerical Computations

Example One:

Given the following equally spaced crisp values $x, f(x)$ for the function $f(x) = \cos(x)$

x	-0.4	-0.2	0	0.2	0.4	0.6	0.8
$f(x)$	0.920	0.980	1.00	0.980	0.921	0.825	0.697

Table 1 Equally spaced values $x, f(x)$ for the function $\cos(x)$

MATLAB Simulink [14] features used to describe and design a fuzzy logic controller to determine the interpolating value i.e. the values of $f(x) = \cos(x), x \in [-0.4, 0.8]$. the x values are assumed as an input crisp values, and $f(x)$ as output crisp value. The x-input and y-output fuzzy descriptors are shown in Table 2, seven descriptors were used for the x values, five descriptor for the output interpolating variable. The triangular shape functions representing the input and output are shown in Fig 2 and Fig 3 respectively. The centroid defuzzification option and non-uniform triangular memberships in MATLAB [14] are used for the output. The output variable is the action that should be taken to satisfy the function *f(x)* at the input values i.e. $\mu_A(x_i) = 1, i = 0,1,2, \dots, n$.

Table 3 and Fig. 4 show the approximated y values of cos(x) obtained for a selected values x in the given domain. the graph agree well with the graph of cos(x) in the interval [-0.4, 0.8]. The output values are considered as an action that should be taken due to the given input values. The membership functions and the fuzzy rules are designed such that the approximated y values are consistent with the neighbouring given values. According to the defined membership functions and fuzzy rules, fig 4 show the MATLAB computations for an input x= 0 .42, the output result is y=0.905, where the true value is 0.913, the error is 0.008.

The input fuzzy descriptors for x	**The output fuzzy descriptors y**
S1X: Extremely small x values, S2X: Very small x values, SX: Small x values, M1X: Medium small x values, M2X: Medium x values, LX: Large x values, VLX: Very large x values.	VSY: Very small interpolation value, SY: Small interpolation value, SMY: Small Medium interpolation value, MY: Medium interpolation value , LY: Large interpolation value,

Table 2 The *x*-input and *y*-output fuzzy descriptors

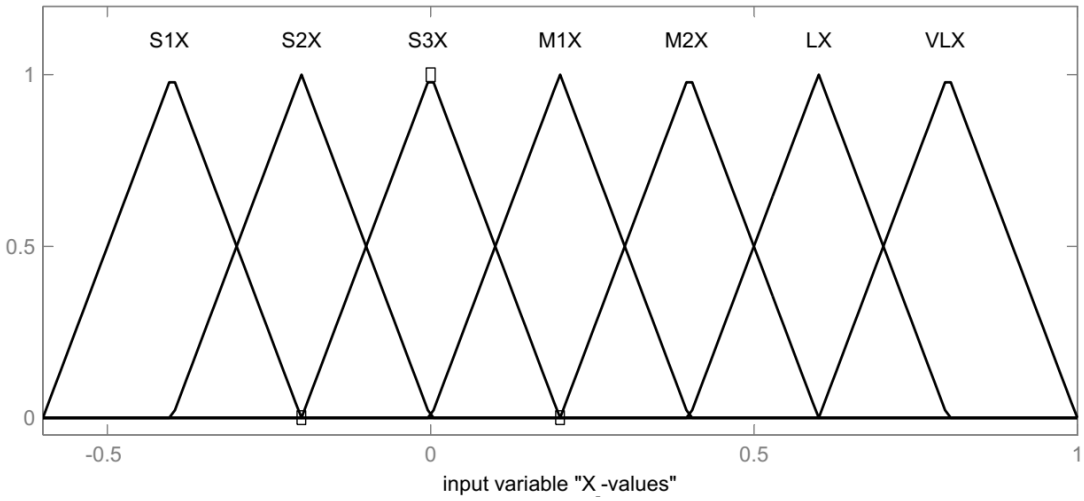

Fig. 2 The uniform triangular membership functions used to define the input.

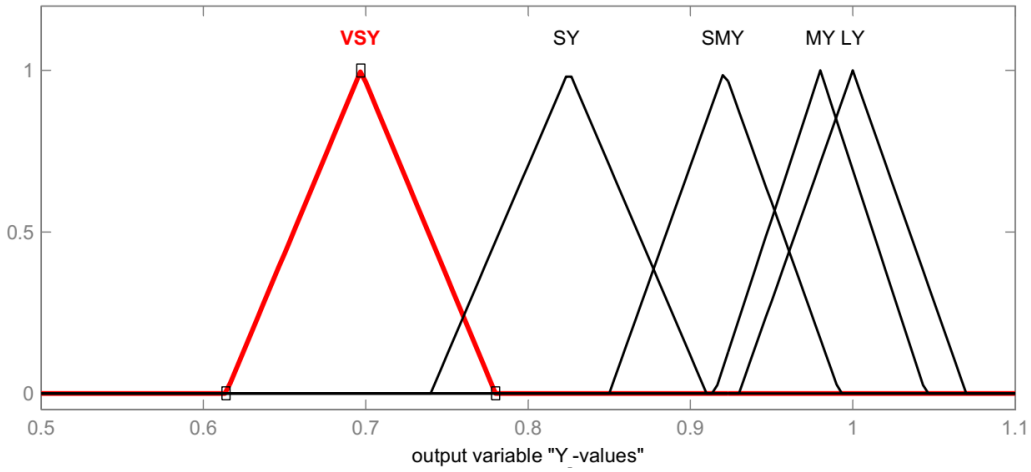

Fig. 3 The non-uniform triangular membership functions used to define the output.

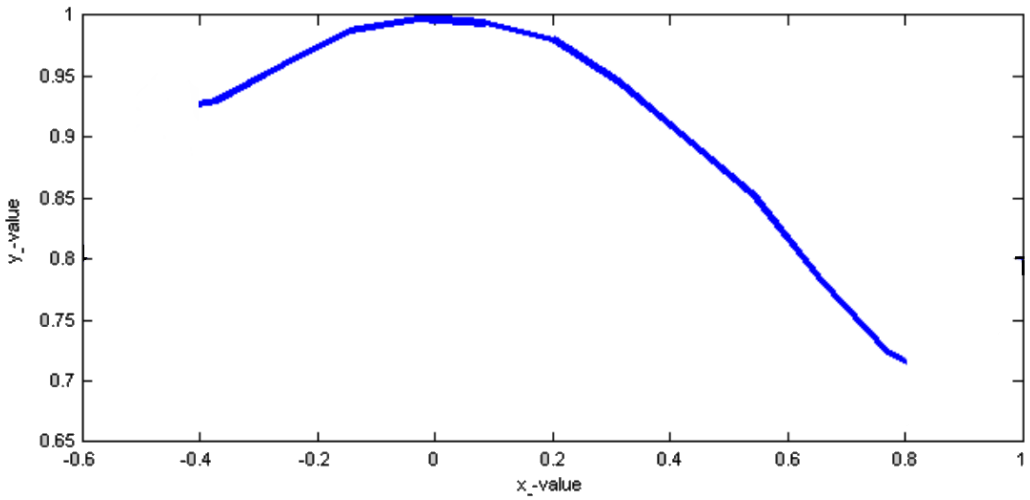

Fig. 4 A two dimensional surface figure for the input and output parameters.

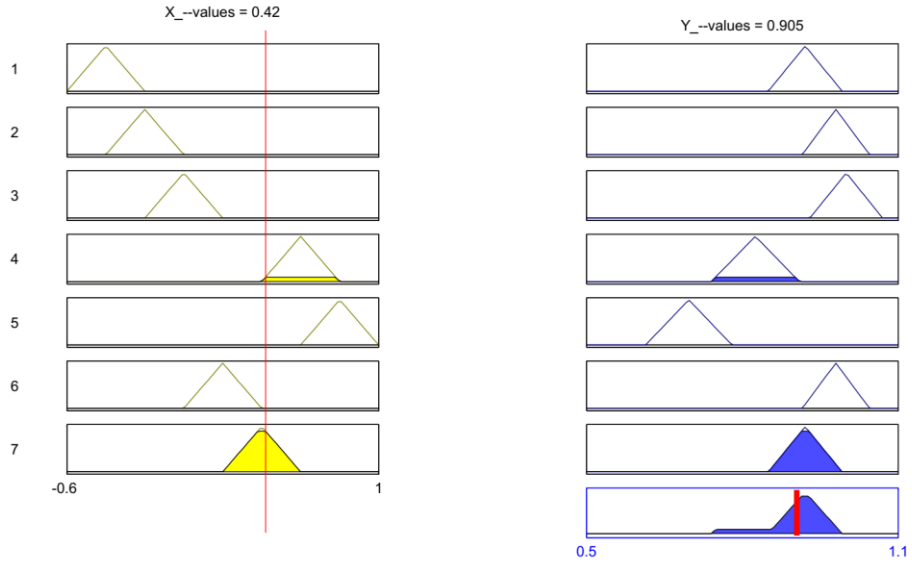

Fig. 5 MATLAB Rule figure for the interpolation at x=0.42

x	Interpolation value	Abs. actual error	x	Interpolation value	Abs. actual error
-0.35	0.936	0.003	0.39	0.924	0.001
-0.3	0.948	0.007	0.42	0.905	0.008
-0.25	0.961	0.008	0.57	0.841	0.001
-0.1	0.992	0.003	0.62	0.806	0.008
-0.15	0.986	0.003	0.69	0.766	0.005
0.1	0.992	0.003	0.7	0.762	0.003
0.15	0.986	0.003	0.73	0.748	0.003
0.25	0.961	0.008	0.75	0.738	0.006
0.3	0.948	0.007	0.77	0.725	0.007
0.35	0.936	0.003	0.79	0.708	0.004

Table 3 The interpolation crisp output, the inputs x and the absolute actual error.

s	x	f	Δf	$\Delta^2 f$	$\Delta^3 f$	$\Delta^4 f$
-1	-0.4	0.921	0.059	-0.039	-0.001	0.002
0	-0.2	0.980	0.020	-0.04	0.001	0.001
1	0.0	1.000	-0.02	-0.039	0.002	0.003
2	0.2	0.980	-0.059	-0.037	0.005	
3	0.4	0.921	-0.096	-0.032		
4	0.6	0.825	-0.128			
	0.8	0.697				

Table 4 Difference table for $f(x) = \cos(x)$

In order to calculate interpolating value $f(0.15)$ from the data in table 4, we use third degree Newton-Gregory polynomial [6, 9]

$$p_n(x_s) = f_\circ + s\Delta f_\circ + \frac{s(s-1)}{2!}\Delta^2 f + \frac{s(s-1)(s-2)}{3!}\Delta^3 f_\circ \ldots \frac{s(s-1)\ldots(s-n+1)}{n!}\Delta^n f_\circ,$$

Where $s = \frac{x-x_\circ}{h}$, with $h = \Delta x$, the uniform spacing in x-values.

In order to centre the x-value around x=0.15, we must use the four entries beginning with x = - 0.2. That makes $x_\circ$= - 0.2 and $s = \frac{0.15+0.2}{0.2} = 1.75$. Inserting the proper values into the expression for Newton-Gregory polynomial, we get

$$f(0.15) = 0.980 + \frac{(1.75)(0.75)}{2!}(0.020) + \frac{(1.75)(0.75)(-0.25)}{3!}(-0.04)$$

$$= 0.980 + 0.0131 + 0.002 = 0.995$$

The function is actually for $f(x) = \cos(x)$, so we know that the true value of $f(0.15) = 0.989$; the absolute error is 0.006. Fuzzy logic computation results indicates that the error at x = 0.15 is 0.003, which indicates that our estimate is very good compared to the true value and even for Newton-Gregory polynomial approximation.

Example Two

The data given in Table 4 is the numerical solution for the steady-state equation-flow equation, u(x, y) are the temperatures at the nodes of a gridwork constructed in the domain of interest [9]. As in example one, we used MATLAB [14] Simulink features to describe and design the interpolation with fuzzy controller method to calculate the

temperatures at points other than the nodes of the grid. The *x*-values are assumed as the first input crisp values, the y values are assumed as the second input crisp values and the temperature will be considered as the output values

Six descriptors were used for the first input x values, Five descriptors were used for the second input y-values, and 10 descriptors used for the output interpolating Temperature variable. The input and output membership functions are chosen such that the given values are satisfied at most given nodes and very close to the others. The centroid defuzzification option and non-uniform triangular memberships in MATLAB [14] Simulink features are used for the output. The *x*-input and *y*-output fuzzy descriptors are indicated in table 6, and the fuzzy rules are in table 7.

X / y	**0.0**	**0.5**	**1.0**	**1.5**	**2.0**	**2.5**
0.0	0.0	5.00	10.00	15.00	20.00	25.00
0.5	5.00	7.51	10.05	12.00	15.67	20.00
1.0	10.00	10.00	10.00	10.00	10.00	10.00
1.5	15.00	12.51	9.95	7.32	4.33	0.00
2.0	20.00	15.00	10.00	5.00	0.00	-5.00

Table 5 Temperatures at the nodes of a gridwork constructed in the domain of interest for the steady-state equation.

The first input fuzzy descriptor x	**The second input fuzzy descriptor y**	**The output (Temperature) fuzzy descriptor**
VSX: Very small x values, SX: Small x values, MSX: Medium small x values, MX: Medium x values, LX: Large x values, VLX: Very large x values.	VSY: Very small y values, SY: Small y values, MY: Medium y values, LY: Large y values, VLY: Very large y values.	VVST: Extremely small T values, VST: Very small T values, S1T: Small 1 T values, ST: Small T values, M1T: Medium1 T values, M2T: medium 2 T values. M3T: Medium 3 T values, L1T: Large 1 T values, L2T: Large 2 T values, VLT: Very large T values

Table 6 The two inputs and output fuzzy descriptors

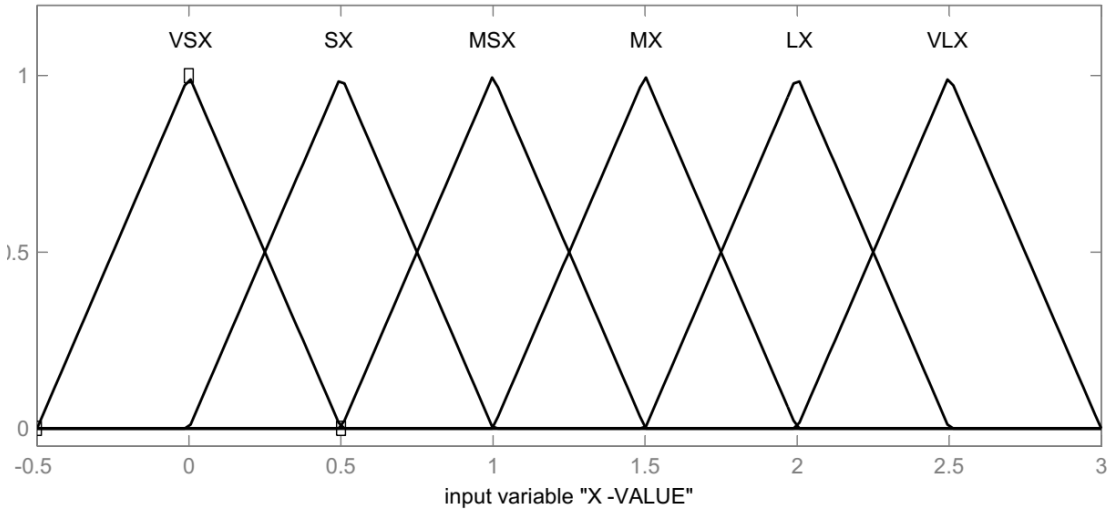

Fig. 6 The uniform triangular membership functions used to define the first input.

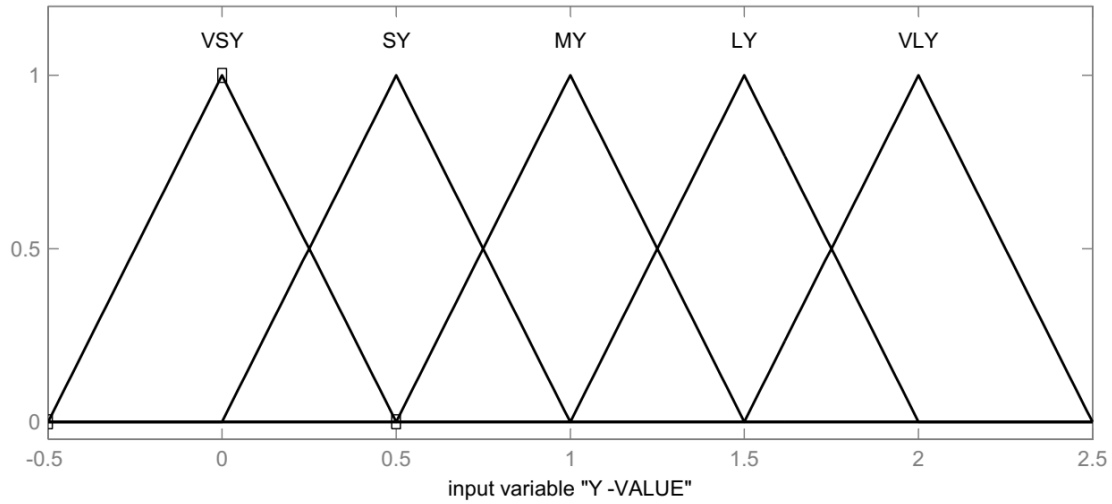

Fig. 7 The uniform triangular membership functions used to define the second input.

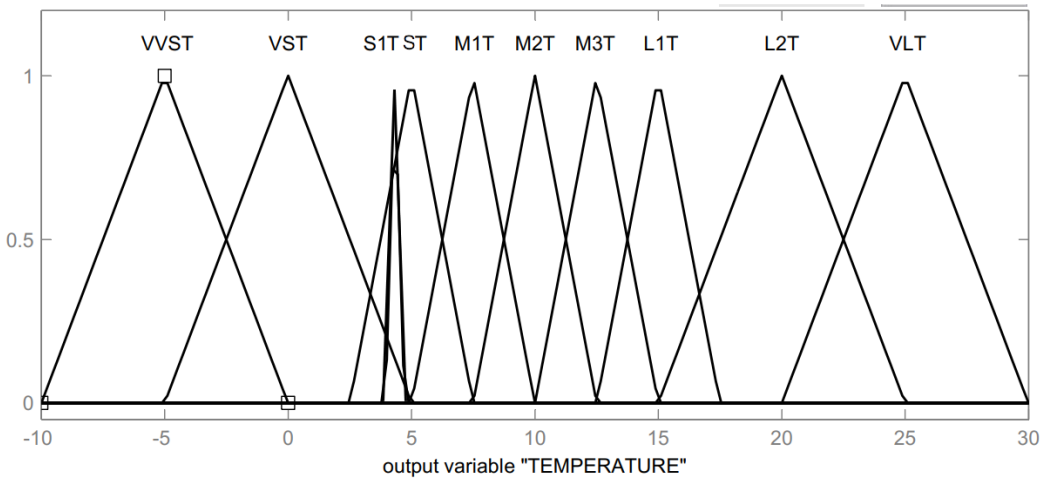

Fig. 8 The non-uniform triangular membership functions used to define the output.

x/y	VSX	SX	MSX	MX	LX	VLX
VSY	VST	ST	M2T	L1T	L2T	VLT
SY	ST	M1T	M2T	M3T	L1T	L2T
MY	M2T	M2T	M2T	M2T	M2T	M2T
LY	L1T	M3T	M2T	M1T	S1T	VST
VLY	L2T	L1T	M2T	ST	VST	VVST

Table 7 The applied fuzzy rules

x/y	0.3	0.8	1.3	1.6	1.9	2.1	2.4
0.4	4.10	8.26	12.1	15.7	16.6	19.2	23.8
0.9	8.47	9.33	10.7	11.4	11.4	14.3	14.3
1.2	12.2	11.1	8.92	8.75	8.94	5.34	3.96
1.6	15.9	11.6	7.92	4.29	2.74	-1.87	-1.17
1.8	17.1	12.2	7.19	3.83	1.45	-1.21	-2.86

Table 8 The x, y inputs and the interpolation crisp output (Temperature).

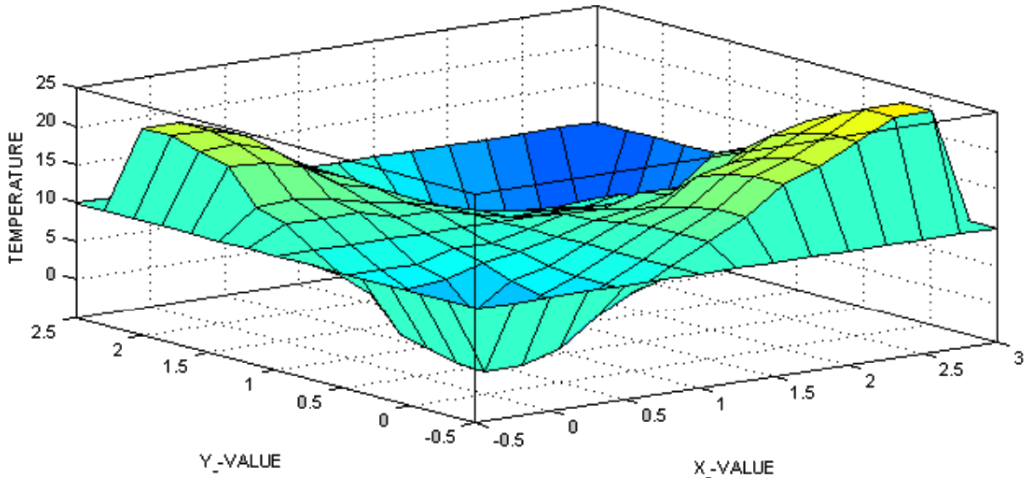

Fig 9 A surface graph for the x, y inputs and temperature output.

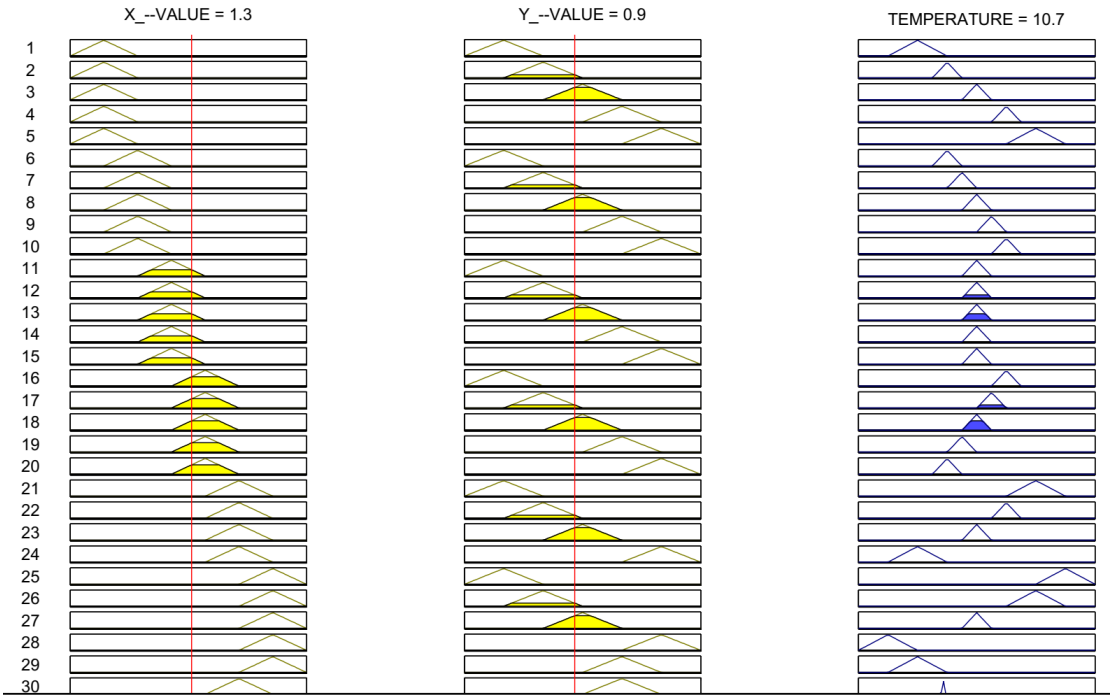

Fig 10 MATLAB rules figure for the output interpolation at x = 1.3, and y = 0.9

Fig. 9 is a three dimensional graph for *x*, *y* inputs and the approximated temperatures obtained at selected nodes in the given domain. Table 8 contains the computed result at random nodes. The membership functions and the fuzzy rules are designed such that the obtained temperature values are consistent with the neighbouring given values. Fig 10 is an example showing the MATLAB computation at the node (1.3, 0.9) according to the given membership functions and fuzzy rules.

Conclusions

Fuzzy logic is considered to be an elite approximation tool, it provides an effective means of capturing the approximate, inexact nature of the real word. It's a scientific revolution that made our linguistic wards accessible and processed easily by computers In this paper, we presented a method which used a given experimental or mathematical data, fuzzy logic, uniform and non-uniform triangular membership functions to approximate and interpolate unknown values. The proposed interpolation method is proved to be applicable and flexible rather than the other existing methods which depends mainly on numerical approximation algorithms (i.e. polynomials or splines). The presented method can introduce a new and fundamental change in dealing with mathematical fuzzy terms and vagueness phenomenon (i.e. close, very close, large, small, very small,….etc.). It provided us with a tool to avoid different actions or values for almost same input values (same behaviour). The method can be easily used and implemented for different mathematical and physical data. Results show the efficiency of fuzzy logic and fuzzy controller system (FCS) in approximating values satisfying mathematical relations. It's shown that obtained approximated values in the domain of the given values are very close to the exact values, and by using proper number and suitable memberships, the error could be even less than the error from known traditional

methods. Choosing and designing suitable membership functions and rules plays a crucial role, we recommend more study and on that side.

CONFLICT OF INTERESTS

The author declare that there is no conflict of interests.

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UNIVARIATE TIME SERIES ANALYSIS OF CONSUMER PRICE INDEX ON FOOD AND NON-ALCOHOLIC BEVERAGES

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Abstract

The paper examined univariate time series forecast of consumer price index on the consumption of food and nonalcoholic beverages in Nigeria. It filled the knowledge gap by explicitly modeling and forecasting consumer price index in Nigeria using the univariate ARIMA model. The work was restricted to Nigerian Consumer Price Index. It was also restricted to food consumption (FC) data and food & nonalcoholic beverages consumption (FNBC) data from 1995-2021. This paper analyses were carried out using gretl 2019c, Minitab 16 and Micro software Excel (2010). The monthly and yearly means plots were done, so as to have a better understanding of the series behaviours. The series plots points to the fact that there is possibility that the time series are integrated of order 1 for food consumption series with no seasonality, while integrated of order 1 for food & nonalcoholic beverages consumption series with seasonality of order 12. Stationarity after second difference of the first differencing was obtained for both series. A suitable ARIMA Model was obtained for both series and was used for models forecast computation. Hence, the computed result suggested that ARIMA(0,1,1) and ARIMA(0,1,1)$(0,0,0)_{12}$ model were the best model for estimating and forecasting the two time series, using model selection criteria and accuracy measures. The plots of the forecasts generated for the FC and FNBC shows that the two variables are dependent and also shows that any gradual increase in the food consumption tends to pave way for increase in the food & nonalcoholic beverages consumption or a drastic drop in the food consumption will also drop the food & nonalcoholic beverages consumption in the same manner. It seems reasonable to conclude that there is significant relationship between the food consumption and food & nonalcoholic beverages consumption series. It was recommended that more detailed work should be carried out in the area of co-integration analysis of the two variables to enhance a better understanding and prediction distribution in Nigeria.

Keywords*:* *Consumer Price Index; Time Series Analysis; Non-alcoholic Beverages; Food Consumption; ARIMA Model*

1. Introduction

The environment of any organization is the aggregate of all conditions, events, and influences that surround and affect organizations (Levy, 1992). It is an aspiring sesame that economic environment in Nigeria is an embodiment of macro-economic variables associated with the

factors of production of goods and services. The environment is dynamic and constantly changing. This then creates opportunities and threats for food and beverage sub sector of Nigeria. Prominent stakeholders and players in the Nigerian Food and Beverage subsector include Nestle Food Nigeria Plc, Cadbury Nigeria Plc, Nigeria Bottling company Plc, and Dangote flourmill Plc to mention but a few. An analysis of inflation in a country needs to be carried out to prevent and control inflation, to determine the causes of inflation, and as a basis for economic policy making to prevent an economic crisis, to maintain economic stability and to ensure the welfare of society (Siswanah, 2021).

Decision makers must make forecasts to help in decision making. To conduct these forecasts, most central banks take a number of variables into account. However, it is not an easy task, especially in developing countries, where economic processes are highly unstable and volatile. Moreover, the macroeconomic data on developing countries can be unreliable due to many reasons: measurement error, imperfect methods of measuring, etc. Nevertheless, there exist a number of empirical studies on inflation factors in developing countries. These studies show that inflation is a country-specific phenomenon, and its determinants differ across countries (Adams, 2014). Therefore, an effective monetary policy depends largely on the ability of economists to develop a reliable model that could help understand the on-going economic processes and predict future developments. In this regard, this study is important since it is aimed at forecasting Consumer Price Index (CPI), which is a component of inflation in the Nigeria economy. Consumer price index (CPI) is a measure that examines the weighted average of price of a basket of consumer goods and services, such as transportation, food and medical care; it is one of the most frequently used statistics for identifying period of inflation or deflation (Adams, 2014). Consumption of non-alcoholic beverages (NABs) such as Juice and carbonated drinks has been a basic form of refreshment among Nigerians of all ages, tribes and socioeconomic backgrounds (Phillip, 2013).

Forecasting is a global important part of econometric analysis, the most important probably for most people. How do we forecast economic variables, such as Food and non-alcoholic beverages, GDP, inflation, exchange rates, stock process, unemployment rates and myriad? Other economic variables problems involved in forecasting prices of financial assets, such as food and food & non-alcoholic beverages are of great concern. These asset prices are characterized by the phenomenon known as vitality clustery (Adams, 2014). It is no longer news that the global economic crisis has brought about shortage of financial resources and to a general down tone in consumption price index of food and food & non-alcoholic beverages across the globe. Therefore, forecasting consumption prices (food and food & non-alcoholic beverages) will help provide a way to expect and maybe avoid the risk of large change in prices.

Time series analysis is a statistical technique that deals with time series data, or trend analysis. Time series data means that data is in a series of particular time periods or intervals. The data is considered in three types: (1) Time series data: A set of observations on the values that a variable takes at different times. (2) Cross-sectional data: Data of one or more variables, collected at the same point in time. (3) Pooled data: A combination of time series data and cross-sectional data. (4) Exponential smoothing in time series analysis: This method predicts the one next period value based on the past and current value. It involves averaging of data such that the nonsystematic components of each individual case or observation cancel out each other. The exponential smoothing method is used to predict the short term predication. Alpha, Gamma, Phi, and Delta are the parameters that estimate the effect of the

time series data. Alpha is used when seasonality is not present in data. Gamma is used when a series has a trend in data (Durbin, 2012). Delta is used when seasonality cycles are present in data. A model is applied according to the pattern of the data.

ARIMA modeling has been successfully used in various food and nonalcoholic beverage-market activities. With the growing economy, people need more funds to meet up the rapid expansion. At the same time people source for money in order to meet up with the numerous challenges. As such the food and nonalcoholic beverage market serves as an important tool in mobilizing and allocating savings among users who are critical to the growth and efficiency of the growing economy. This has led researchers to explore efficient ways of predicting food and nonalcoholic beverage market activities to enhance the benefit derived from them. No method has been discovered to accurately predict price movement of food and nonalcoholic beverage after numerous attempts by researchers.

As a result of insecurity and risk in some parts of Nigeria, manufacturing sector including food and beverage manufacturing sub-sector have lost substantial portion of their sales as it becomes problematic to penetrate some parts of the country. In view of these environmental challenges of Boko Haram vis-a-vis insurgency in the country, most Food and Beverage companies have got to relocate. This leads to decrease in Gross Domestic Product (GDP), promotes imports and demotes export among others. As earlier stated above, poor infrastructure associated with unstable power supply affect Food and Beverage subsector. This results in high cost of operation and low profit margin in the sub-sector.

In addition, consumer price index on the consumption of food and nonalcoholic beverage has become one of the well-known investments in the recent past due to its higher returns. It has become a great part of the global economy in the food and nonalcoholic beverage market influences both personal and corporate lives and economic life of a country. The Nigerian consumer price index forecasting is known more by its failure than success since its prices reveals the judgment and what investors expects base on the available information.

Base on this, the accuracy in forecasting the consumer price index on the consumption of food and nonalcoholic beverage or predicting the trend accurately is of importance of for anyone who wishes to invest in the dynamic global economy. Over the years, economists and financial analysts have constantly maintained that a market price that is not regulated is the best and stick to prove the true scarcity of a commodity or its worth. It is easy for one to evaluate the consumption of food and nonalcoholic beverage (CF and NABC) performance by the use of consumer price index or returns. You can predict consumer price returns from a variety of financial and macroeconomic variables which has been any attraction for equity investors. Of recent, attention has increased on the shift to the consumer price index as a way of measuring a sector of the consumption of food and nonalcoholic beverage market. The investing public has to a large extent an important indicator used by a bench mark by which investor or fund management compares the returns of his portfolio (Senol, 2012).

Thus, the need to predict the consumer price in order to meet the basic objectives of operators and investors of the consumer market for gaining more benefits cannot be overemphasized. This issue has brought to focus the attentions of statisticians and researchers all over the world. Consumer market is affected by numerous factors and this has created high controversy in the field. Many methods and approaches for models are present in the interaction. This study

exclusively deals with the time series forecasting model and in particular the Autoregressive Integrated Moving Average (ARIMA) models which were described by Box-Jenkins.

This paper fills the lacuna by explicitly modeling and forecasting the consumer price index of food and nonalcoholic beverage consumption using the ARIMA model. It contributes to knowledge of forecasting consumer returns and expands forecasting literature in Nigeria. It may also spur further studies aimed at either sustaining or debunking its forecast model. The particular objectives are:

1. Describe the series plots, yearly mean plots, and monthly mean plots and obtain the stationarity of the series.
2. Determine the year with the highest Food Consumption (FC) and Food & Non-alcoholic Beverage Consumption (FNBC) rate.
3. Obtain a suitable model to fit the consumer price index of food and nonalcoholic beverage (Food Consumption and Non-alcoholic Beverage consumption) using the model selection criteria (AIC and BIC).
4. Estimate the forecasts from the obtained models considered (ARIMA model) covering the period of 2022-2023 using accuracy measures of forecast values.

1.2 Statement of the Problem

It is obvious that the global economic crises have led to a general down turn in consumer prices and shortage of financial resources across the globe. Most of the world's reports on consumer markets highlight substantial decline in consumer price.

As a result of insecurity and risk in some parts of Nigeria, manufacturing sector including food and non-alcoholic beverage manufacturing sub-sector have lost substantial portion of their sales as it becomes problematic to penetrate some parts of the country. In view of these environmental challenges of BokoHaram vis-a-vis insurgency in the country, most Food and non-alcohol beverage companies have got to relocate. This leads to decrease in Gross Domestic Product (GDP), promotes imports and demotes export among others. As earlier stated above, poor infrastructure associated with unstable power supply affect Food and Beverage subsector. This results in high cost of operation and low profit margin in the sub-sector.

The purpose of this study is to forecast and model the Consumer Price Index (CPI) on the consumption rate of food and consumption rate of food & non-alcoholic beverage using ARIMA time series. Forecasts of CPI of consumption food and non-alcoholic beverages are important because they affect many economic decisions. Without knowing future CPI rates, it would be difficult for lenders to price loans, which would limit credit and investments in turn have a negative impact on the economy. Investors need good CPI forecasts, since the returns to stocks and bonds depend on what happens to CPI. Businesses need CPI forecasts to price their goods and plan production. Homeowners' decisions about refinancing food and non-alcoholic beverage also depend on what they think will happen to CPI. Modeling CPI on the consumption rate of food and non-alcoholic beverage is important from the point of view of poverty alleviation and social justice.

It is also evident that consumer price index of food and nonalcoholic beverage market has been one of the most popular investments in Nigeria and the globe at large due to its high returns. Also, fluctuation in this market influences personal and cooperate financial hires and the

economic health of a country**.** Accurately forecasting the consumer price index of food and nonalcoholic beverage consumption is of crucial importance for any future investor. Thus, there is need to predict the consumer price index to meet the fundamental objectives of investors and operators of the food and nonalcoholic beverage market. This has attracted the attention of Researchers so as to fill in this gap.

1.3 Significance of the Research

The study is significant to policy makers as it guides them in formulating macroeconomic policies by providing them with a long term perspective of food consumption and non-alcoholic beverage. Optimal policy will depend on optimal food and non-alcoholic beverage consumption forecasts. CPI uses include;

- As the main estimator of the rate of inflation. The percentage change of the CPI over a one-year period is what is usually referred to as the rate of inflation.
- A macroeconomic indicator. The CPI is used for general economic/social analysis and policy formulation particularly since it conveys important information about indirect tax revenue.
- As a tool in wage negotiation and indexation. CPI is used to adjust taxes and to determine, among other things, wage levels in the event of trade disputes, social security benefits, public service remuneration and pensions.
- As a deflator of expenditure. The prevailing CPI can be used to establish the real/constant value by deflating nominal values (previous cost) of goods and services.

Forecasting inflation in the food and non-alcoholic beverage industries generally improves financial planning in both the corporate and private sectors. Inflation affects actual cost of expenses and stock valuations on the corporate level. Forecasting changes can therefore help investors understand risks and hedge investments.

1.4 Scope of the Research

This paper focused on univariate time series forecast of consumer price index of food and non-alcoholic beverage consumption rate. It uses the components of food consumption rate of the consumer price index (CPI) and the food & non-alcoholic beverage consumption rate which will comprise the monthly value of the both. It fills this lacuna by explicitly modeling and forecasting consumer price index returns in Nigeria using the bivariate ARIMA model. The box Jenkins approach of model of identification, parameter estimation and diagnostic checking will be adopted in the analyses.

2. Literature Review

Many authors among who are: Lirby (2007), Malkeil (2013), and Durbin (2012) have compared, estimated, and forecasted for the future consumptions rate and commodities dealing with auto-correlation.

Abdullahi and Yakubu (2013) conducted a study on ‘Determinants of Non Alcoholic Beverages (NAB) Consumption in North-Western Nigeria: A study of Sokoto Metropolis’. This study seeks to determine the role played by socioeconomic characteristics in driving consumption of some selected NABs particularly juice and carbonated soda in Sokoto metropolis of Nigeria. To adequately capture the variables of interest, specifically, location and ethnicity variables, the metropolis was grouped into four clusters, namely Peri-urban, Sokoto

main, G R As, and Resident community. In each of the clusters one hundred households were randomly selected to arrive at a sample size of four hundred households. Descriptive, OLSMR and probit analytical tools were used to analyse the data collected. The results shows that about 59% and 71% of the respondents consumed juice and soda respectively while the per capita consumption of juice was slightly higher than that of soda which were 7.57 and 7.32 litres respectively.

Partick et al. (2016) studied economic environment and performance of food and beverage sub-sector of a developing economy: Nigeria. This study examines the implications of economic environment on the performance of food and beverage sub-sector of Nigeria. The economic environment is an embodiment of dynamic variables characterized by significant challenges impacting on the food and beverage sub-sector. Performance in this sector is measured in terms of profitability, exchange rate, interest rate, current asset, turnover, market share and return on investment among others. This study therefore serves as report of investigation into the implications of these variables on the performance of food and beverage sub sector. The ordinary least square technique is adopted in the methodology and the result reveals a significant relationship between economic environmental variables and the food and beverage sub-sector. The study advocates a strong public private partnership between governments and the sector as well as encouragement of stable exchange rate so as to foster economic growth.

Phillip et al. (2013) studied the demand for non-alcoholic beverages among urban households in South-West, Nigeria. This study examined the roles of income, prices and household demography in household demand for non-alcoholic beverages (NABs) in two cities – Abeokuta and Ibadan in Southwest Nigeria. The study was based on primary data obtained from a cross-section of 407 households (211 from Abeokuta and 198 from Ibadan) drawn by multistage sampling technique across six Local Government Areas (LGAs) and 60 National Population Commission (NPC) enumeration areas (EAs). A structured questionnaire was used to collect data on households NABs expenditure, income, prices and other relevant socio-economic variables. The data were analysed within the framework of a linear approximation of an Almost Ideal Demand System. The study found that an average household, consisting of five (5) members, expended an average N5, 235.89 per month on NABs (approximately US$34.21 at N153.06/US$1 exchange rate in 2010). The bulk (67%) of the NABs expenditures were devoted to purchase of dairy products (36%) and cocoa-based products used in preparing chocolate drinks (31%). The estimated income elasticity of demand for the six categories of NABs studied were positive while all the own price elasticity of demand were negative. Demand for dairy products and cocoa based drinks were found to be price elastic, while those of carbonated drinks, malt drinks, fruit juice and other NABs were price inelastic. Increase in education of the household heads was found to be associated with significant increase in the budget shares of dairy products ($p<0.01$) and fruit juice ($p<0.10$), but a significant reduction in budget shares of cocoa-based products ($p<0.05$), carbonated drinks ($p<0.01$) and malt drinks ($p<0.05$). The study concludes that policies aimed at promoting increased demand and healthy choices of NABS must pay some attention to raising real income and increasing level of education among the citizenry. Profitability of business enterprises involved in dairies and cocoa-based products would also be better enhanced if the firms adopt cost saving strategies as against price hikes in a bid to enhance performance.

Nkpordee and Nduka (2018) conducted a research on application of time series analysis on the forecasting of the outbreak of malaria epidemic in Nigeria. The secondary data used for the

study was obtained from National Bureau of Statistics (nbs), Social Statistics in Nigeria. The research work employed Box-Jenkins (1976) methodology to construct a suitable mathematical model by putting the ACF and PACF correlogram into consideration. The model; ARIMA(1,0,1) was used to forecast the monthly reported cases of malaria with a 16 months lead, which shows a gradual increase and decrease in the series. The research also evaluated the monthly mean of the reported cases of malaria. It was recommended that Government should ensure the provision of treated bed mosquito nets, insecticides, anti-malaria drugs etc. in the rural areas in Nigeria.

Al-Shiab (2006) examined the univariate ARIMA forecasting model on the Amman Stock Exchange (ASE) general daily index between 4/1/2004 and 10/8/2004; with out-of-sample testing undertaken on the following seven days. Four diagnostic tests were performed to select the best model describing the data, namely: R-square, adjusted R-square, Akaike's Information Criterion (AIC), and Schwarz Information Criterion (SIC). On the basis of these four diagnostic tests, ARIMA (*4, 1, 5*) model was chosen as the best model that explains the data and is suitable for accurate forecasting. The selected model predicted that the ASE would continue to grow by 0.195% for seven days starting on 11/8/2004. This forecast, however, was not consistent with actual performance during the period of the prediction since the ASE declined by 0.003%. He concluded that the forecast error implies the ASE tends towards weak form efficiency.

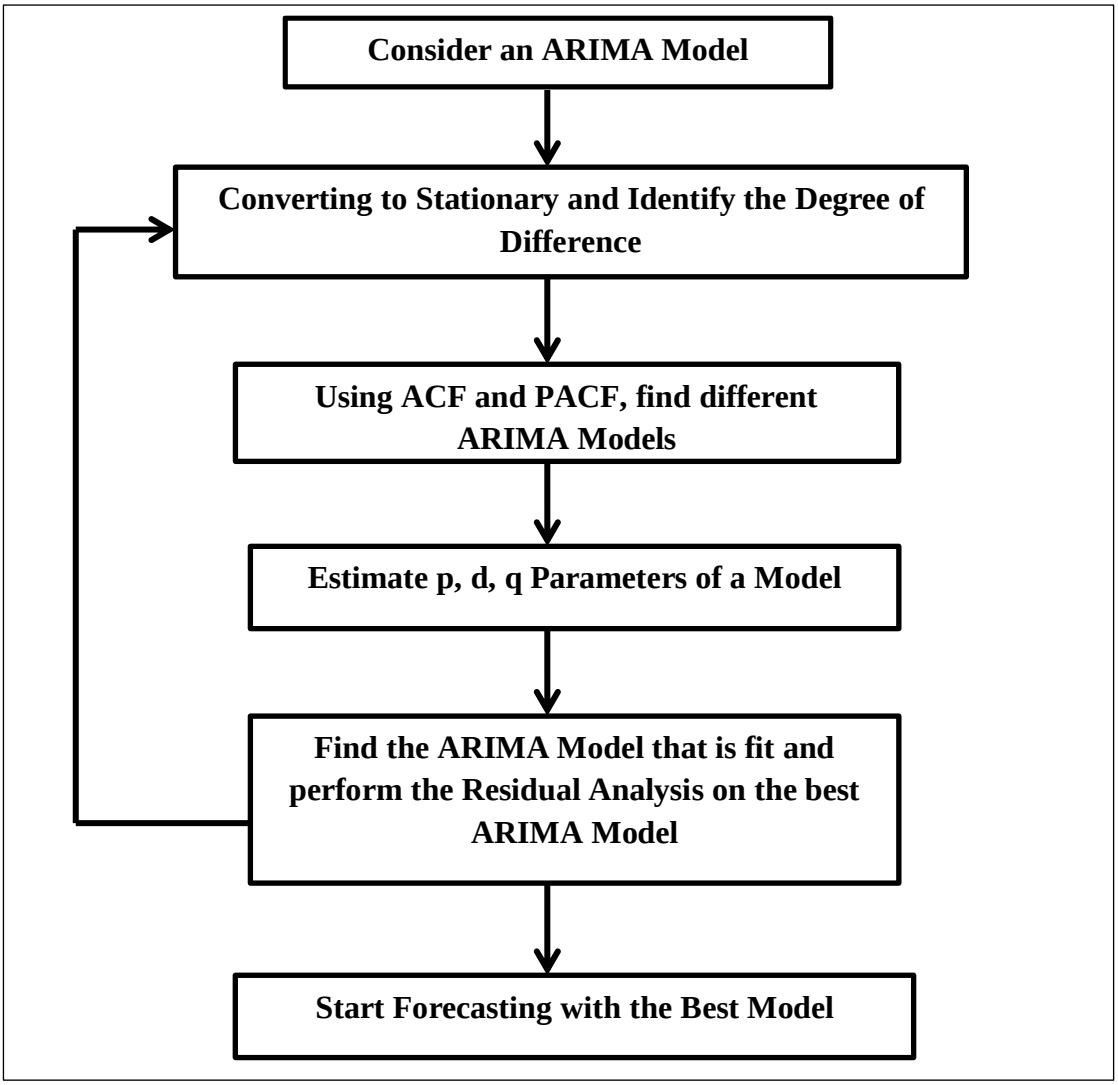
Stanley, Biu and Enegesele (2020) conducted a study on comparison of univariate and bivariate time series forecasts of Nigerian stock exchange variables. The study examined univariate and bivariate time series forecast of Nigerian stock exchange variables: All Share Index (ASI) and the External Reserves (ER) which comprise of monthly value from 1985 to 2018 of them both. It filled the lacuna by explicitly modeling and forecasting stock returns in Nigeria using the univariate ARIMA and bivariate VAR models. The monthly and yearly means plots were done, to have a better understanding of the series behaviours. The order of the regular autoregressive and moving avenge model that is necessary to adequately represent the time series model was determined. The series plots showed that ASI series is integrated of order 1 without seasonality while ER series is integrated of order 1 with the seasonality of order 12. A suitable ARIMA and VAR Model were obtained for both series using model selection criteria (MSC) and the models were used to generate forecasts. The univariate and bivariate model forecasts were compared and the result shows that the bivariate model is better to predict the two series than the univariate model from the result of forecast accuracy measures (i.e. MAPE and MSE).

3. Materials and Methods

3.1 Research Design: The study seeks to examine univariate time series forecast of consumer price index of food and non-alcoholic beverage consumption rate, using the bivariate ARIMA model. The Box-Jenkins approach of model identification, parameter estimation, and diagnostic checking will be adopted in the analyses. This study is restricted to the consumer price index of Nigeria. It is also restricted to food consumption (FC) and non-alcoholic beverage consumption (FNBC) rate data. The secondary data used for the study were collected from the National Bureau of Statistics (NBS) Statistical Bulletin. It is the monthly data of food consumption (FC) and non-alcoholic beverage consumption (FNBC) on the consumer price index ranging from 1995 to 2021.

3.2 Nature and Source of Study Data: The accumulated and used data for this study is a secondary statistics extracted from survey information performed in Nigeria on the consumer price index (specifically on the components of food consumption rate and the food & non-alcoholic beverage consumption rate) from the National Bureau of Statistics from January 1995 to December 2021. Implicit composite consumer price index table of the rebased GDP figures (Base Period: November 2009 = 100) evaluating several interest sectors of the Nigeria economy data was used. The data are presented in Appendix A and B. One impartial bivariate model has been proposed to predict the trend in both food consumption rate and food & non-alcoholic beverage. The following programmes are used to acquire the parameters which constitute the model; a number of which include Gretl 2019c and MINITAB 16. To facilitate records evaluation, the researcher made use of Microsoft Excel 2010, Gretl 2019c and MINITAB 16. Gretl 2019c and MINITAB 16 had been utilized in estimating the parameters for the ARIMA model.

3.4 Flowchart of Method



3.5 Model Specification

Autoregressive Model AR (p)

An autoregressive process is denoted by AR (P) and this can be denoted by

$$X_t = \sum_{k=1}^{p} \phi k(B^{k}) X_t + \varepsilon_t \quad (3.1)$$

where

$$\phi(B) = 1 - \sum_{k=1}^{p} \phi \varepsilon_t B^k = 1 - (\phi_1 B + \phi_2 B^2 + ... + \phi_k B^k)$$

but since $\phi(B) X_t = \varepsilon_t$ for an AR(p) process we have $(1 - \phi_1 B - \phi_2 B^2 - ... - \phi_k B^k) X_t = \varepsilon_t$

$$\Rightarrow X_t - \phi_1 X_{t-1} - \phi_2 X_{t-2} - ... - \phi_k X_{t-k} = \varepsilon_t$$

$$X_t = \phi_1 X_{t-1} + \phi_2 X_{t-2} + ... + \phi_k X_{t-k} + \varepsilon_t \quad (3.2)$$

$\phi_1, \phi_2, ..., \phi_k$ are constants and ε_t is a sequence of independent (uncorrelated) random variables with mean 0 and variance σ^2 such sequence of random variables is called the White noise $\varepsilon_t \approx N(0, \sigma^2$

Moving Average Model MA(q)

The moving average of order q is denoted by

$$X_t = \sum_{k=1}^{p} \theta_k (B^k) \varepsilon_t \quad (3.3)$$

$$\theta(B) = 1 + \sum_{k=1}^{p} \theta_k B^k = 1 + \theta_1 B + \theta_2 B^2 + ... + \theta_k B^k$$

But since $X_t = \theta(B) \varepsilon_t$ for an MA process, then we have

$$X_t = (1 + \theta_1 B + \theta_2 B^2 + ... + \theta_q B^q) \varepsilon_t$$

$$= \varepsilon_t + \theta_1 \varepsilon_{t-1} + \theta_2 \varepsilon_{t-2} + ... + \theta_q \varepsilon_{t-q} \quad (3.4)$$

Where $\theta_1, \theta_2, ..., \theta_n$ are the fixed constants with $\theta_0 = 1$ and ε_t is a sequence of independent (uncorrelated) random variables with mean 0 and variance σ^2 such sequence of random variables is called the White noise $\varepsilon_t \approx N(0, \sigma^2)$.

Autoregressive Moving Average (ARMA) Model

Box and Jenkins (1976) noted that the mixed autoregressive moving average model is the combination of *AR (p) and MA (q).* Let's say that X_t is the deviation from the mean μ, and then ARMA (p, q) model can be written as

$$y = \phi_1 x_{t-1} + \phi_p x_{t-p} + \theta_1 \varepsilon_{t-1} + ... + \theta_q \varepsilon_{t-q} . \quad (3.5)$$

Using the backward shift operator

$\phi(L)x_t = \theta(L)\varepsilon_t$, where $\phi(L)$ and $\theta(L)$ are polynomial of order p and q respectively. Thus,

$$\phi(L) = 1 - \phi_1 L - ... - \phi_p L^p .$$

If the root of the equation, $\phi(L) = 1 - \phi_1 L - ... - \phi_p L^p$, lies outside the unit circle, then the ARMA(p,q) model is stationary.

Mixed Autoregressive Integrated Moving Average (ARIMA) Model

The seasonal ARIMA model incorporated non-seasonal and seasonal factors in the multiplicative model. One shorthand notation for the model is ARIMA $(p,d,q)*(P,D,Q)_s$. Where P = non-seasonal AR order, d = non-seasonal differencing, q = non-seasonal MA order, s = time of the repeating seasonal pattern.

Without differencing operations the model could be written more formally as:

$$\phi_1(B)^p \phi_2(B^s)^P \nabla^d \nabla^D X_t = \theta_1(B)^q \theta_2(B^s)^Q \varepsilon_t \tag{3.6}$$

The non-seasonal components are:

AR: $\phi(B) = 1 - \phi_1 B - ... - \phi_p B^p$

MA: $\theta(B) = 1 + \theta_1 B + ... + \theta_q B^q$

The seasonal components are:

Seasonal AR: $\phi(B^s) = 1 - \phi_1 B^s - ... - \phi_p B^{Ps}$

Seasonal MA: $\theta(B^s) = 1 + \theta_1 B^s + ... + \theta_q B^{Qs}$

Box and Jenkins (1976) noted that the mixed autoregressive moving average model is the combination of *AR (p) and MA (q).* Let's say that X_t is the deviation from the mean μ, and then ARMA (p, q) model can be written as

$$X_t - \phi_1 x_{t-1} - \phi_2 x_{t-2} - ... - \phi_p x_{t-p} = \varepsilon_t - \theta_1 \varepsilon_{t-1} - \theta_2 \varepsilon_{t-2} - ... - \theta_q \varepsilon_{t-q} \tag{3.7}$$

Thus,

$$\phi(B)x_t = \theta(B)\varepsilon_t \tag{3.8}$$

The equation (3.7) can be written as

$$x_t = \phi^{-1}(B)\theta(B)\varepsilon_t$$

$$= \frac{\theta(B)}{\phi(B)}\varepsilon_t = \frac{1 - \theta_1 B - ... - \theta_q B^q}{1 - \phi_1 B - ... - \phi_p B^p}\varepsilon_t \tag{3.9}$$

The ARIMA model is based on prior values in the autoregressive terms and the error made by the previous prediction. The order of ARIMA model is given by *p, d, q* where, *p* represents the

autoregressive component, *d* stands for the differencing to achieve stationarity and *q* is the order of the moving average.

Seasonal Autoregressive Integrated Moving Average (SARIMA) model applies to time series with seasonal and non-seasonal behavior. SARIMA model has a multiplicative and additive part. The multiplicative is so applied because of the assumption that there exists a significant parameter resulting from the multiplication between nonseasonal parameters. By the use of ∇ and *B* notation, ARIMA (*p, d, q*) model can be written as

$$\phi(B)w_t = \theta(B)\varepsilon_t \tag{3.10}$$

where the polynomial in *B* is given as

$$\phi(B) = 1 - \phi_1(B) - ... - \phi_p B^p and \theta(B) = 1 - \theta_1(B) - ... - \theta_q B^q$$

This study focused on the multiplication model because of the assumption that there is a major parameter between the non-seasonal and seasonal models. This is denoted by *ARIMA (p,d,q)*× *(P,D,Q)*$_s$ written as

$$\phi_p(B)\phi_P(B^s)\nabla^d\nabla_s^{\ D}\chi_t = \theta_q(B)\theta_Q(B^s) \tag{3.11}$$

Where

$$\phi(B) = 1 - \phi_1 B - \phi_2 B^2 - ... - \phi_p B^p; \varphi(B^s) = 1 - \Phi_{1,} sB^s - \Phi_{2,} sB^{ss} - ... - \varphi_{p,} sB^{ps}$$

$$\nabla^d = 1 - B - B^2 - ... - B^d; \nabla_s^{\ D} = 1 - B^s - B^{2s} ... - B^{2D}$$

$$\theta(B) = 1 - \theta_1 B - \theta_2 B^2 - ... - \theta_q B^q; \Theta(B^s) = 1 - \Theta_{1,} sB^s - \Theta_2 sB^{2s} - ... - \Theta_{q,} sB^{Qs}$$

where χ_t is the time series at period *t*, ε_t stands for the white noise, *B* represents the backshift operator, is the duration of the seasonal model which could be weekly, quarterly, or yearly, *p* is the autoregressive parameter, *P* is the seasonal autoregressive parameter, *d* is the order of the monthly difference (quarterly difference), *D* is the order of seasonal difference, *q* is the moving average parameter and *Q* is the seasonal moving average parameter.

3.2 Model Selection Criteria (MSC)

The AR and MA order p and q have to be determined by examining the regular and seasonal autocorrelation and partial autocorrelation function; ACF, PACF, SACF, and SPACF for Yt. before an ARMA (p, q) is estimated. The idea is to fit all ARMA (p,q) models with order $p \leq p_{max}$ and $q \leq q_{max}$ and choose the value of p and q which minimizes some model selection criteria. For ARMA (p, q), the model selection criteria are given by

$$MSC(p,q) = Ln(\sigma^2(p,q)) + C_T.\varphi(p,q) \tag{3.12}$$

where $\sigma^2(p,q)$ is the MLE of $\text{var}(\varepsilon_t) C_T$ is a sequence indexed by the sample size T, and $\varphi(p,q)$ is a penalty function that penalizes large ARMA(p, q) model.

The three most common information criteria for selection models are the Akaike Information Criteria (AIC), Schwarz-Bayesian Information Criteria (BIC), and Hannan-Quinn Information Criteria.

Akaike Information Criteria

The AIC is a measure of the relative goodness of fit of a statistical model. The AIC value is given by

$$AIC = T\ln\left[\frac{RSS}{T}\right] + 2P \tag{3.13}$$

where T is the number of data points (observations); ln is the natural logarithm; RSS is the residual sum of square (σ^2) or the error variance of the model which is an unbiased estimator of the true variance and p is the number of parameters in the model (Akaike, 1983).

Schwartz-Bayesian Information Criteria (SBIC or BIC)

The BIC is a model selection criterion that involves selections among a finite set of models. The BIC is given by

$$BIC = T\ln\left[\frac{RSS}{T}\right] + P\ln(T) \tag{3.14}$$

where the parameters are defined as previous Equation (2.10)

Hunnan and Quinn (HQ) criterion

Hunnan and Quinn (1979) developed a procedure which is known as HQ criteria. Statistic of the procedure can be represented as

$$\text{HQ}=\left(\frac{ESS}{T}\right)(InT)\left(\frac{2K}{T}\right) \tag{3.15}$$

The value of HQ will decrease provided there are at least 16 observation (Ramanathan. 1995)

3.3 Model Accuracy Measures

To gauge the accuracy of our estimates, the estimated errors will be used to compare the two models forecasts. This is done by subtracting the estimated forecast values (EFV) from the original values or [actual values (AV)] to obtain the estimate errors. The estimated error is denoted by

$$e_i = AV_i - EFV_i, i = 1,2,...,V \tag{3.16}$$

where v is the number of forecast values

Then accuracy measures considered in this paper are: Mean Error (ME), Mean Absolute Error (MAE), and Mean Square Error (MSE).

Mean Error (ME)

The first descriptive Statistics of Error used is called the Mean Error. It indicates the deviation between the actual values and estimates, Mean Error is given as

$$ME = \left[\frac{1}{V} \sum_{i=1}^{v} e_i \right] \tag{3.17}$$

Mean Square Error (MSE)

MSE also indicates the fluctuations of the deviations and it can be calculated as

$$MSE = \left[\frac{1}{V} \sum_{i=1}^{v} e_i^{\,2} \right] \tag{3.18}$$

Mean Absolute Percentage Error (MAPE)

This accounts for the percentage of deviation between the actual values and estimates. This can be obtained as

$$MAPE = 100 \times \left[\frac{1}{V} \sum_{i=1}^{V} \left| \frac{e_i}{AV_i} \right| \right] \qquad (AV_i \neq 0) \tag{3.19}$$

Mean Absolute Error (MAE)

$$MAE = \frac{1}{n} \sum_{t=1}^{n} \left| \hat{e}_t \right| \tag{3.20}$$

4. Results

This research work collected two data sets from the National Bureau of Statistics on Consumer Price Index on Food Consumption Report (1995-2021) and Consumer Price Index on Consumption of Food & Non Alcoholic Beverage (1995-2021). The data sets are Consumer Price Index on Food Consumption: 1995-2021 and Consumer Price Index on Consumption of Food & Non Alcoholic Beverage: 1995-2021; which were used to study the univariate time series analysis of series forecasts in Nigeria.

Monthly Plot, Yearly Plot and Series of the Data Sets

In this section, the monthly mean, yearly mean and the series plots (Consumer Price Index on Consumption of Food and Non Alcoholic Beverage) were done to examine the relationships, trend component and seasonality effect, if present in the data sets.

4.1 Monthly Plot, Yearly Plot and Series plot of Food Consumption

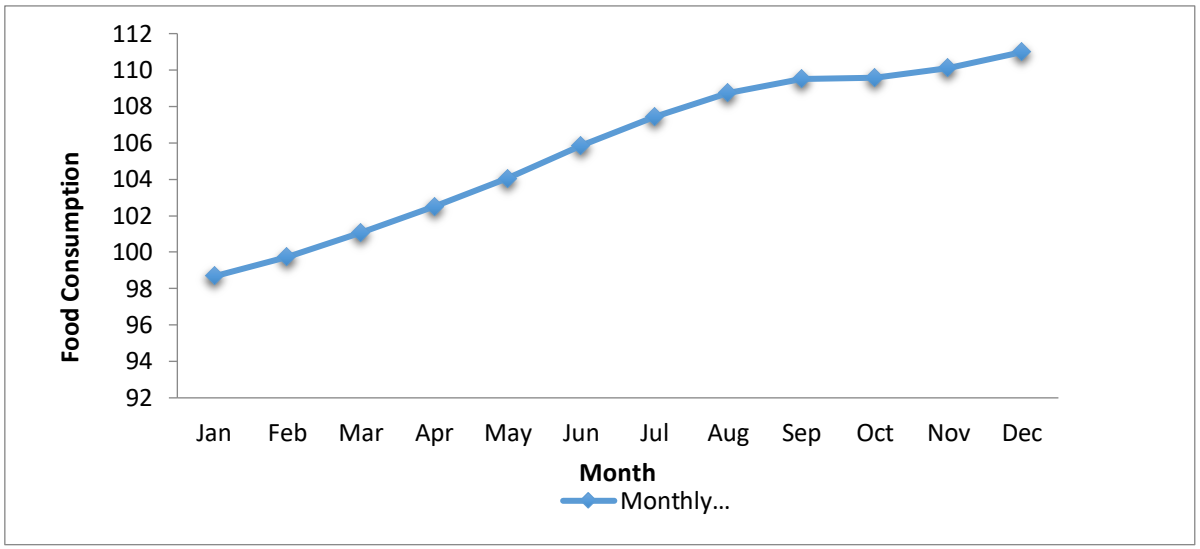


Figure 1: Monthly Means Plot of Food Consumption

Figure 1 shows the monthly mean behaviour of the food consumption index, where the peak is in December and least food consumption rate is in January. However, the monthly mean series show an increase from the beginning (or swing upward); January to December.

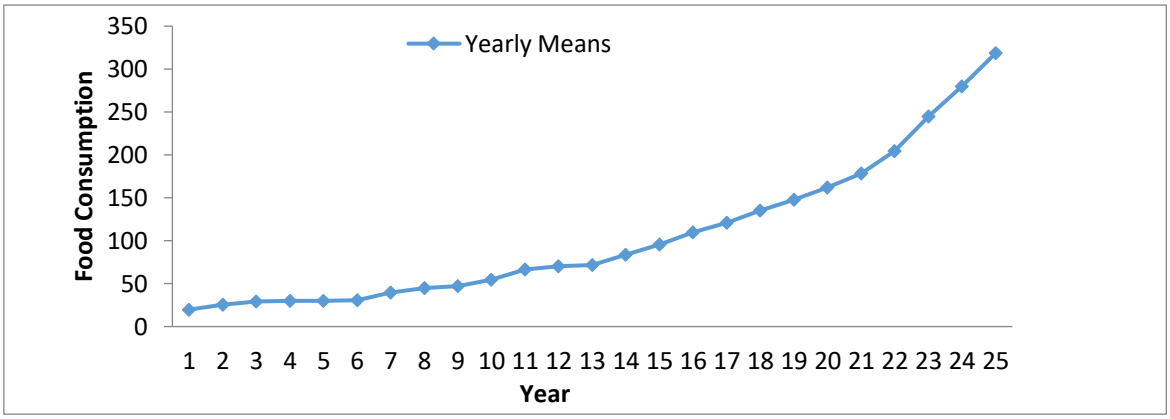


Figure 2: Yearly Means Plot of Food Consumption

Figure 2 shows an upward trend and then upward movements in a random manner. There seem to be evidence of peak in the year 2021 represented by 25 and increases almost all through the periods.

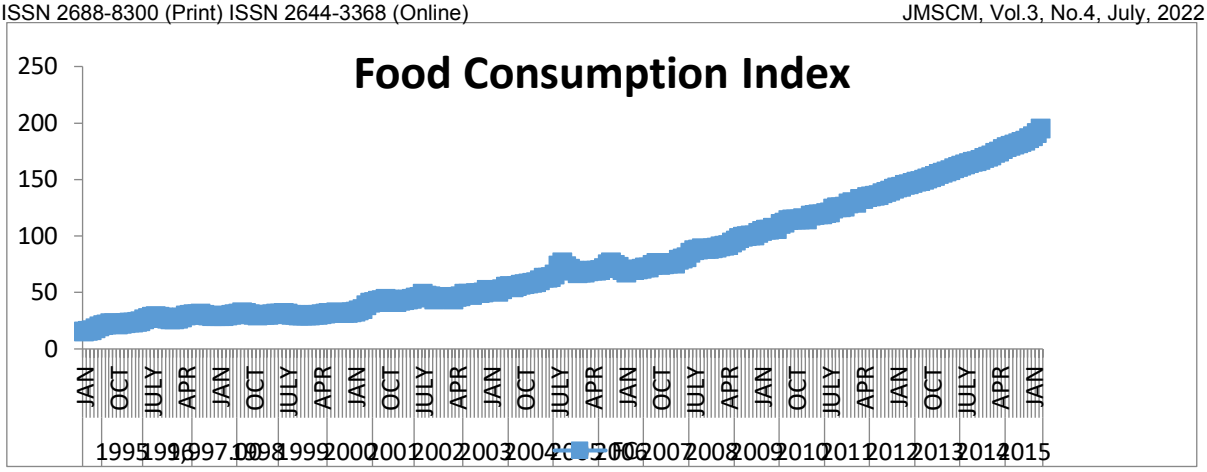


Figure 3: Series Plot of total Food Consumption

Figure 3 shows an upward trend from the year 1995 to 2021. There seem to be an evidence of peak in the year 2021 and show continuous swing upward movements almost all through the latter periods.

4.2 Monthly Plot, Yearly Plot and Series Plot of Food & Non-Alcoholic Beverage Consumption

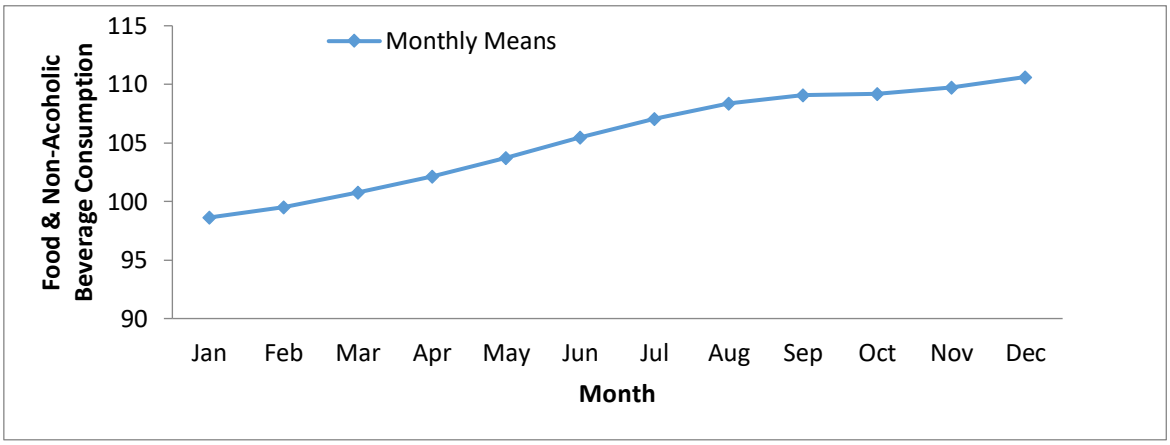


Figure 4: Monthly Means Plot of Food & Non-Alcoholic Beverage Consumption

Figure 4 shows the monthly mean behavior of the food & non-alcoholic beverage consumption, where the peak is in December and least food & non-alcoholic beverage consumption rate is in January. However, the monthly mean series shows an increase from the beginning (or swing upward) of January and a decrease in October. Then, show an upward movement in a random manner from November to December.

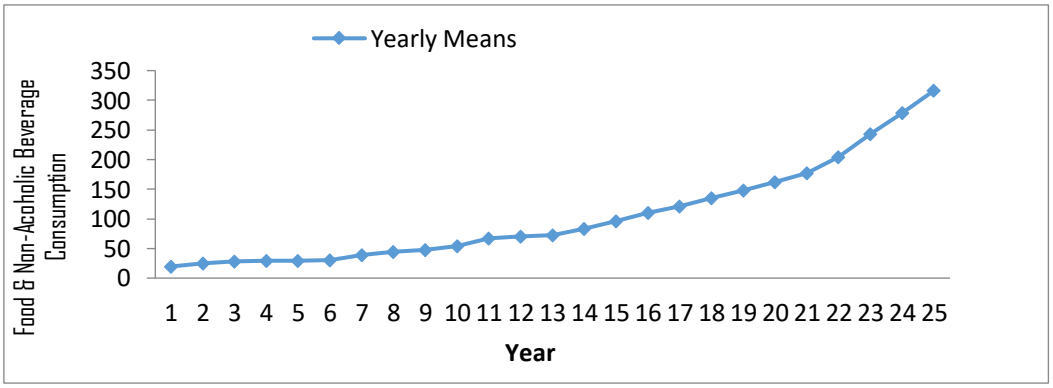


Figure 5: Yearly Means Plot of Food & Non-Alcoholic Beverage Consumption

Figure 5 is similar to Figure 2, which shows an upward trend and then upward movements in a random manner at the last years. There seem to be an evidence of peak in the year 2007 and depressions almost all through the latter periods.

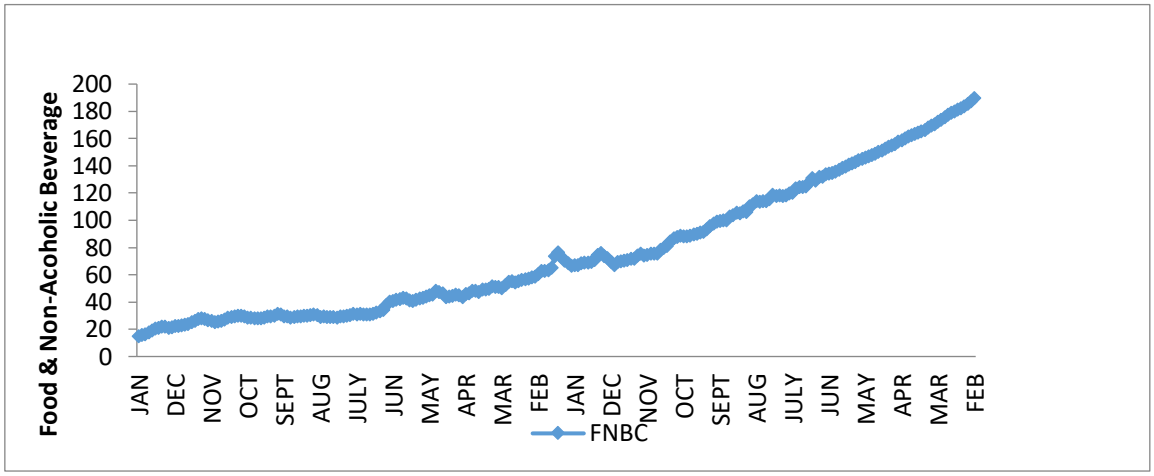


Figure 6: Series Plot of Food & Non-Alcoholic Beverage Consumption

Also Figure 6 is similar to Figure 3, which shows an upward trend from the year 1995 to 2009, then show upward movements in a random manner from 2009 to 2021. There seem to be an evidence of peak in the year 2021 and show continuous swing upward and downward movements almost all through the latter periods. In addition, continuous swing upward and downward movements in a random manner in the early years seem to indicate that these series have seasonal variation. Next, we compare the two plots in Figure 7;

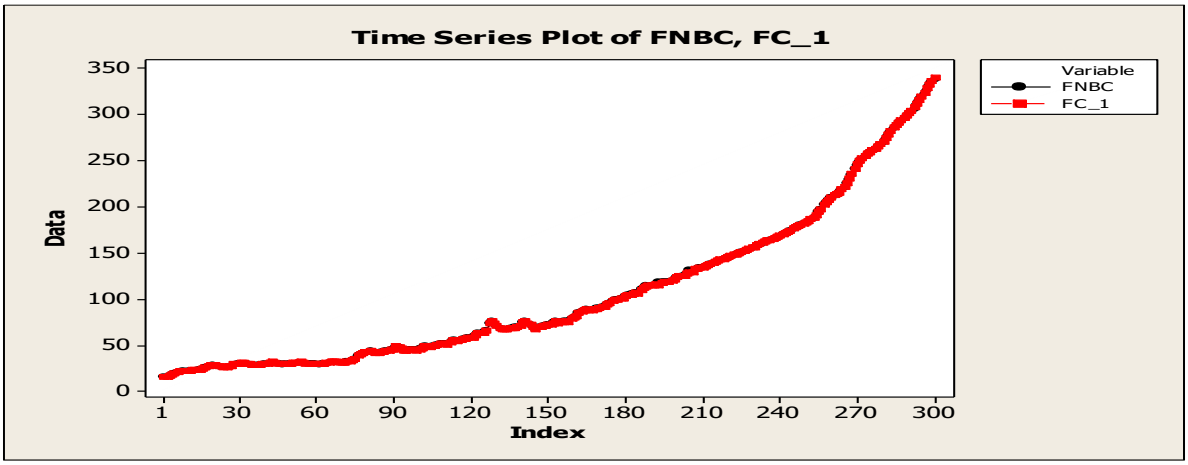


Figure 7: The Food Consumption and Food & Non-Alcoholic Beverage comparison

In Figure 7, comparing the two series, it is noticed that there is a similar behaviour between the two series which show an upward trend component. It also indicated that the two series are not stationary.

4.3: Stationarity of the Data Sets

Next, since these series are not stationary, then first difference was applied to obtained stationarity below (Figure 8 to 9).

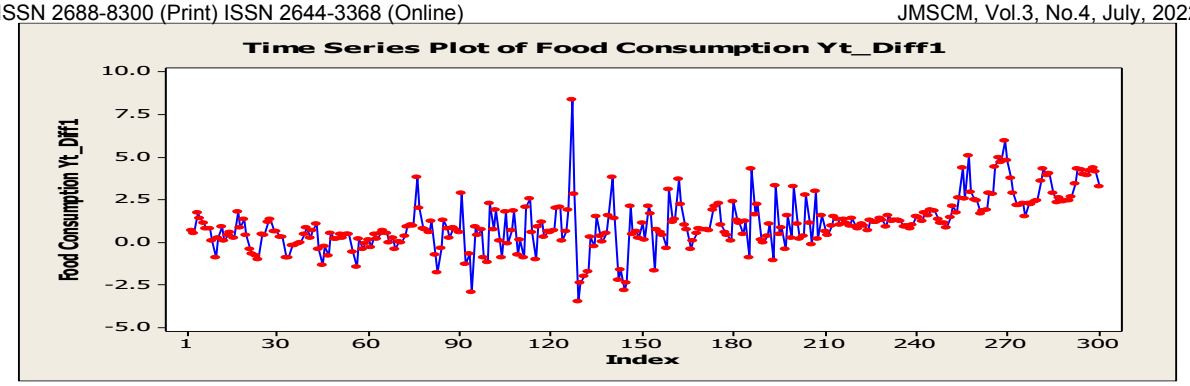


Figure 8: Food Consumption first difference series

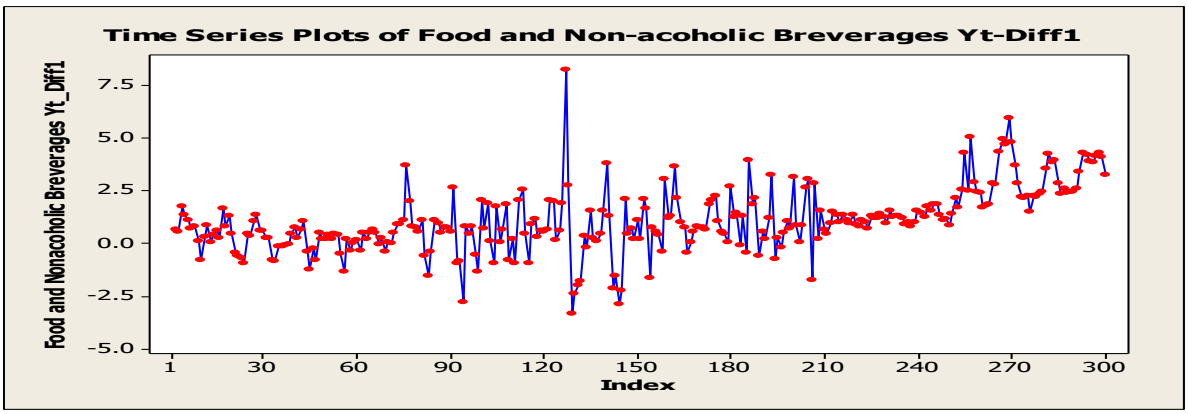


Figure 9: Food & Non-Alcoholic Beverage first difference series

Figure 8 and 9 are the first difference series which shows the series are sine wave pattern in nature with mean greater than zero and non-constants variance. However, both the series are not stationary after first difference (or do not behave better).

Next, since these series are not yet stationary, then, difference of the first difference was applied to obtained stationarity below (Figure 10 to 11).

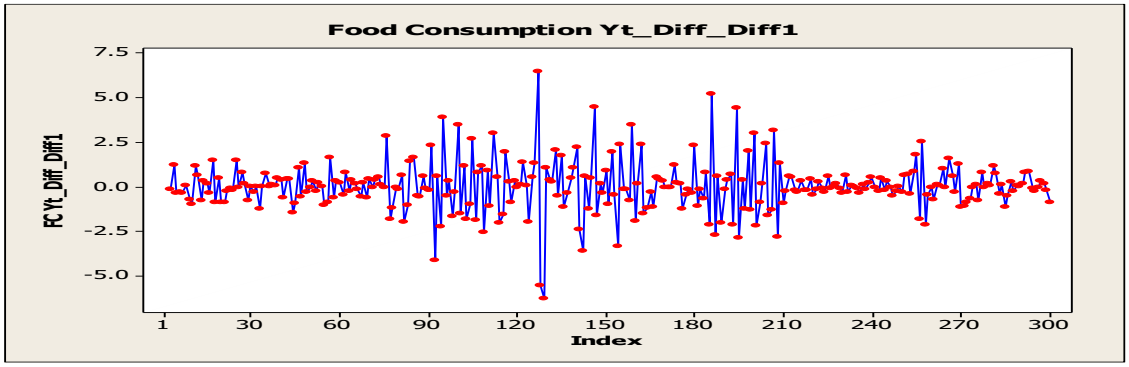


Figure 10: Food Consumption difference of the first difference series

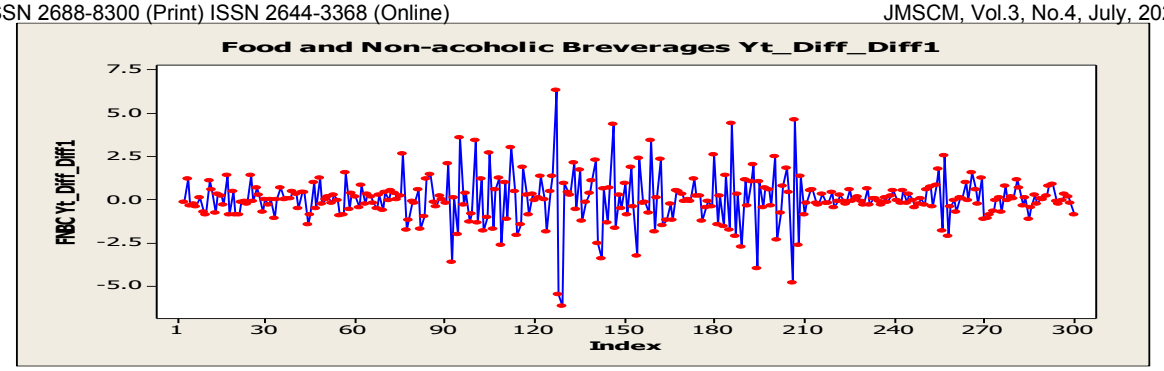


Figure 11: Food & Non-Alcoholic Beverage difference of the first difference series

Figure 10 and 11 are the difference series which shows the series are sine wave pattern in nature with mean zero and constant variance. However, both the series are now stationary after first difference (or behaves much better).

4.4 Discussion of Results

Parameters Estimates and ARIMA Model Identification

The ACF and PACF Plots for the actual series and the difference series for both food consumption and food & nonalcoholic beverages series were obtained in Figure 12, 13, 14 and 15 for food consumption and Figure 16, 17, 18 and 19 for food & nonalcoholic beverages.

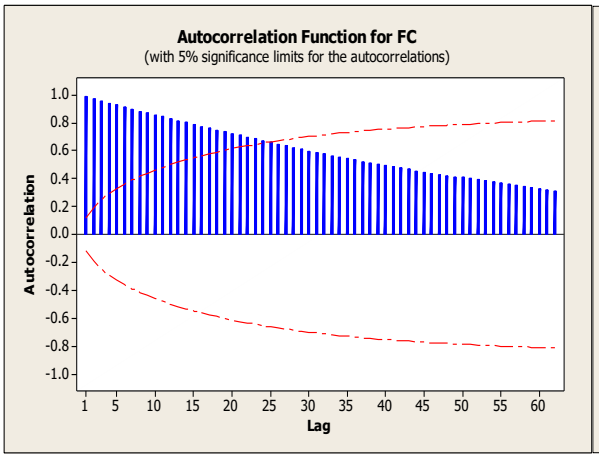


Figure 12: ACF for FC

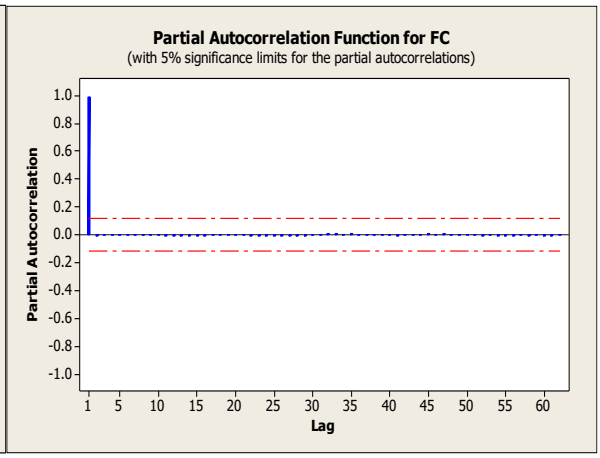


Figure 13: PACF for FC

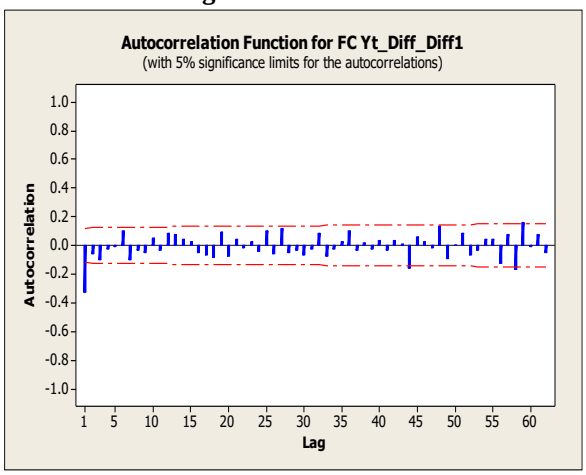


Figure 14: ACF for FC difference of the first difference Series

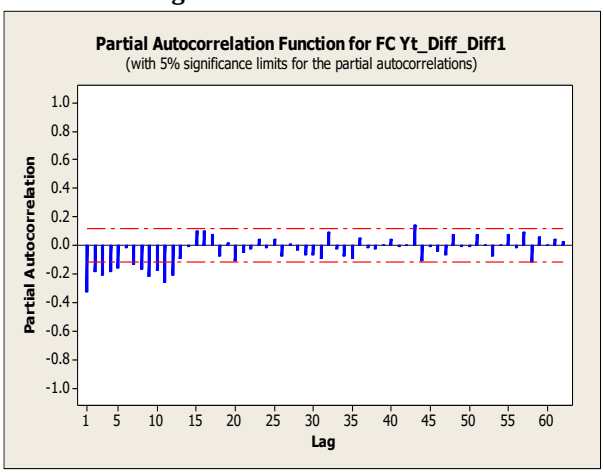


Figure 15: PACF for FC difference of the first difference Series

The ACF plot in Figure 12 spikes dies down extremely slowly indicating AR(p) process (where p=1 or p=2), which is indicated in the difference of the first difference series in Figure 4.14. In addition, PACF plots spikes are close to white noise except lags 1 or 2 cut on, which is also an indication of AR(P); where p=1 or p=2) and MA(q), where q=1.

However, Various ARMA(p,q) models were fitted to the food consumption series with respective residuals as white noise and is summarized in Table 1. The model selection criteria used to select the best model amongst models is AIC and BIC is also detailed out in Table 1.

Table 1:

ARIMA Models	AR(p) Estimates ϕ_1	ϕ_2	ϕ_3	MA(q) Estimates θ_1	θ_2	θ_3	Modified Box-Pierce (Ljung-Box) Chi-Square statistic k=12	k=24	k=36	k=48
ARIMA(1,1,0)	-0.6002 (0.000***)						65.0 (10)	74.4 (22)	93.7 (34)	114.2 (46)
ARIMA(2,1,0)	-0.6292 (0.000***)	-0.3823 (0.086*)					66.8 (9)	81.9 (21)	105.0 (33)	118.8 (45)
ARIMA(3,1,0)	-0.9537 (0.000***)	-0.6528 (0.000***)	-0.3263 (0.000***)				58.9 (8)	78.6 (20)	97.1 (32)	119.8 (44)
ARIMA(0,1,1)				**1.0000 (0.000***)**			**44.0 (10)**	**57.1 (22)**	**77.1 (34)**	**95.5 (46)**
ARIMA(2,1,1)	-1.5849 (0.000***)	-0.6026 (0.000***))		-0.9809 (0.000***)			65.7 (8)	74.4 (20)	93.0 (32)	112.8 (44)

ARIMA Models	**RSS (σ^2)**	**AIC**	**Rank**	**BIC**	**Rank**	**Average Rank**
ARIMA(1,1,0)	968.940	353.37	4	357.07	4	4
ARIMA(2,1,0)	827.777	308.45	3	315.85	3	3
ARIMA(3,1,0)	740.036	277.06	2	288.15	2	2
ARIMA(0,1,1)	**576.684**	**198.74**	**1**	**202.44**	**1**	**1**
ARIMA(2,1,1)	961.829	355.18	5	366.27	5	5

FOOTNOTE: ***-sig. at 1%, **-sig. at 5%,*-sig. at 10%.

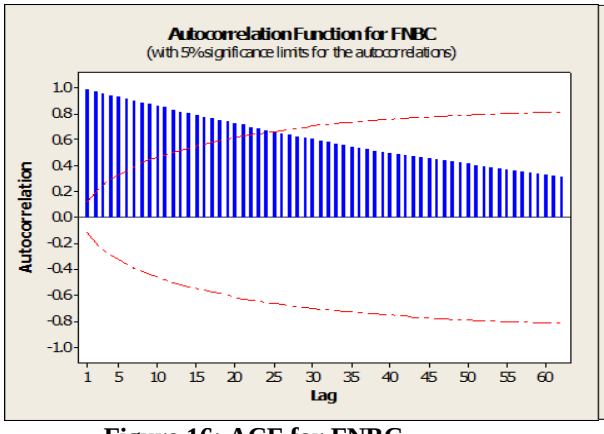


Figure 16: ACF for FNBC

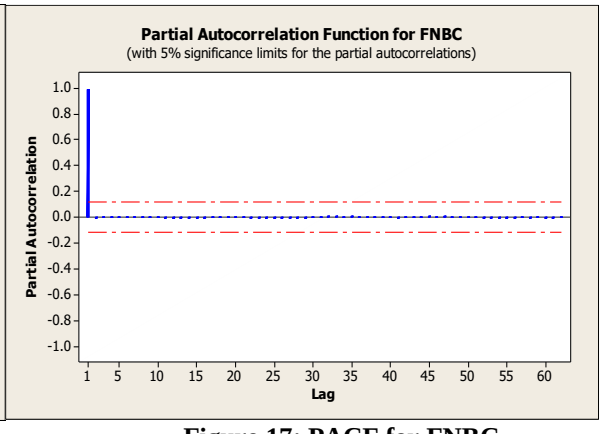


Figure 17: PACF for FNBC

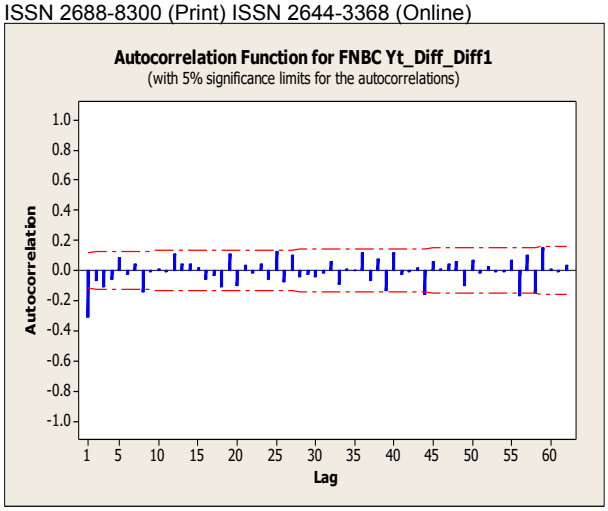


Figure 18: ACF for FNBC difference of the first difference Series

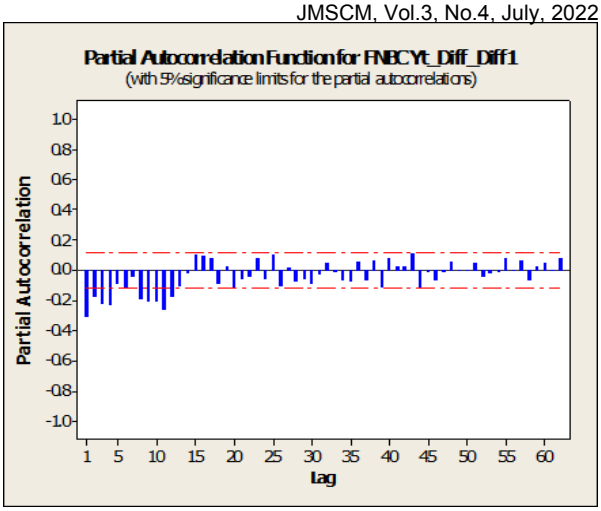


Figure 19: PACF for FNBC difference of the first difference Series

The ACF plot in Figure 16 spikes dies down extremely slowly indicating AR(p) process (where p=1 or p=2), which is also indicated in the difference of the first difference series in Figure 18. In addition, PACF plots spikes are close to white noise except lags 1 or 2 cut on, which is also an indicating of AR(P); where p=1 or p=2) and MA(q), where q=1. Also the early lags indicated that these series have seasonal variation of order 12. However, Various SARIMA(P, D, Q) models were fitted to the Food & Nonalcoholic beverage consumption series with respective residuals as white noise and it is summarized in table 4.2. The model selection criteria used to select the best model amongst models is AIC and BIC is also detailed out in Table 2.

Table 2:

	AR(p) Estimates			MA(q) Estimates			Modified Box-Pierce (Ljung-Box) Chi-Square statistic			
SARIMA 0f Order 12	ϕ_1	ϕ_2	ϕ_3	θ_1	θ_2	θ_3	k=12	k=24	k=36	k=48
$ARIMA(1,1,0)(1,0,0)_{12}$	0.0810 (0.165)					$ARIMA(1,1,0)(1,0,0)_{12}$	61.9 (9)	17.7 (21)	86.1 (33)	104.2 (45)
$ARIMA(0,1,0)(1,0,0)_{12}$	0.0713 (0.221)					$ARIMA(0,1,0)(1,0,0)_{12}$	120.0 (10)	148.4 (22)	187.4 (34)	236.1 (46)
$ARIMA(0,1,0)(2,0,0)_{12}$		-0.1232 (0.035***)				$ARIMA(0,1,0)(2,0,0)_{12}$	116.7 (9)	144.7 (21)	171.5 (33)	228.9 (45)
$ARIMA(1,1,0)(0,0,1)_{12}$	-0.5950 (0.000***)			-0.0915 (0.117)		$ARIMA(1,1,0)(0,0,1)_{12}$	62.1 (9)	72.0 (21)	86.6 (33)	105.0 (45)
$ARIMA(0,1,0)(0,0,1)_{12}$				-0.0979 (0.092*)		$ARIMA(0,1,0)(0,0,1)_{12}$	118.8 (10)	147.0 (22)	187.6 (34)	237.0 (46)
$ARIMA(0,1,0)(0,0,2)_{12}$				-0.1133 (0.051*)	-0.1270 (0.030***)	$ARIMA(0,1,0)(0,0,2)_{12}$	114.6 (9)	142.0 (21)	168.6 (33)	227.1 (45)
$ARIMA(1,1,0)(0,0,0)_{12}$	-0.5947 (0.000***)					$ARIMA(1,1,0)(0,0,0)_{12}$	62.1 (10)	70.9 (22)	83.6 (34)	101.4 (46)
$ARIMA(0,1,1)(0,0,0)_{12}$				**1.0000 (0.000***)**		**$ARIMA(0,1,1)(0,0,0)_{12}$**	**46.3 (10)**	**62.1 (22)**	**82.8 (34)**	**109.6 (46)**

SARIMA 0f Order 12	**RSS (σ^2)**	**AIC**	**Rank**	**BIC**	**Rank**	**Average Rank**
$ARIMA(1,1,0)(1,0,0)_{12}$	930.206	341.22	2	344.91	2	2
$ARIMA(0,1,0)(1,0,0)_{12}$	1440.81	471.61	8	475.31	7	7.5
$ARIMA(0,1,0)(2,0,0)_{12}$	1418.91	469.04	6	476.44	8	7
$ARIMA(1,1,0)(0,0,1)_{12}$	929.447	342.97	3	350.37	4	3.5

ARIMA(0,1,0)(0,0,1)$_{12}$	1438.17	471.06	7	474.76	5	6
ARIMA(0,1,0)(0,0,2)$_{12}$	1412.09	467.61	5	475.0	6	5.5
ARIMA(1,1,0)(0,0,0)$_{12}$	936.353	343.18	4	346.88	3	3.5
ARIMA(0,1,1)(0,0,0)$_{12}$	**556.385**	**188.06**	**1**	**191.76**	**1**	**1**

FOOTNOTE: ***-sig. at 1%, **-sig. at 5%,*-sig. at 10%.

From table 1, it was reveals that the ARIMA(0,1,1) was the best and suitable model that should be used to forecast the behaviour of Food consumption in Nigeria which was determined by equation below;

$$Y_{1t} = 1.0000 X_{1t-1} + \varepsilon_{1t}$$

From table 2, it is reveals that the ARIMA(0,1,1)(0,0,0)$_{12}$ was the best and suitable model that should be used to forecast the behaviour of Food & Nonalcoholic Beverages consumption in Nigeria which was determined by equation below;

$$Y_{2t} = 1.0000 X_{2t-1} + \varepsilon_{2t}$$

4. 5: Forecast

In this stage, the ARIMA models identified can now be used to generate forecast. We have observed 298 data points, the start of the origin is 1 and the end is 298. Appendix A shows the fitted values and the forecasts for t = 299, 300, to 322. Twenty- four (24) months' forecast values with confidence intervals of the forecasts (i.e. January 2022 to December 2023).

The plots of the forecasts generated for the food consumption and food & nonalcoholic beverages consumption are shown in Appendices G and H respectively. The graph shows that the two variables are dependent and also shows that any gradual increase in the food consumption tends to pave way for increase in the food & nonalcoholic beverages consumption or a drastic drop in the food consumption will also drop the food & nonalcoholic beverages consumption in the same manner.

1. Models Forecasts (The ARIMA Model Identified Forecasts)

To gauge the accuracy of our estimates, the estimated errors were used to compare the two models forecasts in Section 3.3 and 3.5. This is done by subtracting the estimated forecast values (F_i) from the original values or [actual values (A_i)] to obtain the estimate errors. The ARIMA model identified forecasts in Section 4.5 is compared to determine the suitable model between the two models for forecasting food consumption and food & nonalcoholic beverages consumption, using accuracy measures

Accuracy Measures.

Variable		ARIMA model
Food Consumption (FC)	MAE	0.0456
ARIMA(0,1,1)	MSE	1.955
	MAPE	1.092%
Food & Nonalcoholic Beverages Consumption (FNBC)	MAE	0.7805
	MSE	1.886
ARIMA$(0,1,1)(0,0,0)_{12}$	MAPE	1.020%

The univariate is better to predict the two series, using accuracy measure of (i.e. MAPE, MAE and MSE).

5. Conclusion

It is reasonable to conclude that there is significant relationship between the food consumption and food & nonalcoholic beverage consumption series. In addition, univariate model such as (**ARIMA(0,1,1)** and **ARIMA$(0,1,1)(0,0,0)_{12}$**) seems better to predict the two series, using accuracy measure of univariate (i.e. MAPE, MAE and MSE).

6. Recommendation

It is recommended that more detailed work should be carried out in the area of co-integration analysis of the two variables to enhance a better understanding and prediction distribution in Nigeria. This will help in creating a strong and adequate model that can improve the consumption of food and food & nonalcoholic beverages in Nigeria in the nearest future.

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APPENDIX A

Forecasts from period 298

		95% Limits		
Period	Forecast	Lower	Upper	Actual
299	-0.19993	-2.89221	2.49235	
300	-0.20086	-2.89314	2.49142	
301	-0.20179	-2.89407	2.49049	
302	-0.20272	-2.89500	2.48956	
303	-0.20365	-2.89593	2.48863	
304	-0.20458	-2.89686	2.48770	
305	-0.20551	-2.89779	2.48677	
306	-0.20644	-2.89872	2.48584	
307	-0.20737	-2.89965	2.48491	
308	-0.20830	-2.90058	2.48398	

309	-0.20922	-2.90150	2.48305
310	-0.21015	-2.90243	2.48213
311	-0.21108	-2.90336	2.48120
312	-0.21201	-2.90429	2.48027
313	-0.21294	-2.90522	2.47934
314	-0.21387	-2.90615	2.47841
315	-0.21480	-2.90708	2.47748
316	-0.21573	-2.90801	2.47655
317	-0.21666	-2.90894	2.47562
318	-0.21759	-2.90987	2.47469
319	-0.21852	-2.91080	2.47376
320	-0.21945	-2.91173	2.47283
321	-0.22037	-2.91265	2.47191
322	-0.22130	-2.91358	2.47098

ON THE MULTIVARIATE ANALYSIS OF STUDENTS' ACADEMIC PERFORMANCE IN WASSCE IN PUBLIC SENIOR SECONDARY SCHOOLS IN RIVERS STATE (2018-2020)

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Abstract

The paper examined students' performance in six subjects from WAEC examination from 2018 to 2020 using multivariate analysis through Hotelling T^2 distribution and paired t-test statistics. Four objectives where formulated and used for this study. Based on the factors in the objectives, relevant related literatures were reviewed. A secondary data extracted from the WAEC results from the public senior secondary schools under study were used for this study and the analyses of the data were done using Hotelling T^2 distribution, Quadratic form, and paired t-test statistics. All computations were done via Microsoft Excel 2010, SPSS (version 23.0) and MINITAB (version 16.0). The Hotelling T^2 statistics results between the students' academic performance for (2018 & 2019), (2019 & 2020) and (2018 & 2020) were all significant. Paired t-test statistics results showed a decrease in the Students' average performance for four subjects (Mathematics, English Language, Marketing and Biology), while an increase in the Students' average performance for Economics and Civic Education subjects. It was discovered that students' average performances in Economics and Civic Education subjects were better than other subjects. This research recommend the effective implementation of the Nigeria education policies that emphasizes on teachers qualification, years of teaching experience and the UNESCO policy on Teacher-Students ratio (this policy stipulates that the maximum number of students that should be in a secondary class is 25), since there is significant difference between students' average performance for four subjects.

Keywords*:* *Multivariate Analysis; Academic Performance; WASSCE; Hotelling T^2 Distribution; Paired t-test Statistics*

1. Introduction

Education, as a tool for change, lies at the heart of every country's desire for developing human capital for effective societal functioning. In Nigeria, education is a priceless tool for advancing the country's political, social, economic, scientific, and technological growth. Secondary education, which is the foundation of the entire educational system in Nigeria, is rapidly losing importance, owing, among other things, to students' unsatisfactory and bad performance in public examinations. In Nigeria, education is a “par excellence” tool for achieving national development. It has seen active participation from non-governmental organizations, communities, and individuals as well as government interventions. As a result, it is desirable for the country to state clearly and unequivocally the philosophy and goals that underpin its investment in education.

Several issues have recently been recognized by researchers and stakeholders in the education industry as the causes of students' low performance in public examinations. Poor school location, constant changes in government policies, school closures based on teachers' strike action, home-school distance, high student-teacher ratio, lack of supervision, monitoring, and evaluation machinery, lack of good textbooks, poor content and context of instruction are some of the factors identified, poor and nonconductive environment among others (Adeboyej et al., 2003; Adepoju, 2003).

In order to ensure that their children score better in the SSCE and, as a result, secure admission to universities of their choosing, some parents and guardians have made a specific choice of secondary school for their children, disregarding the school's location or financial implications. However, the distribution of secondary schools in both urban and rural areas (the urban-rural dichotomy) has a significant impact on students' private costs and academic achievement. For instance, secondary schools should be planned such that students living in all parts of a state can have cheap means of transport and easy access to them. In order to lower private costs, school size must be proportional to the prospective population of pupils within various towns or zones. The establishment of adjacent schools will surely aid in increasing enrolment rates and therefore bridging educational inequities within the state.

The importance of English Language and Mathematics as prerequisite subjects for admission to higher education in Nigeria and some West African countries such as Ghana, The Gambia, Sierra Leone, and Liberia (These countries share a colonial history and jointly established the WAEC) has made the two subjects compulsory or mandatory to pass at credit level by secondary schools students in public examinations. In the Nigerian setting, a credit level in either of the subjects has been utilized as one of the criteria for measuring and establishing a candidate's brilliancy. Of course, low performance in SSCE English Language and Mathematics by secondary school students has made it difficult for the majority of pupils to gain entrance to higher education institutions in recent years. According to Adepoju (2002), approximately 93 percent of secondary school leavers fail to qualify for university education in any given year. He also stated that 7.7% of students received credit in English Language in 1988, 9.0% in 1989, and 6.3 percent in 1990. The fall in students' academic achievement was more pronounced in Mathematics.

Academic achievement of pupils, according to Nwokocha and Amadike (2005), is the criterion for assessing a nation's educational excellence. As a result, it is necessary to maintain a high level of performance in internal and, for the most part, external examinations. Student's performance has been a subject of discussions and debate among scholars; because it is the most vital educational policy and indicator that stakeholders are interested in (Alaka, 2011). Xinyi (2006) informed that student's performance has been a subject of national interest and comparative studies among countries since the beginning of educational theory.

While stressing the importance of academic performance in the educational system, Aremu et al. (2001) believed that academic performance is a fundamental criterion by which all teaching-learning activities are measured, using some standards of excellence and the acquisition of specific grades in examinations to measure candidate's ability, mastery of the content, and skills in the classroom. Scholars agree, according to Arief (2019), that a student's academic attainment or performance is a 'net outcome' of their cognitive and non-cognitive traits, as well as the sociocultural framework in which the learning process occurs. Students' academic success is an important aspect of schooling (Anthony, 2018). It is regarded as the hub around which the entire educational system revolves. According to Abaidoo (2018), the success or failure of any academic institution is determined by the academic achievement of students. Similarly, some experts believe that a student's academic success serves as the foundation for acquiring knowledge and developing future talents. Additionally, some emphasized that the topmost priority of all educators is the academic performance of students.

Abdullah (2016) defined academic performance as the knowledge gained which is assessed by marks by a teacher and/or educational goals set by students and teachers to be achieved over a specific period of time. He went on to say that these objectives are assessed by ongoing evaluation or examination results. WASSCE is a standardized test that is administered in West African countries. Students who pass the exam receive a diploma indicating that they have completed secondary school. The West African Examination Council (WAEC) administers it, and it is only available to candidates who live in Anglophone West African countries.

Many Nigerian state governments have tried several times to make it a policy in public secondary schools not to have more than 30 students per class to improving the performance of students in public examinations. The major goal of this research was to determine the trend in students' SSCE performance in English Language and Mathematics in typical urban and rural secondary schools in Oyo State, Nigeria. It also sought to find out the percentage of those students who obtained grades from A1 – E8 as well as A1 – C6 with a view to providing useful data on the strengths and weaknesses of students' performance in the two subject areas in typical urban and rural schools for educational planners, educational policy makers, and curriculum planners

Multivariate statistical analysis is the study and solution of multi-index theories and methodologies using mathematical statistics methods. The past 20 years, with the computer application technology and the urgent need for research and production, multivariate

statistical analysis techniques are widely used in geology, meteorology, hydrology, medicine, industry, agriculture, economics, and a variety of other fields have evolved to effectively solve practical problems. Simplified system architecture to explore the system kernel, can use principal component analysis, factor analysis, correspondence analysis and other methods, a number of factors in each variable to find the best subset of information from a subset of the description found in multivariable system results, as well as the impact of various factors on the system.

Controlling for the model's prediction has two categories in multivariate analysis. The prediction model, which frequently uses multiple linear regression, stepwise regression analysis, discriminant analysis, or stepwise regression analysis in double screening modeling, is one example. The other is a descriptive model, which is a type of cluster analysis modeling technique that is widely utilized. Many prior studies have shown that a multivariate analysis system requires a similar nature of things or events grouped together in order to find the relationships between them and the underlying regularity are mostly qualitative treatment by a single factor, so the results do not ref the general characteristics of the system. For instance, a numerical classification model built using cluster analysis and discriminant analysis techniques, or a general classification model developed utilizing cluster analysis and discriminant analysis techniques.

Multivariate analysis (MVA) is based on multivariate statistics principles. MVA is typically utilized in cases where several measurements are taken on each experimental unit and the relationships between these measurements and their structures are critical. MVA is classified in a modern, overlapping manner as follows:

1. Multivariate normal and universal models, as well as distribution theory
2. The study and measurement of relationships
3. Probability computations of multidimensional regions
4. Data structures and patterns are investigated.

The desire to add physics-based analysis to compute the impacts of variables for a hierarchical "system-of-systems" can complicate multivariate analysis. Studies that want to apply multivariate analysis are frequently stymied by the problem's dimensionality. Surrogate models, which are very precise representations of the physics-based code, are frequently used to alleviate these difficulties. Surrogate models can be evaluated fast since they take the form of an equation. This becomes a key enabler for large-scale MVA studies: a Monte Carlo simulation spanning the design space, which is challenging with physics-based codes, becomes straightforward with this method when evaluating surrogate models, which often take the form of response-surface equations. In consumer and market research, quality control and quality assurance, process optimization and process control, and research and development, multivariate approaches are used to investigate datasets. Because social scientists are unable to conduct randomized laboratory experiments like those used in medical and natural sciences, these procedures are especially crucial in social science research. Multivariate approaches can be used to statistically estimate relationships between several

variables, as well as correlate how essential each one is to the final outcome and where dependencies exist.

The T-squared distribution of Hotelling is significant because it arises as a distribution of a series of statistics that are natural generalizations of the statistics underlying the T distribution of students. In particular, the distribution arises in multivariate statistics in undertaking tests of the differences between the (multivariate) means of different populations, where tests for univariate problem would make use of a t-test. It is proportional to the F distribution. Harold Hotelling created the distribution as a generalization of the student t-distribution.

If the notation T^2 p.m. is used to denote a random variable having Hotelling's T-squared distribution with parameters p and m then, if a random variable x has Hotelling's T-squared distribution X ~ T^2 p.m.

Then,

$$\frac{m-p+1}{pm} X \sim F_{p,m-p+1} \qquad (1.1)$$

where $F_{p,m-p+1}$ is the F= distribution with parameters $PandM - P + 1$

where Fp.m – P + 1is the F = distribution with parameters P and M – P + 1

Hotelling's T-squared statistics is a generalization of student's t statistics that is use in multivariate hypothesis testing and is defined as follows. Let Np (N,Σ) denote a p- variate normal distribution with location μ and covariance Σ Let $X_1, \dots, X_n$ ~ Np (N, Σ) be *an* independent random variables, It can be represented as a P x 1 real number column vector. It can be shown that $n(\bar{X} - N)^T \Sigma^{-1} (\bar{X} - N) \sim \varphi^2$ Where $\varphi^2 p$ is the chi-squared distribution with p degrees of freedom.

Multivariate techniques are complex and involve high level mathematics that requires a statistical program to analyze the data. These statistical programs are generally expensive. The results of multivariate analysis are not always easy to interpret and tend to be based on assumptions that may be difficult to assess. Multivariate approaches require a large sample of data to get meaningful conclusions; otherwise, the results are useless due to excessive standard errors. Standard errors define how confident you can be in the results, and the results from a large sample are more reliable than those from a small sample. Running statistical programs is very simple, but deciphering the results requires the assistance of a statistician.

The aim of the research is to investigate students' performance in six subjects' areas in WAEC examination results from 2018 to 2020 using multivariate analysis through Hotelling T^2 distribution and paired t-test statistics. Specifically, this paper seeks to achieve the following objectives:

i. To estimate the mean vector, covariance matrix and correlation matrix for each of the years of the six subjects over the years (2018-2020).

ii. Estimate a quadratic form for each of the years using their covariance matrix, which will be used to show the homogenous function that consists of all possible second order terms.

iii.To test for significant difference between the students' academic performance for (2018 & 2019), (2019 & 2020) and (2018 & 2020) using Hotelling T^2 statistics.

iv.Using Paired t-test statistics to determine the difference between subjects (Students' performance in Mathematics, English Language, Marketing, Economics, Civic Education and Biology in WASSCE for; 2018, 2019 and 2020).

1.2 Statement of the Problem

Educators, parents, and the government have been concerned about the poor performance of students who sat for the West African Examination Council (WAEC). The flaws that are contributing to the downward trend in teaching and learning are numerous. Students find it increasingly difficult to pay attention to the teaching of subjects such as Mathematics in their schools, claiming that the topic is too difficult to master. The number of students who offer to learn subjects in the real sense and make career of it is diminishing progressively, as they have begun to lose interest in it.

Poor management of public schools on students' performance in Senior Secondary Schools in West African Examination council (WASSCE) is becoming worrisome to educational development in Nigeria and particularly in Rivers State. According to observations and complaints from examination boards, a substantial percentage of public secondary school students continue to do poorly in Senior School Certificate Examinations due to inefficient resource management. Over the years, the majority of students that sat for the May/June West African Examinations Council (WASSCE) have been recording mass failure, not only in the area of overall performance of the students but also in the core subjects like English, Mathematics, and other compulsory subjects like Civic Education, Marketing, Economics and Biology.

Armed robbery, rapping, cultism, abduction, and other social vices would rise in a state or country where a greater number of teenagers drop out of school. Some graduates of today's secondary education system are unable to function in society or progress to further education without the assistance of their parents or forgery. They are incapable of thinking for themselves or of respecting others' opinions and feelings. Except for items that would make them money quickly, they do not regard the dignity of labour. Every year, the number of pupils and teachers in the classroom decreases. It is based on these factors that this study is designed to address the situation of poor performance of students in WAEC in some selected secondary schools (Public Schools) in Rivers State; Akuku Toru LGA as a case study. The study also looked out for the factors responsible for the poor performance and the way out. These, therefore, have been a source of concern to the researcher in taking a decision to examine students' academic performance in WASSCE over the recent years (from 2018 to 2020).

1.3 Significance of the Research

The findings of this study will aid in the efficient implementation of Nigerian education policies that place a premium on teacher qualifications, years of experience, and the UNESCO teacher-student ratio policy (this policy stipulates that the maximum number of students that should be in a secondary class is 25). It will also encourage curriculum revision and improvement to ensure that the subject's content and scope are adequately covered.

It will also assist students to recognize that the bulk of key subjects learning are vested on their innate urge and willingness to perform and do well. The essential premise that learning or teaching is child-centered underpins all of their other activities and businesses.

Given the importance of secondary education in the Nigerian educational system and the rise in WASSCE failure in public senior secondary schools, it is necessary to discuss some of the issues basic truth to staff of public Secondary Schools in Rivers State which will in turn enlighten them to know the level of performance of students within the Zones, whether they are performing very well or below expectation.

It would also be of significance to the respective school authorities in the state to take corrective measures within their authority through adequate planning of resources to meet the demands and guide their action and future of public schools‟ staffs and students for successful teaching and learning process in schools. It would also be of significance to parents to supervise their child's/children's work at home in order to improve their performances. Finally, the study would also be of significance to future researchers who may be interested in carrying out further research in this current area.

1.4 Scope of the Research

This study examines the analysis of students' academic performance in West African Secondary Schools Certificate Examination (WASSCE) in Public Senior Secondary Schools which implied that this work is limited to a Public Senior Secondary Schools in Akuku Toru LGA of Rivers State, Nigeria (CCS Abonima) between the years 2018-2020. The scope of the study is restricted to one examination center among the public schools under the Education Zone that have presented students for WASSCE examination for at least three years. Due to the large number of Public Senior Secondary Schools in this Zone, the researchers will not cover all public schools in the educational Zone of the State.

2. Literature Review

According to Nwaozuzu (2012), poor teacher quality is responsible for pupils' poor performance in WAEC exams. She emphasized that previous research had revealed a concerning rate of pupil failure in the English language. She pointed out that some English Language teachers in secondary schools, particularly in private schools, are primarily secondary school graduates with little prior teaching experience. However, things have changed as many private and public schools have qualified teachers as per the certificate they possess but these teachers are still not able to deliver as expected. Their poor teaching inability to handle the teaching and learning of English is still very significant.

According to Adebayo (2008), most pupils who attend public schools have low standardized test results. He went on to say that children at private schools perform better in English than students in public schools because certain private schools indulge in examination misconduct.

Abdullahi (2009) pointed out that mathematics like an Octopus has its numerous tentacles in all branches of knowledge. Previous researches conducted show that there have been mass failures in Mathematics. Some studies also show that students' negative attitude towards Mathematics has led to poor performance in the Subject. With regards sex factor as an influence in attitude and performance of students in Mathematics, it was discovered that when males and females performance were compared in an analysis, there existed a sex factor.

Adeniran (2009) investigated the many reasons that contribute to pupils' low performance in mathematics, as well as potential solutions. Students' negative attitudes toward mathematics, as well as their performance in the subject, are examples of such factors. Another issue contributing to students' poor performance in mathematics is certain teachers' inability to effectively teach the subject and convey its abilities. Out of all the subjects in the school curriculum, mathematics has been the hurdle or hindrance to many students' advancement. In publicly administered examinations, mathematics records the most valuable and heartbreakingly outcomes. All stakeholders in the educational system, including the government, educators, proprietors, principals, teachers, and guardians, have been concerned, worried, and anxious about kids' dismal poor performance in mathematics year after year.

Umoru (2010) opines that the development of any nation depends on advancement in Science and technology. He stressed that people of the world are living in a changing world where Science and technology have been part of the world's tradition and any country that fails to recognize this fact at the risk of being technologically backward. As a result, the National Policy on Education emphasizes the need of students being well-trained in order to fulfill the demands of the current age of science and technology. Students should have a strong grounding in science subjects in order to reach this goal.

Reginald (2009) evaluated students' performance in WAEC Science Subjects in a few selected schools in the southeast and discovered that more students scored better in Biology and Chemistry than Physics. In his opinion, the low performance of pupils in Physics is primarily due to schools' failure to employ excellent and competent Physics teachers as well as inadequate laboratory equipment for physics practical.

Lamenting on students poor performance in Physics, Chemistry and Biology DanAzumi (2008), reiterated that one of the most repeatedly mentioned problem causing poor performance in these Subjects since the introduction of SSCE is lack of equipment and materials to conduct practical. Lawal (2006) found no significant link between laboratory equipment adequacy and student academic performance in Science (Physics, Chemistry, and Biology) in SSCE in a multivariate research his study on availability and impact of material resources on performance in Physics, Chemistry and Biology in selected secondary schools in Zaria metropolis.

Leonard (2012) who conducted a study of students' performance in WAEC in Art Subjects in a few selected schools in the south cast, and it was discovered that 70% of the students that sat for the WAEC performed tremendously very well while 30% of the population of the students failed. Also in 2012, it was published in a media that a girl had nine (9) Al in WASSCE examination in Art Subjects from St. Louis Secondary School Umuahia. From the analysis of some of the WAEC results on students' performance in Art Subjects it can be deduced that students performed fairly in Art Subjects than Science Subjects.

Nwobia (2007) conducted a study of students' performance in WAEC in Art Subjects from 2007 to 2010 in a few selected secondary schools in Southeast and it was discovered that 75% of students obtained Al - A3 in Art Subjects while the remaining 25% all in the category of pass and fail. Also an analysis of WAEC result by WAEC office as at December 2012 shows that 62.03% students performed poorly in art Subjects especially English language.

Ojo (2009) opines that teacher quality matters. In fact, it is the single most important school-related element impacting pupils' art achievement. Teacher competency in teaching Art Subjects can contribute to positive achievement on students' performance. According to David (2007), the Arts Subject contributes to children's growth, and it is critical that students are well-taught in order to attain success.

Omekara and Kelechi (2012) evaluated Multivariate Analysis of the performance of students using Hotelling T^2 Statistic. The goal of this study is to see if there is any evidence of a substantial difference in academic achievement between two groups of pupils. Its goal is to see how effective the Hotelling's T-square test statistic is at determining such a difference, establishing this distinction will aid in identifying the high-performing group for a more in-depth investigation into the causes for the disparities. This will help education researchers who are working to improve student performance. The study used data from Michael Okpara University of Agriculture (MOUA), Umudike's College of Natural and Applied Sciences (CNAS). Two departments (Chemistry and Statistics) were chosen at random and their first year students of 2009/2010 academic were considered of which their results were analyzed. Hotelling's T-square test statistic was used to analyze the performance of 135 and 120 first-year students from the Departments of Chemistry and Statistics, respectively. The results reveal a huge disparity in the students' performances in the two Departments. The much improved performance of statistics students suggests that this technique could be used to examine comparative performance of students in order to better understand the better performers and uncover variables that contribute to their superior performance.

Atanda (2011) conducted A Survey of Secondary Students Achievement in English Language and Mathematics in Nigeria: Lessons for Secondary School Administrators in Nigeria. Secondary school education prepares students for institutions of higher learning. Notably, the transition to the institutions of higher learning depends on their level of performance in at least five subjects, Mathematics and English language inclusive in most cases. Thus, the study investigated the senior secondary school students' performance in the two core subjects in Senior secondary school certificate examination (SSCE) conducted by the West African Examination Council (WAEC) with the view to draw lessons for effective secondary school

administration. The descriptive research design was adopted while secondary data on statistics of student academic performance in SSCE were used. The data were analysed with simple percentages. The study revealed poor academic performance in the two subjects in the six geopolitical zones in Nigeria. Only 21% of the candidates passed both Mathematics and English in South-West, 29% passed in South-South; 21% passed in South- East, 7% passed in North-Central; 7.3% passed in North-East; while 11.7% passed in North-West. The performance was poorer in the three geopolitical zones in the northern part of Nigeria. The study recommended among others moderate average student-teacher ratio, good guidance counselling service, provision of regular feedback to the students, provision of adequate instructional materials and encouragement of participatory method of teaching.

Christian (2015) investigated panel data analysis on students' academic performance in West Africa senior school certificate examination (WASSCE). This work focused on panel data analysis on students' academic performance. This study is significant because it is necessary to understand some of the variables that have contributed to the drop in student performance in West Africa Senior High School Certificate Examination (WASSCE) over the years. The effects of student-teacher ratio and teacher years of experience on academic achievement of chosen secondary school pupils in Lagos, Nigeria were explored in this study. Ten Senior Secondary Schools in Lagos' Ajeromi Ifelodun Local Government Area were chosen using a simple random sampling method. The study was guided by three research questions and hypotheses. They were analyzed using the fixed and random effect models at 0.05 level of significance. The findings revealed that there is a link between student-teacher ratio and academic achievement, as well as the years of experience of the teachers. The results show that the student-teacher ratio has a significant impact on student performance in these selected schools, and that as class sizes grow students' performance declines. Similarly, the results show that teachers' years of experience have a considerable impact on their students' performance. This suggests that as the years of experience increases, the students are may likely perform very well. In addition, when class sizes grew larger, students' performance declined across the board. On this note, it can be deduced that when the class size keep increase, learning process becomes difficult which in turn affects the performance of the students. The findings of comparing the fixed and random effects models indicated that the fixed effects model best fit the data. On the basis of these findings, proposals for the government and educational stakeholders on how to address this dreadful situation were developed.

Adepoju and Oluchukwu (2011) conducted a researcher on a study of secondary school students' academic performance at the senior school certificate examinations and implications for educational planning and policy in Nigeria. Between 2005 and 2007, this study assessed and investigated secondary school students' academic performance in two main subjects (English Language and Mathematics) at the Senior School Certificate Examinations (SSCE) in ten secondary schools in five randomised Local Government Areas of Oyo State, Nigeria. A descriptive survey research design was used in this study. The study's data was gathered using a tool called the Students' Academic Performance in English Language and Mathematics Questionnaire (SAPEMQ). The ten secondary schools that

participated were chosen using a basic random sampling technique and statistical techniques employed were such as Percentages, mean scores, and multiple regression which were used to analyze the data (backward procedure). The study was guided by four research questions and one null hypothesis. The results revealed, among other things, that students in urban and rural schools performed significantly differently on the SSCE, with impressive mean scores obtained in urban schools (Urban = 69.8, 54.4, and 60.2 in 2005, 2006, and 2007 respectively; Rural = 36.4, 24.9, and 23.8 in 2005, 2006, and 2007). The findings were reviewed in terms of their significance for educational planning and policy in Nigeria.

Ali and Bisandu (2018) examined the application of Hotelling's t-squared statistic and two-way ANOVA model. This study was on the application of Hotelling's T-Squared Statistic and Two-way analysis of variance without replication on the comparison between boarding and day school student performance using the selected subjects; Mathematics, English Language, Biology and Economics, from 2014 to 2017. The paper's main goal is to see if there is a substantial difference in average performance between boarding and day school pupils taking a school test operating both systems. The data collected were presented in tabular form. The data were further analyzed using the above mentioned statistical tools. According to Hotelling's T-Squared test, there is a considerable difference in average performance between boarding and day students. The two-way analysis of variance, on the other hand, shows a significant variation in student performance across subjects but provides little evidence to support the conclusion that there is significant difference between the years under study.

From the literature reviewed on analysis of students' performance in WAEC, it is of great importance that schools, both Public and Private should monitor their teachers to make sure that students are well-taught, and teachers should use appropriate instructional resources when instructing them. Also, the public schools should try to discourage students and teachers from engaging in examination malpractice because it has been observed that students generally depend on cheating in examinations and as a result develop nonchalant attitude towards studying and reading their books which usually lead to poor performance of students in WAEC examination.

Related literatures from several scholarship study such as Christian (2015), Adepoju and Oluchukwu (2011), Oluwatoyin (2015), Ali and Bisandu (2018), Atanda (2011), Omekara and Kelechi (2012) were reviewed to support this present paper. From the researcher's observation, none of these researchers conducted their research study on the same population, study area, uses same tools, same statistical models, same period and the same title as this present study. The theory that was used in this present study was based on multivariate analysis specifically; Hotelling's T^2 Distribution propended by Hotelling (1931).

3. Materials and Methods

The researchers used secondary data extracted from the WAEC results from the schools under study for this work. The researchers personally went to the schools to collect the WAEC results from the head of the schools (Principal's office). To determine an adequate

Hotelling T^2 distribution to checkmate the students' performance in each of the subject, a yearly statistics data of a periodic range of 3 years 2018 – 2020 was used. The data used for this study can be provided on demand.

The following programmes are used to obtain the parameters which constitute the models; some of which include MINITAB (version 16.0), Microsoft Excel (2010) and SPSS (version 23.0). To facilitate data analysis, the researcher made use of Microsoft Excel (2013), MINITAB (version 16.0) and SPSS (version 23.0). Microsoft Excel 2010 and MINITAB (version 16.0) were used in estimating the parameters for covariance matrix, correlation matrix, Hotelling T^2 statistics and F-value. SPSS (version 23.0) was used to determine the parameters estimates for paired t-test and goodness of fit parameters.

3.1 Method of Data Analysis and Model Specification

3.1.1 Mean Vectors

If $x_1, x_2, ..., x_n \neg \text{Np}(\mu, \Sigma)$, with the samples independently drawn from two or more multivariate normal distribution with same mean, where

$$\boldsymbol{X}_i = \begin{bmatrix} x_{i1} \\ x_{i2} \\ \vdots \\ x_{ip} \end{bmatrix} \tag{3.1}$$

where

x_1 = Students' score in Mathematics

x_2 = Students' score in English Language

x_3 = Students' score in Marketing

x_4 = Students' score in Economics

x_5 = Students' score in Civic Education and

x_6 = Students' score in Biology

The sample mean vector $\bar{x}$ can be found either as the average of the n observation vectors or by calculating the average of each of the ρ values separately:

$$\bar{\boldsymbol{X}}_1 = \frac{1}{n_1}\sum_{i=1}^{n_1} \boldsymbol{X}_{i1} = \begin{bmatrix} \bar{x}_1 \\ \bar{x}_2 \\ \vdots \\ \bar{x}_p \end{bmatrix} \tag{3.2}$$

Where, for example,

$$\bar{X}_2 = \frac{1}{n_2}\sum_{i=1}^{n_2} X_{i2} = \begin{bmatrix} \bar{x}_1 \\ \bar{x}_2 \\ \vdots \\ \bar{x}_p \end{bmatrix} \text{ and so on} \tag{3.3}$$

Again, the mean of *x* over all possible values in the population is called *population mean vector* or the expected value of *x.* It is defined as a vector of expected values of each variable,

$$\mathrm{E}(x) = \mathrm{E}\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_p \end{bmatrix} = \begin{bmatrix} \mathrm{E}(x_1) \\ \mathrm{E}(x_2) \\ \vdots \\ \mathrm{E}(x_p) \end{bmatrix} = \begin{bmatrix} \mu_1 \\ \mu_2 \\ \vdots \\ \mu_p \end{bmatrix} = \mu \tag{3.4}$$

Where μ_j is the population mean of the j[th] variable. Therefore, we say that $\bar{x}$ is an unbiased estimator of μ.

3.1.2 Covariance Matrix

The sample covariance matrix $S = (S_{jk})$ is the matrix of sample variance and covariance of the p variables:

$$S = (S_{jk}) = \begin{bmatrix} s_{11} & s_{12} & \cdots & s_{1p} \\ s_{21} & s_{22} & \cdots & s_{2p} \\ \vdots & \vdots & \vdots & \vdots \\ s_{p1} & s_{p2} & \cdots & s_{pp} \end{bmatrix} \tag{3.5}$$

To obtain *S*, we simply calculate the individual elements in S_{jk}.

$$S_{jj} = S_j^2 = \frac{1}{n-1}\sum_{i=1}^{n}\left(x_{ij} - \bar{x}_j\right)^2 \tag{3.6}$$

$$= \frac{1}{n-1}\left(\sum_{i=1}^{n} x_{ij}^2 - n\bar{x}_j^2\right) \tag{3.7}$$

The sample covariance matrix *S* can also be expressed in terms of the observation vectors:

$$S = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})(x_i - \bar{x})' \tag{3.8}$$

$$= \frac{1}{n-1}\left(\sum_{i=1}^{n} x_i x_i' - n\bar{x}\bar{x}' \right)$$

(3.9)

If x is a random vector taking on any possible value in a multivariate population the population covariance matrix is defined as

$$\Sigma = Cov(x) = \begin{bmatrix} \sigma_{11} & \sigma_{12} & \cdots & \sigma_{1p} \\ \sigma_{21} & \sigma_{22} & \cdots & \sigma_{2p} \\ \vdots & \vdots & \vdots & \vdots \\ \sigma_{p1} & \sigma_{p2} & \cdots & \sigma_{pp} \end{bmatrix} \tag{3.10}$$

The diagonal elements $\sigma_{jj} = \sigma_j^2$ are the population variance of the *x's* and the off-diagonal elements σ_{jk} are the population covariances of all possible pairs of *x's.* The population covariance matrix in (3.10) can also be found as

$$\Sigma = \mathrm{E}\left[(x_i - \mu)(x_i - \mu)' \right] \tag{3.11}$$

Since $\mathrm{E}(S_{jk}) = \sigma_{jk}$ for all *j, k,* the sample covariance matrix *S* is an unbiased estimator for $\sum$:

$$\mathrm{E}(S) = \Sigma \tag{3.12}$$

3.1.3 Correlation Matrix

The sample correlation between the j^{th} and k^{th} variables is calculated as follows:

$$r_{x_1x_2} = \frac{S_{x_1x_2}}{S_{x_1} S_{x_2}} = \frac{\sum_{i=1}^{n} (x_{i1} - \bar{x}_1)(x_{i2} - \bar{x}_2)}{\sqrt{\sum_{i=1}^{n} (x_{i1} - \bar{x}_1)^2 \sum_{i=1}^{n} (x_{i2} - \bar{x}_2)^2}} \tag{3.13}$$

Which can be further defined as

$$r_{jk} = \frac{S_{jk}}{\sqrt{S_{jj} S_{kk}}} = \frac{S_{jk}}{S_j S_k} \tag{3.14}$$

The sample correlation matrix is similar to the covariance matrix, but instead of covariances, it has correlations:

$$R = (r_{jk}) = \begin{bmatrix} 1 & r_{12} & \cdots & r_{1p} \\ r_{21} & 1 & \cdots & r_{2p} \\ \vdots & \vdots & \vdots & \vdots \\ r_{p1} & r_{p2} & \cdots & 1 \end{bmatrix} \tag{3.15}$$

Again, the population correlation of two random variables x_1 and x_2 is

$$r_{x_1x_2} = Corr(x_1, x_2) = \frac{\sigma_{x_1x_2}}{\sigma_{x_1}\sigma_{x_2}} = \frac{\mathrm{E}\left[\left(x_1 - \mu_{x_1}\right)\left(x_2 - \mu_{x_2}\right)\right]}{\sqrt{\mathrm{E}\left(x_1 - \mu_{x_1}\right)^2}\sqrt{\mathrm{E}\left(x_2 - \mu_{x_2}\right)^2}} \tag{3.16}$$

3.1.4 Quadratic Form (Q.F)

A quadratic form in p variables $X_1, X_2, \ldots, X_p$ is a homogenous function that consists of all possible second order terms.

$$Q(x) = a_{11}X_1^2 + a_{22}X_2^2 + \ldots + a_{pp}X_p^2 + a_{12}X_1X_2 + \ldots + a_{p-1}X_{p-1}X_p = \sum_i \sum_j a_{ij}X_iX_j = X^TAX \tag{3.17}$$

Note: A quadratic form is called positive definite if;

$Q(x) = X^TAX \rangle 0 \ \forall \ x \neq 0$.

It is called positive semi-definite if

$Q(x) = X^TAX \geq 0$.

where

$$A = (n-1)S \text{ as } S = \frac{A}{n-1} \tag{3.18}$$

3.1.5 Multivariate Test Statistics (Hotelling T^2 Distribution)

The Hotelling T^2 Distribution is the multivariate extension of the student distribution.

1. **One Sample Test**

Hypothesis: $H_0 : \bar{x} = \mu_0$

VS

$H_1 : \bar{x} \neq \mu_0$

Test Statistics:

$$T^2 = n(\bar{x} - \mu_0)^T S^{-1}(\bar{x} - \mu_0) \tag{3.19}$$

where :

$\bar{x}$ is the sample mean vector

μ_0 is the known population mean vector

n is the total sample size

Decision Rule:

Reject $H_0 : \bar{x} = \mu_0$ if $T^2 \geq \left[\frac{p(n-1)}{n-p} \right] F^{\alpha}_{p,(n-p)}$, otherwise accept H_0.

where;

P is the number of variables

n is the sample size and

n-p is the degree of freedom.

2. Two Sample Multivariate Test

Hypothesis: $H_0 : \bar{x}_1 = \bar{x}_2$

VS

$H_1 : \bar{x}_1 \neq \bar{x}_2$

Test Statistics:

$$T^2 = \frac{n_1 n_2}{n_1 + n_2 - 2} (\bar{x}_1 - \bar{x}_2)^T \Sigma^{-1} (\bar{x}_1 - \bar{x}_2) \tag{3.20}$$

where

$$\Sigma = \frac{(n_1 - 1)\Sigma_1 + (n_2 - 1)\Sigma_2}{n_1 + n_2 - 2} \tag{3.21}$$

Decision Rule:

Reject H_0 if $T^2 \geq \left[\frac{p(n_1 + n_2 - p - 1)}{n_1 + n_2 - 2} \right] F^{\alpha}_{p,(n_1+n_2-2)}$, otherwise accept H_0.

where;

P is the number of variables

n_1 is the sample size of the first variable

n_2 is the sample size of the second variable and

$n_1 + n_2 - 2$ is the degree of freedom.

3.1.6 Paired t-test

1. Hypotheses

The null hypothesis for a paired t-test is: H_0: □$_d$ = □$_0$ where:

□ □$_d$ = the population mean of the differences

□ □$_0$ = the hypothesized mean of the differences

You can choose from three different hypotheses:

H$_1$: □$_d$ > □$_0$ One-tailed test

H$_1$: □$_d$ < □$_0$ One-tailed test

H$_1$: □$_d$ □$_0$ Two-tailed test

2. Test Statistic

$$t = \frac{\overline{X} - \mu_0}{S_d / \left(\sqrt{n}\right)} \tag{3.22}$$

where:

μ_0 = the hypothesized population mean of the differences

$\overline{X}$ = the average of the differences between paired samples

S_d = is the sample standard deviation of the paired sample differences

n = the sample size.

3. Confidence Interval

$$\overline{X} - t_{\alpha/2}\left(S_d / \sqrt{n}\right) \text{ to } \overline{X} + t_{\alpha/2}\left(S_d / \sqrt{n}\right)$$

where:

$$S_d = \sqrt{\frac{\Sigma\left(x - \bar{x}\right)^2}{n-1}}$$

(3.23)

$\overline{X}$ = Σ X / n, where X = x$_1$ - x$_2$ and x$_1$ and x$_2$ are paired observations from populations 1 and 2, respectively

$t_{\alpha/2}$ = the inverse cumulative probability of a t distribution with n-1 degrees of freedom at 1- $\alpha/2$; α = 1 - confidence level /100

S_d = the standard deviation of the differences

n = number of pairs of values

3.2 Percentage and Data samples from WAEC results Collected

The secondary data extracted from the WAEC results from the various schools under study percentage and data sample determined in this work. The data sets are presented in tables below were used to determine the sample used for each years. The Chi-square test for each year's result can be seen in Appendix V.

YEAR	TOTAL NO OF STUDENTS	TOTAL NO. THAT WROTE MARKETING	% OF STUDENTS THAT WROTE MARKETING
2018	115	110	110/115 X 100 = 95.65%
2019	119	86	86/119 X 100% = 72.27%
2020	199	195	195/199 X 100% = 97.99%

YEAR	TOTAL NO OF STUDENTS	TOTAL NO. THAT WROTE ECONOMIC	% OF STUDENTS THAT WROTE ECONOMIC
2018	115	110	110/115 X 100 = 95.65%
2019	119	86	86/119 X 100% = 72.27%
2020	199	195	195/199 X 100% = 97.99%

YEAR	TOTAL NO OF STUDENTS	TOTAL NO. THAT WROTE BIOLOGY	% OF STUDENTS THAT WROTE BIOLOGY
2018	115	110	110/115 X 100 = 95.65%
2019	119	110	110 X 100 = 92.44%
2020	199	195	195/199 X 100% = 97.99%

YEAR	TOTAL NO OF STUDENTS	TOTAL NO. THAT WROTE ENGLISH LANGUAGE	% OF STUDENTS THAT WROTE ENGLISH LANGUAGE
2018	115	110	110/115 X 100 = 95.65%
2019	119	110	110 X 100 = 92.44%
2020	199	195	195/199 X 100% = 97.99%

YEAR	TOTAL NO OF STUDENTS	TOTAL NO. THAT WROTE MATHEMATICS	% OF STUDENTS THAT WROTE MATHEMATICS
2018	115	113	113/115 X 100% = 98.26%
2019	119	86	86/119 X 100% = 72.27%
2020	199	195	195/199 X 100% = 97.99%

YEAR	TOTAL NO OF STUDENTS	TOTAL NO. THAT WROTE CIVIC EDUCATION	% OF STUDENTS THAT WROTE CIVIC EDUCATION
2018	115	110	110/115 X 100 = 95.65%
2019	119	86	86/119 X 100% = 72.27%
2020	199	195	195/199 X 100% = 97.99%

Note: The smallest number of students that wrote each subjects was used as the sample sizes, that is $n_1 = 110$, $n_2 = 86$ and $n_3 = 195$.

4. Results

4.1 Presentation of Data

This section deals with the results for description of the variable; mean vector, covariance matrix and correlation matrix for each of the years of the six subjects, quadratic form for each of the years using their covariance matrix, Hotelling T^2 statistics, Paired t-test statistics and discussion of findings. However, the descriptive statistics of the variables and test for significant difference between the students' academic performance for each of the subjects for (2018-2019), (2019-2020) and (2018-2020) were also done.

4.2 Data Analysis

4.2.1 Mean Vectors for 2018, 2019 and 2020

$$\overline{X}_{2018} = \begin{bmatrix} 61.13 \\ 55.082 \\ 63.762 \\ 60.712 \\ 76.10 \\ 53.654 \end{bmatrix}, \quad \overline{X}_{2019} = \begin{bmatrix} 67.535 \\ 54.209 \\ 64.73 \\ 70.52 \\ 73.52 \\ 55.708 \end{bmatrix}, \quad \overline{X}_{2020} = \begin{bmatrix} 65.954 \\ 61.102 \\ 68.194 \\ 56.370 \\ 68.026 \\ 67.780 \end{bmatrix}$$

4.2.2 Covariance Matrix for 2018, 2019 and 2020

$$S_{2018} = \begin{bmatrix} 136.63 & 30.934 & 21.591 & -0.435 & 3.374 & 21.898 \\ 30.934 & 60.515 & 11.544 & 5.9344 & -3.454 & 11.905 \\ 21.591 & 11.544 & 58.606 & -5.955 & -8.38918 & 14.929 \\ -0.435 & 5.9344 & -5.955 & 37.062 & -5.734 & 0.9272 \\ 3.374 & -3.454 & -8.38918 & -5.734 & 117.04 & -1.775 \\ 21.898 & 11.905 & 14.929 & 0.9272 & -1.775 & 48.411 \end{bmatrix}$$

$$S_{2019} = \begin{bmatrix} 18.063 & 3.640 & 5.450 & 5.797 & -10.760 & -1.583 \\ 3.640 & 29.626 & 1.0395 & 6.361 & 5.191 & 0.087 \\ 5.450 & 1.0395 & 131.60 & 0.241 & 8.078 & 1.108 \\ 5.797 & 6.361 & 0.241 & 82.90 & 16.82 & 4.166 \\ -10.760 & 5.191 & 8.078 & 16.82 & 189.51 & -16.81 \\ -1.583 & 0.087 & 1.108 & 4.166 & -16.81 & 23.741 \end{bmatrix}$$

$$S_{2020} = \begin{bmatrix} 66.618 & 11.087 & -4.534 & 2.7426 & 30.043 & 3.2461 \\ 11.087 & 29.959 & -2.471 & -3.9949 & 14.6725 & -0.359 \\ -4.534 & -2.471 & 61.928 & 1.4997 & -4.986 & -4.741 \\ 2.7426 & -3.9949 & 1.4997 & 45.349 & 7.2702 & -1.0428 \\ 30.043 & 14.6725 & -4.986 & 7.2702 & 134.025 & 5.4347 \\ 3.2461 & -0.359 & -4.741 & -1.0428 & 5.4347 & 20.042 \end{bmatrix}$$

4.2.3 Correlation Matrix for 2018, 2019 and 2020

$$R_{2018} = \begin{bmatrix} 1 & 0.34017 & 0.24128 & -0.00611 & 0.02667 & 0.26922 \\ 0.34017 & 1 & 0.19386 & 0.12531 & -0.04104 & 0.21995 \\ 0.24128 & 0.19386 & 1 & -0.12778 & -0.10129 & 0.28029 \\ -0.00611 & 0.12531 & -0.12778 & 1 & -0.08705 & 0.02189 \\ 0.02667 & -0.04104 & -0.10129 & -0.08705 & 1 & -0.02358 \\ 0.26922 & 0.21995 & 0.28029 & 0.02189 & -0.02358 & 1 \end{bmatrix}$$

$$R_{2019} = \begin{bmatrix} 1 & 0.157 & 0.112 & 0.15 & -0.184 & -0.076 \\ 0.157 & 1 & 0.017 & 0.128 & 0.069 & 0.003 \\ 0.112 & 0.017 & 1 & 0.002 & 0.051 & 0.02 \\ 0.15 & 0.128 & 0.002 & 1 & 0.134 & 0.094 \\ -0.184 & 0.069 & 0.051 & 0.134 & 1 & -0.25 \\ -0.076 & 0.003 & 0.02 & 0.094 & -0.25 & 1 \end{bmatrix}$$

$$R_{2020} = \begin{bmatrix} 1 & 0.248 & -0.07 & 0.05 & 0.318 & 0.089 \\ 0.248 & 1 & -0.06 & -0.11 & 0.232 & -0.01 \\ -0.07 & -0.06 & 1 & 0.028 & -0.05 & -0.13 \\ 0.05 & -0.11 & 0.028 & 1 & 0.093 & -0.03 \\ 0.318 & 0.232 & -0.05 & 0.093 & 1 & 0.105 \\ 0.089 & -0.01 & -0.13 & -0.03 & 0.105 & 1 \end{bmatrix}$$

4.2.4 Quadratic Form for 2018, 2019 and 2020 to show the Homogenous Function

1. Quadratic Form for 2018

$$X_{2018} = \begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \\ X_5 \\ X_6 \end{bmatrix}, \quad n=110 \qquad X^{T}{}_{2018} = [X_1, X_2, X_3, X_4, X_5, X_6]$$

$$S_{2018} = \begin{bmatrix} 136.63 & 30.934 & 21.591 & -0.435 & 3.374 & 21.898 \\ 30.934 & 60.515 & 11.544 & 5.9344 & -3.454 & 11.905 \\ 21.591 & 11.544 & 58.606 & -5.955 & -8.38918 & 14.929 \\ -0.435 & 5.9344 & -5.955 & 37.062 & -5.734 & 0.9272 \\ 3.374 & -3.454 & -8.38918 & -5.734 & 117.04 & -1.775 \\ 21.898 & 11.905 & 14.929 & 0.9272 & -1.775 & 48.411 \end{bmatrix}$$

$$Q(x) = a_{11}X_1^2 + a_{22}X_2^2 + \ldots + a_{pp}X_p^2 + a_{12}X_1X_2 + \ldots + a_{p-1}X_{p-1}X_p = \sum_i \sum_j a_{ij}X_iX_j = X^TAX$$

and p = 6; **t**hen,

$$X^TAX = 136.6X_1^2 + 60.5X_2^2 + \ldots + 48.4X_6^2 + 61.9X_1X_2 + 43.2X_1X_3 - 0.87X_1X_4 + \ldots - 3.55X_5X_6$$

2. Quadratic Form for 2019

$$X_{2019} = \begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \\ X_5 \\ X_6 \end{bmatrix}, \quad n=86 \qquad X^{T}{}_{2019} = [X_1, X_2, X_3, X_4, X_5, X_6]$$

$$S_{2019} = \begin{bmatrix} 18.063 & 3.640 & 5.450 & 5.797 & -10.760 & -1.583 \\ 3.640 & 29.626 & 1.0395 & 6.361 & 5.191 & 0.087 \\ 5.450 & 1.0395 & 131.60 & 0.241 & 8.078 & 1.108 \\ 5.797 & 6.361 & 0.241 & 82.90 & 16.82 & 4.166 \\ -10.760 & 5.191 & 8.078 & 16.82 & 189.51 & -16.81 \\ -1.583 & 0.087 & 1.108 & 4.166 & -16.81 & 23.741 \end{bmatrix}$$

Similarly, the Quadratic Form for 2019

$$X^T AX = 18.1X_1^2 + 29.6X_2^2 + ... + 23.7X_6^2 + 7.3X_1X_2 + 10.9X_1X_3 + 11.8X_1X_4 + ... - 33.6X_5X_6$$

3. Quadratic Form for 2020

$$X_{2020} = \begin{bmatrix} 12927 \\ 11976 \\ 11934 \\ 9898 \\ 11891 \\ 8337 \end{bmatrix}, \quad n=196$$

$$S_{2020} = \begin{bmatrix} 66.618 & 11.087 & -4.534 & 2.7426 & 30.043 & 3.2461 \\ 11.087 & 29.959 & -2.471 & -3.9949 & 14.6725 & -0.359 \\ -4.534 & -2.471 & 61.928 & 1.4997 & -4.986 & -4.741 \\ 2.7426 & -3.9949 & 1.4997 & 45.349 & 7.2702 & -1.0428 \\ 30.043 & 14.6725 & -4.986 & 7.2702 & 134.025 & 5.4347 \\ 3.2461 & -0.359 & -4.741 & -1.0428 & 5.4347 & 20.042 \end{bmatrix}$$

Likewise, the Quadratic Form for 2020

$$X^T AX = 66.6X_1^2 + 29.96X_2^2 + ... + 20.04X_6^2 + 22.1X_1X_2 - 9.1X_1X_3 + 5.5X_1X_4 + ... + 10.86X_5X_6$$

Note: the quadratic form obtained for each years can be used to determine if $Q(x) = X^T AX > 0, \ \forall\ x \neq 0$ that is the covariance matrix is positive definite or positive semi-definite or not.

4.2.5 Multivariate Test Statistics (Hotelling T^2 Distribution)

1. Hotelling (T^2) Statistics Summary of the Significant Difference Between the Students' Academic Performance for 2018 and 2019

Hypothesis 1: $H_0 : \bar{x}_{2018} = \bar{x}_{2019}$

VS

$H_1 : \bar{x}_{2018} \neq \bar{x}_{2019}$

Test Statistics:

$$T^2 = \frac{n_1 n_2}{n_1 + n_2 - 2}(\bar{x}_{2018} - \bar{x}_{2019})^T \Sigma^{-1}(\bar{x}_{2018} - \bar{x}_{2019})$$

n_1=110, n_2=86,

$$\bar{X}_{2018} = \begin{bmatrix} 61.13 \\ 55.082 \\ 63.762 \\ 60.712 \\ 76.10 \\ 53.654 \end{bmatrix}, \quad \bar{X}_{2019} = \begin{bmatrix} 67.535 \\ 54.209 \\ 64.73 \\ 70.52 \\ 73.52 \\ 55.708 \end{bmatrix},$$

$$(\bar{x}_{2018} - \bar{x}_{2019}) = \begin{bmatrix} 61.13 \\ 55.082 \\ 63.762 \\ 60.712 \\ 76.1 \\ 53.654 \end{bmatrix} - \begin{bmatrix} 67.535 \\ 54.209 \\ 64.73 \\ 70.52 \\ 73.52 \\ 55.708 \end{bmatrix} = \begin{bmatrix} -6.405 \\ 0.873 \\ -0.968 \\ -9.808 \\ 2.58 \\ -2.054 \end{bmatrix}$$

$$(\bar{x}_{2018} - \bar{x}_{2019})^T = -6.405 \quad 0.873 \quad -0.968 \quad -9.808 \quad 2.58 \quad -2.054$$

$$\Sigma = \frac{(n_1 - 1)\Sigma_1 + (n_2 - 1)\Sigma_2}{n_1 + n_2 - 2}$$

$$\Sigma = \frac{\begin{bmatrix} 14892.67 & 3371.806 & 2353.419 & -47.415 & 367.766 & 2386.882 \\ 3371.806 & 6596.244 & 1258.296 & 646.8496 & -376.486 & 1297.645 \\ 2353.419 & 1258.296 & 6388.054 & -649.095 & -914.421 & 1627.261 \\ -47.415 & 646.8496 & -649.095 & 4039.758 & -625.006 & 101.0648 \\ 367.766 & -376.486 & -914.42062 & -625.006 & 12757.36 & -193.475 \\ 2386.882 & 1297.645 & 1627.261 & 101.0648 & -193.475 & 5276.799 \end{bmatrix} + \begin{bmatrix} 1535.355 & 309.4 & 463.25 & 492.745 & -914.6 & -134.555 \\ 309.4 & 2518.21 & 88.3575 & 540.685 & 441.235 & 7.395 \\ 463.25 & 88.3575 & 11186 & 20.485 & 686.63 & 94.18 \\ 492.745 & 540.685 & 20.485 & 7046.5 & 1429.7 & 354.11 \\ -914.6 & 441.235 & 686.63 & 1429.71 & 5853.35 & -1428.85 \\ -134.555 & 7.395 & 94.18 & 354.11 & -1428.85 & 2017.985 \end{bmatrix}}{194}$$

$$\Sigma = \frac{\begin{bmatrix} 16428.03 & 3681.206 & 2816.669 & 445.33 & -546.834 & 2252.327 \\ 3681.206 & 9114.454 & 1346.654 & 1187.535 & 64.749 & 1305.04 \\ 2816.669 & 1346.654 & 17574.05 & -628.61 & -227.791 & 1721.441 \\ 445.33 & 1187.535 & -628.61 & 11086.26 & 804.69 & 455.1748 \\ -546.834 & 64.749 & -227.791804.694 & 28610.71 & -1622.33 & \\ 2252.327 & 1305.04 & 1721.441455.1748 & -1622.325 & 7294.784 & \end{bmatrix}}{194}$$

$$\Sigma = \begin{bmatrix} 84.68054 & 18.97529 & 14.51891 & 2.295515 & -2.8187 & 11.609 \\ 18.97529 & 46.98172 & 6.941513 & 6.121312 & 0.334 & 6.727 \\ 14.51891 & 6.941513 & 90.58791 & -3.24026 & -1.174 & 8.873 \\ 2.295515 & 6.121312 & -3.24026 & 57.14566 & 4.1479 & 2.346 \\ -2.81873 & 0.333758 & -1.17418 & 4.147907 & 147.478 & -8.363 \\ 11.60993 & 6.72701 & 8.873407 & 2.346262 & -8.3625 & 37.601 \end{bmatrix}$$

$$\Sigma^{-1} = \begin{bmatrix} 0.013558 & -0.00483 & -0.00151 & 1.42E-06 & 9.101E-05 & -0.00294 \\ -0.00483 & 0.024031 & -0.00091 & -0.00231 & -0.00023 & -0.0025 \\ -0.00151 & -0.00091 & 0.011596 & 0.000911 & -8.392E-05 & -0.00218 \\ 1.42E-06 & -0.00231 & 0.000911 & 0.017881 & -0.00055 & -0.00104 \\ 9.1E-05 & -0.00023 & -8.4E-05 & -0.00055 & 0.00689 & 0.001599 \\ -0.00294 & -0.0025 & -0.00218 & -0.00104 & 0.00160 & 0.028886 \end{bmatrix}$$

$$\Sigma^{-1}(\bar{x}_{2018} - \bar{x}_{2019}) = \begin{bmatrix} -0.08332 & -0.08332 & -0.08332 & -0.08332 & -0.0833 & -0.08332 \\ 0.080022 & 0.080022 & 0.080022 & 0.080022 & 0.08002 & 0.080022 \\ -0.007 & -0.007 & -0.007 & -0.007 & -0.00699 & -0.007 \\ -0.17757 & -0.17757 & -0.17757 & -0.17757 & -0.17757 & -0.17757 \\ 0.01917 & 0.01917 & 0.01917 & 0.01917 & 0.01917 & 0.019172 \\ -0.02622 & -0.02622 & -0.02622 & -0.02622 & -0.026219 & -0.02622 \end{bmatrix}$$

$$\frac{n_1 n_2}{n_1+n_2-2}=\frac{110*86}{110+86-2}=\frac{9460}{194}=48.7629$$

$$T^2_{cal.}=119.7237$$

$$T^2_{Crit.}=\left[\frac{p(n_1+n_2-p-1)}{n_1+n_2-2}\right]F^{\alpha}_{p,(n_1+n_2-2)}=\left[\frac{6(110+86-6-1)}{110+86-2}\right]*F^{0.05}_{6,194}=5.845361*2.14$$

$$T^2_{Crit.}=12.50907$$

Decision Rule:

Since the calculated T^2 of 119.7237 is greater than the critical T^2 of 12.50907, we reject the null hypothesis at 0.05 level of significance. This implies that there is significant difference between the students' academic performance in WAEC for 2018 and 2019.

1. **Hotelling (T^2) Statistics Summary of the Significant Difference Between the Students' Academic Performance for 2019 and 2020**

Hypothesis 2: $H_0: \bar{x}_{2019}=\bar{x}_{2020}$

VS

$H_1: \bar{x}_{2019}\neq\bar{x}_{2020}$

Test Statistics:

$$T^2=\frac{n_1 n_2}{n_1+n_2-2}(\bar{x}_{2019}-\bar{x}_{2020})^T\Sigma^{-1}(\bar{x}_{2019}-\bar{x}_{2020})$$

n_1=86, n_2=196,

$$\bar{X}_{2019}=\begin{bmatrix}67.535\\54.209\\64.73\\70.52\\73.52\\55.708\end{bmatrix},\quad \bar{X}_{2020}=\begin{bmatrix}65.954\\61.102\\68.194\\56.370\\68.026\\67.780\end{bmatrix},$$

$$(\bar{x}_{2019} - \bar{x}_{2020}) = \begin{bmatrix} 67.535 \\ 54.209 \\ 64.73 \\ 70.52 \\ 73.52 \\ 55.708 \end{bmatrix} - \begin{bmatrix} 65.954 \\ 61.102 \\ 68.194 \\ 56.370 \\ 68.026 \\ 67.780 \end{bmatrix} = \begin{bmatrix} 1.581 \\ -6.893 \\ -3.464 \\ 14.15 \\ 5.494 \\ -12.072 \end{bmatrix}$$

$$(\bar{x}_{2019} - \bar{x}_{2020})^T = 1.581 \text{ -6.893 } \text{-3.464 } 14.15 \quad 5.494 \quad \text{-12.072}$$

$$\Sigma = \frac{(n_1 - 1)\Sigma_1 + (n_2 - 1)\Sigma_2}{n_1 + n_2 - 2}$$

$$\Sigma = \frac{\begin{bmatrix} 1535.355 & 309.4 & 463.25 & 492.745 & -914.6 & -134.555 \\ 309.4 & 2518.21 & 88.3575 & 540.685 & 441.235 & 7.395 \\ 463.25 & 88.3575 & 11186 & 20.485 & 686.63 & 94.18 \\ 492.745 & 540.685 & 20.485 & 7046.5 & 1429.7 & 354.11 \\ -914.6 & 441.235 & 686.63 & 1429.71 & 5853.35 & -1428.85 \\ -134.555 & 7.395 & 94.18 & 354.11 & -1428.85 & 2017.985 \end{bmatrix} + \begin{bmatrix} 12990.51 & 2161.965 & -884.13 & 534.807 & 5858.385 & 632.9895 \\ 2161.965 & 5842.005 & -481.845 & -779.006 & 2861.138 & -70.005 \\ -884.13 & -481.845 & 12075.96 & 292.4415 & -972.27 & -924.495 \\ 534.807 & -779.006 & 292.4415 & 8843.055 & 1417.689 & -203.346 \\ 5858.385 & 2861.138 & -972.27 & 1417.689 & 26134.88 & 1059.767 \\ 632.9895 & -70.005 & -924.495 & -203.346 & 1059.767 & 3908.19 \end{bmatrix}}{280}$$

$$\Sigma = \frac{\begin{bmatrix} 14525.87 & 2471.365 & -420.88 & 1027.552 & 4943.785 & 498.4345 \\ 2471.365 & 8360.215 & -393.4875 & -238.321 & 3302.373 & -62.61 \\ -420.88 & -393.488 & 23261.96 & 312.9265 & -285.64 & -830.315 \\ 1027.552 & -238.321 & 312.9265 & 15889.56 & 2847.389 & 150.764 \\ 4943.785 & 3302.373 & -285.64 & 2847.389 & 41988.23 & -369.084 \\ 498.4345 & -62.61 & -830.315 & 150.764 & -369.084 & 5926.175 \end{bmatrix}}{280}$$

$$\Sigma = \begin{bmatrix} 51.878 & 8.826 & -1.503 & 3.670 & 17.656 & 1.780 \\ 8.826 & 29.858 & -1.405 & -0.851 & 11.794 & -0.224 \\ -1.503 & -1.405 & 83.078 & 1.118 & -1.020 & -2.965 \\ 3.670 & -0.851 & 1.118 & 56.748 & 10.169 & 0.538 \\ 17.656 & 11.794 & -1.020 & 10.169 & 149.958 & -1.318 \\ 1.780 & -0.224 & -2.965 & 0.538 & -1.318 & 21.165 \end{bmatrix}$$

$$\Sigma^{-1} = \begin{bmatrix} 0.0210 & -0.0055 & 0.00021 & -0.0012 & -0.0020 & -0.0019 \\ -0.0055 & 0.0361 & 0.00049 & 0.00129 & -0.0023 & 0.00074 \\ 0.00021 & 0.00049 & 0.01211 & -0.0003 & 5.2E-05 & 0.0017 \\ -0.0011 & 0.0013 & -0.00027 & 0.0179 & -0.0012 & -0.0005 \\ -0.0020 & -0.0023 & 5.21E-05 & -0.0012 & 0.00717 & 0.00063 \\ -0.0019 & 0.00074 & 0.00169 & -0.0005 & 0.00063 & 0.04770 \end{bmatrix}$$

$$\Sigma^{-1}(\bar{x}_{2019} - \bar{x}_{2020}) = \begin{bmatrix} 0.066963 & 0.066963 & 0.06696336 & 0.066963 & 0.066963 & 0.066963 \\ -0.26215 & -0.26215 & -0.2621548\,8 & -0.26215 & -0.26215 & -0.26215 \\ -0.06902 & -0.06902 & -0.0690206 & -0.06902 & -0.06902 & -0.06902 \\ 0.243195 & 0.243195 & 0.24319537\,4 & 0.243195 & 0.243195 & 0.243195 \\ 0.027197 & 0.027197 & 0.02719734\,2 & 0.027197 & 0.027197 & 0.027197 \\ -0.59294 & -0.59294 & -0.5929433\,8 & -0.59294 & -0.59294 & -0.59294 \end{bmatrix}$$

$$\frac{n_1 n_2}{n_1 + n_2 - 2} = \frac{86*196}{86+196-2} = \frac{16856}{280} = 60.20$$

$$T^2_{cal.} = 776.618$$

$$T^2_{Crit.} = \left\lfloor \frac{p(n_1 + n_2 - p - 1)}{n_1 + n_2 - 2} \right\rfloor F^{\alpha}_{p,(n_1+n_2-2)} = \left\lfloor \frac{6(86+196-6-1)}{86+196-2} \right\rfloor * F^{0.05}_{6,280} = 5.89286 * 2.13$$

$$T^2_{Crit.} = 12.5518$$

Decision Rule:

Since the calculated T^2 of 776.618 is greater than the critical T^2 of 12.5518, we reject the null hypothesis at 0.05 level of significance. This implies that there is significant difference between the students' academic performance in WAEC for 2019 and 2020.

4.2.5.3 Hotelling (T^2) Statistics Summary of the Significant Difference Between the Students' Academic Performance for 2018 and 2020

Hypothesis 3: $H_0 : \bar{x}_{2018} = \bar{x}_{2020}$

VS

$H_1 : \bar{x}_{2018} \neq \bar{x}_{2020}$

Test Statistics:

$$T^2 = \frac{n_1 n_2}{n_1 + n_2 - 2}(\bar{x}_{2018} - \bar{x}_{2020})^T \Sigma^{-1}(\bar{x}_{2018} - \bar{x}_{2020})$$

n_1=110, n_2=196,

$$\bar{X}_{2018} = \begin{bmatrix} 61.13 \\ 55.082 \\ 63.762 \\ 60.712 \\ 76.10 \\ 53.654 \end{bmatrix}, \quad \bar{X}_{2020} = \begin{bmatrix} 65.954 \\ 61.102 \\ 68.194 \\ 56.370 \\ 68.026 \\ 67.780 \end{bmatrix},$$

$$(\bar{x}_{2018} - \bar{x}_{2020}) = \begin{bmatrix} 61.13 \\ 55.082 \\ 63.762 \\ 60.712 \\ 76.10 \\ 53.654 \end{bmatrix} - \begin{bmatrix} 65.954 \\ 61.102 \\ 68.194 \\ 56.370 \\ 68.026 \\ 67.780 \end{bmatrix} = \begin{bmatrix} -4.824 \\ -6.02 \\ -4.432 \\ 4.342 \\ 8.074 \\ -14.126 \end{bmatrix}$$

$$(\bar{x}_{2018} - \bar{x}_{2020})^T = -4.824 \quad -6.02 \quad -4.432 \quad 4.342 \quad 8.074 \quad -14.126$$

$$\Sigma = \frac{(n_1 - 1)\Sigma_1 + (n_2 - 1)\Sigma_2}{n_1 + n_2 - 2}$$

$$\Sigma = \frac{\begin{vmatrix} 14892.67 & 3371.806 & 2353.419 & -47.415 & 367.766 & 2386.882 \\ 3371.806 & 6596.244 & 1258.296 & 646.8496 & -376.486 & 1297.645 \\ 2353.419 & 1258.296 & 6388.054 & -649.095 & -914.421 & 1627.261 \\ -47.415 & 646.8496 & -649.095 & 4039.758 & -625.006 & 101.0648 \\ 367.766 & -376.486 & -914.42062 & -625.006 & 12757.36 & -193.475 \\ 2386.882 & 1297.645 & 1627.261 & 101.0648 & -193.475 & 5276.799 \end{vmatrix} + \begin{vmatrix} 12990.51 & 2161.965 & -884.13 & 534.807 & 5858.385 & 632.9895 \\ 2161.965 & 5842.005 & -481.845 & -779.006 & 2861.138 & -70.005 \\ -884.13 & -481.845 & 12075.96 & 292.4415 & -972.27 & -924.495 \\ 534.807 & -779.006 & 292.4415 & 8843.055 & 1417.689 & -203.346 \\ 5858.385 & 2861.138 & -972.27 & 1417.689 & 26134.88 & 1059.767 \\ 632.9895 & -70.005 & -924.495 & -203.346 & 1059.767 & 3908.19 \end{vmatrix}}{304}$$

$$\Sigma = \frac{\begin{vmatrix} 27883.18 & 5533.771 & 1469.289 & 487.392 & 6226.151 & 3019.872 \\ 5533.771 & 12438.25 & 776.451 & -132.156 & 2484.652 & 1227.64 \\ 1469.289 & 776.451 & 18464.014 & -356.654 & -1886.69 & 702.766 \\ 487.392 & -132.156 & -356.6535 & 12882.81 & 792.683 & -102.281 \\ 6226.151 & 2484.652 & -1886.691 & 792.683 & 38892.24 & 866.2915 \\ 3019.872 & 1227.64 & 702.766 & -102.281 & 866.2915 & 9184.989 \end{vmatrix}}{304}$$

$$\Sigma = \begin{vmatrix} 91.72099 & 18.20319 & 4.8331875 & 1.603263 & 20.48076 & 9.933788 \\ 18.20319 & 40.91529 & 2.554115132 & -0.43472 & 8.173196 & 4.038289 \\ 4.833188 & 2.554115 & 60.73688816 & -1.1732 & -6.20622 & 2.31173 \\ 1.603263 & -0.43472 & -1.1732023 & 42.37767 & 2.60751 & -0.33645 \\ 20.48076 & 8.173196 & -6.20621914 & 2.60751 & 127.935 & 2.849643 \\ 9.933788 & 4.038289 & 2.311730263 & -0.33645 & 2.849643 & 30.21378 \end{vmatrix}$$

$$\Sigma^{-1} = \begin{vmatrix} 0.012663 & -0.00493 & -0.00085531 & -0.00048 & -0.00167 & -0.00329 \\ -0.00493 & 0.027056 & -0.00076276 & 0.000486 & -0.00095 & -0.00184 \\ -0.00086 & -0.00076 & 0.016713387 & 0.000417 & 0.00101 & -0.00099 \\ -0.00048 & 0.000486 & 0.000417266 & 0.023661 & -0.00042 & 0.000363 \\ -0.00167 & -0.00095 & 0.001009895 & -0.00042 & 0.008206 & -0.00018 \\ -0.00329 & -0.00184 & -0.00098623 & 0.000363 & -0.00018 & 0.034521 \end{vmatrix}$$

$$\Sigma^{-1}(\bar{x}_{2018} - \bar{x}_{2020}) = \begin{vmatrix} 0.003239 & 0.003239 & 0.003238982 & 0.003239 & 0.003239 & 0.003239 \\ -0.11521 & -0.11521 & -0.11521415 & -0.11521 & -0.11521 & -0.11521 \\ -0.04146 & -0.04146 & -0.04145883 & -0.04146 & -0.04146 & -0.04146 \\ 0.091695 & 0.091695 & 0.091694821 & 0.091695 & 0.091695 & 0.091695 \\ 0.076234 & 0.076234 & 0.076233516 & 0.076234 & 0.076234 & 0.076234 \\ -0.4562 & -0.4562 & -0.45619758 & -0.4562 & -0.4562 & -0.4562 \end{vmatrix}$$

$$\frac{n_1 n_2}{n_1 + n_2 - 2} = \frac{110 * 196}{110 + 196 - 2} = \frac{21560}{304} = 70.9211$$

$$T^2_{cal.} = 590.035$$

$$T^2_{Crit.} = \left\lfloor \frac{p(n_1 + n_2 - p - 1)}{n_1 + n_2 - 2} \right\rfloor F^{\alpha}_{p,(n_1+n_2-2)} = \left\lfloor \frac{6(110 + 196 - 6 - 1)}{110 + 196 - 2} \right\rfloor * F^{0.05}_{6,304} = 5.90132 * 2.12$$

$$T^2_{Crit.} = 12.5108$$

Decision Rule:

Since the calculated T^2 of 590.035 is greater than the critical T^2 of 12.5108, we reject the null hypothesis at 0.05 level of significance. This implies that there is significant difference between the students' academic performance in WAEC for 2018 and 2020.

Table 1: Comparison of the Hotelling (T^2) Statistics Result for the Three Years

Variable	2018-2019	2019-2020	2018-2020
$T^2_{cal.}$	119.724	776.618	590.035
$F^{\alpha}_{p,(n_1+n_2-2)}$	2.14	2.13	2.12
$T^2_{Crit.}$	12.5091	12.5518	12.5108

Table 4.1 shows the results summarized of the hotelling T^2 statistics between the students' academic performance for (2018 & 2019), (2019 & 2020) and (2018 & 2020) are significant. Next, to determine the significant difference between subjects, we applied Paired t-test statistics in the section below.

4.2.6 Paired t-test Analysis

Table 2: Paired t-test Analysis to Determine the Difference between Subjects in WASSCE for; 2018 and 2019

		Mean	Std. Deviation	Std. Error Mean
Pair 1	MATHS 2018	60.8721	12.02526	1.29672
	MATHS 2019	67.5349	4.25011	.45830
Pair 2	ENGLISH 2018	54.4070	8.03870	.86684
	ENGLISH 2019	54.2093	5.44300	.58693
Pair 3	MARKETING 2018	63.6543	7.57654	.84184
	MARKETING 2019	64.7284	11.47172	1.27464
Pair 4	ECONS 2018	60.6812	6.07670	.73155
	ECONS 2019	70.5217	9.10496	1.09611
Pair 5	C. EDU 2018	76.5176	11.12442	1.20661
	C.EDU 2019	73.5176	13.76643	1.49318
Pair 6	BIOLOGY 2018	53.0154	7.05003	.87445
	BIOLOGY 2019	55.7077	4.87251	.60436

Table 3: Summary of the Paired Samples Test between Subjects in WASSCE for 2018 and 2019

		Paired Differences							
					95% Confidence Interval of the Difference				
		Mean	Std. Deviation	Std. Error Mean	Lower	Upper	t	df	Sig. (2-tailed)
Pair 1	MATHS 2018 – MATHS 2019	-6.66	12.595	1.358	-9.363	-3.962	-4.906	85	0.000**
Pair 2	ENGLISH 2018 – ENGLISH 2019	0.198	10.197	1.099	-1.989	2.384	.180	85	0.858
Pair 3	MARKET 2018– MARKET 2019	-1.074	14.088	1.565	-4.189	2.041	-.686	80	0.495
Pair 4	ECONS 2018 – ECONS 2019	-9.841	11.636	1.401	-12.636	-7.045	-7.025	68	0.000**
Pair 5	C.EDU 2018 – C.EDU 2019	3.000	18.444	2.001	-.978	6.978	1.500	84	0.137
Pair 6	BIOLOGY 2018 – BIOLOGY 2019	-2.692	8.132	1.009	-4.707	-0.677	-2.669	64	0.010**

Footnote: p-value **= sig. at 5%.

Table 3 shows that three subjects (Mathematics, Economics and Biology) are significant. It implies a decrease in the Students' average performance in those subjects, since the mean difference between the subjects for the two years is negative.

Table 4: Paired t-test Analysis to Determine the Difference between Subjects in WASSCE for; 2019 and 2020

		Mean	Std. Deviation	Std. Error Mean
Pair 1	MATHS 2019	67.5349	4.25011	.45830
	MATHS 2020	67.0930	7.21050	.77753
Pair 2	ENGLISH 2019	54.2093	5.44300	.58693
	ENGLISH 2020	61.1047	5.85919	.63181
Pair 3	MARKETING 2019	64.7284	11.47172	1.27464
	MARKETING 2020	68.0741	7.19162	.79907

Pair 4	ECONS 2019	70.5217	9.10496	1.09611
	ECONS 2020	57.7826	6.37290	.76721
Pair 5	C. EDU 2019	73.5176	13.76643	1.49318
	C.EDU 2020	71.2588	10.57305	1.14681
Pair 6	BIOLOGY 2019	55.7077	4.87251	.60436
	BIOLOGY 2020	67.7538	4.65373	.57722

Table 5: Summary of the Paired Samples Test between Subjects in WASSCE for 2019 and 2020

		Paired Differences							
		Mean	**Std. Deviation**	**Std. Error Mean**	**95% Confidence Interval of the Difference**		**t**	**df**	**Sig. (2-tailed)**
					Lower	**Upper**			
Pair 1	MATHS 2019–MATHS 2020	0.442	8.904	0.960	-1.467	2.351	0.460	85	0.647
Pair 2	ENGLISH 2019 –ENGLISH 2020	-6.895	8.089	0.872	-8.629	-5.161	-7.905	85	0.000**
Pair 3	MARKET 2019–MARKET 2020	-3.346	13.253	1.473	-6.276	-0.415	-2.272	80	0.026**
Pair 4	ECONS 2019 –ECONS 2020	12.739	10.567	1.272	10.201	15.278	10.014	68	0.000**
Pair 5	C.EDU 2019 –C.EDU 2020	2.259	17.829	1.934	-1.587	6.104	1.168	84	0.246
Pair 6	BIOLOGY 2019 –BIOLOGY 2020	-12.046	7.423	0.921	-13.886	-10.207	-13.083	64	0.000**

Footnote: p-value **= sig. at 5%.

Table 5 shows that four subjects (English Language, Marketing, Economics and Biology) are significant. It implies a decrease in the Students' average performance for three subjects (English Language, Marketing and Biology), since the mean difference between the subjects for the two years is negative. While an increase in the Students' average performance for

Economics subject, since the mean difference between the subjects for the two years is positive.

Table 6: Paired t-test Analysis to Determine the Difference between Subjects in WASSCE for; 2018 and 2020

		Mean	Std. Deviation	Std. Error Mean
Pair 1	MATHS 2018	61.1273	11.68871	1.11447
	MATHS 2020	66.4091	7.11027	.67794
Pair 2	ENGLISH 2018	55.0818	7.77921	.74172
	ENGLISH 2020	60.9091	5.61052	.53494
Pair 3	MARKETING 2018	63.7619	7.65547	.74710
	MARKETING 2020	68.6476	7.71609	.75301
Pair 4	ECONS 2018	60.7115	6.08782	.59696
	ECONS 2020	57.4904	6.55817	.64308
Pair 5	C. EDU 2018	76.1009	10.81832	1.03621
	C.EDU 2020	71.2202	10.37211	.99347
Pair 6	BIOLOGY 2018	53.6538	6.95781	.78782
	BIOLOGY 2020	67.5641	4.46222	.50525

Table 7: Summary of the Paired Samples Test between Subjects in WASSCE for 2018 and 2020

		Paired Differences							
					95% Confidence Interval of the Difference				
		Mean	Std. Deviation	Std. Error Mean	Lower	Upper	t	df	Sig. (2-tailed)
Pair 1	MATHS 2018–MATHS 2020	-5.282	13.907	1.326	-7.910	-2.654	-3.983	109	0.000**
Pair 2	ENGLISH 2018 –ENGLISH 2020	-5.827	8.829	0.842	-7.495	-4.159	-6.923	109	0.000**

Pair 3	MARKET 2018–-4.886 MARKET 2020	11.060	1.079	-7.026	-2.745	-4.526	104	0.000**
Pair 4	ECONS 2018 –3.221 ECONS 2020	9.497	0.931	1.374	5.068	3.459	103	0.001**
Pair 5	C.EDU 2018 –4.881 C.EDU 2020	13.692	1.311	2.281	7.480	3.722	108	0.000**
Pair 6	BIOLOGY 2018 –-13.91 BIOLOGY 2020	7.889	0.893	-15.688	-12.131	-15.572	77	0.000**

Footnote: p-value **= sig. at 5%.

Table 7 shows all WASSCE subjects considered are significant. It implies a decrease in the Students' average performance for four subjects (Mathematics, English Language, Marketing and Biology), since the mean difference between the subjects for the two years is negative. While an increase in the Students' average performance for Economics and Civic Education subjects, since their mean difference positive.

5. Conclusion

A quadratic form was obtained for each year using their covariance matrix, which was used to show the homogenous function that consists of all possible second order terms. The Hotelling T^2 statistics results between the students' academic performance for (2018 & 2019), (2019 & 2020) and (2018 & 2020) are all significant. Paired t-test statistics results a decrease in the Students' average performance for four subjects (Mathematics, English Language, Marketing and Biology), while an increase in the Students' average performance for Economics and Civic Education subjects. It was discovered that students' average performance in Economics and Civic Education subjects better than other subjects.

6. Recommendation

This research recommend the effective implementation of the Nigeria education policies that emphasizes on teachers qualification, years of teaching experience and the UNESCO policy on Teacher-Students ratio (this policy stipulates that the maximum number of students that should be in a secondary class is 25), since there is significant difference between Students' average performance for four subjects.

7. Limitations of the Study

The timeframe for this study was very short, so the researchers focuses their attention on only one WAEC center since they couldn't go round several schools. This is because some of the School to be visited will require crossing of the sea to such schools, and with the activities of sea pirates, they (the researchers) could only go to one public secondary school in the educational zone that has WASSCE examination centre. Another limitation was transportation constraint; this was a serious bottleneck that tends to hinder the completion of this research paper. Thirdly, the behaviour of the school Principal in granting the researchers'

permission to access the WAEC results was another serious problem encountered in the process of completing this study.

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APPENDIX I

CONSENT LETTER

Department of Mathematics/Statistics
Ignatius Ajuru University of Education,
Rumuolumeni, Port Harcourt
March, 2022

Sir/Ma,

Request for Permission to Conduct Research Experiment and obtain Research Information/Gather Data Your School

We are Nkpordee Lekia and Ogolo Ibinabo Magnus, postgraduate students of the above institution conducting a research project on the multivariate analysis of students' academic performance in WASSCE in public senior secondary schools in Rivers State (2018-2020).

To achieve this goal, I plead for your assistance in permitting me to access necessary documents on students' WAEC results for 2018, 2019 and 2020 in order to extract relevant secondary data which will be used for this study only.

I will ensure that whatever information gotten will be treated anonymously.

Thanks for your anticipated co-operation.

Yours faithfully,

NKPORDEE Lekia and OGOLO Ibinabo Magnus

Researchers

APPENDIX II

2018 RESULT

S/N	MATHS	ENGLISH	MARKETING	ECONS	C. EDU	BIOLOGY
1	44	49		54	98	
2	48	68	78	68	59	38
3	39	45	67	60	65	49
4	53	46	65	62	77	45
5	46	45	61	65	86	47
6	49	42	52	53	66	43
7						
8	54	47	66	57		
9					66	
10	51	53	59	66		46
11	54	39	49	54	79	44
12	56	47	65	64	69	45

13	45	55	50	57	74	40
14	65	51	53	50	80	42
15	52	54	58	63	70	41
16	50	52	55	61	68	
17	60	55	56	55	89	40
18	59	58	64	67	95	
19						
20	56	56	51	70	94	49
21	54	49	59	64	79	44
22	59	59	79	69	69	54
23	69	49	54	69	95	49
24	49	44	52	54	59	48
25	79	58	59	59	96	47
26	99	69	58	62	98	54
27	54	69	66	69	95	54
28	52	40	57	61	63	52
29	45	66	64	50	79	
30	59	64	69	64	79	54
31	69	54	69	69	79	59
32	69	54	69	69	69	54
33	69	52	59	59	78	54
34	69	54	54	59	79	49
35	46	48	60	66	70	50
36	58	41	55	60	80	53
37	55	49	58	65	76	
38	67	45	50	67	73	46
39	53	51	65	68	76	58
40	67	53	61	62	72	
41	54	46	67	64	65	61
42	52	54	65	67	90	
43	65	52	66	65	96	
44	69	64	70	56	74	54
45			66			49
46	77	60	65	59	91	
47	51	50	66	56	65	52
48	79	55	70	65	90	64
49	65	63	64		73	61
50	75	59	67	60	68	53
51	61	46	65	61	81	62
52	83	57	72	69	82	67
53	66	55	68	59	73	59
54	54	52	65	48	69	
55	59	54	69		80	55
56	55	50	60		66	56
57	44	49		54	69	54

58	69	54	65	52	89	50
59	69	59	69	58	66	
60	77	55	67	59	89	54
61	54	54	69	64	78	59
62	69	59	65	53	69	52
63	50	49	66	57	80	50
64	55	54	60	55	70	
65	44	45	58	51	65	
66	64	53	77	58	67	56
67	67	51	66	56	68	53
68	59	64	68	57	91	58
69	64	50	62	59	65	
70	61	59		58	94	59
71	53	54	69		96	54
72	49	50		64	68	67
73	69	59	64	54	89	59
74	64	54	64	55	70	
75	79	50	65	57	90	
76	67	97	69	55	72	
77	50	54	54		66	64
78	98	62	60	50	69	60
79	59	60	85	69	70	63
80	60	52	50	54	65	50
81	69	64		53	67	
82	65	53	67	52	87	54
83	54	54	64		82	49
84	70	61	91	51	93	59
85	68	59		54	84	
86	53	55	65	59	78	61
87	58	63	66		68	51
88	49	54	60	58	68	50
89	69	57	69	60	65	66
90	99	65	71	65	70	64
91	59	69	54	55	67	56
92	45	50	63	69	73	
93	59	59	64	66	69	54
94	54	53	68	58	90	59
95	50	64	60	67	66	50
96	65	56	65	68	75	49
97	52	51	60	64	68	
98	51	52	55	61	59	
99	68	58	66	69	72	
100	59	54	59	62	69	
101						
102	64	59	64	69	96	

103	69	50	62	65	70	53
104	95	55	67	65	81	58
105	54	64	65	56	64	
106	50	60	70		75	54
107	59	44	59	69	69	
108	64	54	55	64	73	
109	65	69	93	69	79	57
110	69	65	69	55	90	68
111	61	62	60	54	69	
112	62	59	66	59	77	64
113	67	56	73	57	75	
114	79	53	68	68	69	53
115	69	64	54	58	96	64

Source: Office of the Principal, CCS Abonima

APPENDIX III

2019 RESULT

S/N	MATHS	ENGLISH	MARKETING	ECONS	C. EDU	BIOLOGY
1						
2						
3	68	54	50	65	66	45
4	69	58	74	67	65	
5	65	55	52	68	67	
6	64	50		65	70	52
7	60	55	39		59	57
8	69	49	39	78	68	
9	68	53	48	68	78	
10						
11						
12	68	54	68	95	98	59
13	69	49	57	68	97	64
14	77	49	78	68	78	
15	63	54	67	77	98	49
16	69	64	69	95	98	59
17	78	54	78	97	77	56
18	69	59	68	96	79	
19	59	54		79	98	58
20	68	48	49	58	78	54
21	68	49	56	57	69	53
22	60	64	67	69	97	63
23	68	53	59	69	67	53
24	67	54	53	68	69	54
25	79	67	97	79	68	54
26	68	53	68	67	67	53

27	69	38	54	68	78	54
28	68	64	68	69	97	53
29	66	53	54	67	69	64
30						
31	69	53	67	68	67	
32	67	59	64		79	54
33	69	53	59	97	67	
34						
35	69	54	69	68	67	
36	68	58	67	95	97	54
37	68	59	69	67	68	58
38	67	54	58		79	59
39	67	54	59	78	65	49
40	67	52	65	69	64	
41						
42						
43	69	54	67	73	68	53
44	68	53	68		69	54
45	67	59	77	68	69	60
46	67	58	53		65	57
47	67	53	68	69	98	47
48						
49	69	54	67	65	66	54
50						
51	67	59	64	65	69	54
52						
53						
54						
55	62	46	54	57	53	
56	69	58	48		67	55
57						
58						
59			65		95	
60	65	60	54		75	56
61	68	54		70		52
62						
63			69			
64	78	58	70	66	69	64
65	67	48		69	78	53
66						
67	67	54	95	78		
68	68	45	46	62	53	53
69	67	51	65	65	75	63
70	65	49	73	67	68	
71	69	55	68	65	70	54

72	65	50	69	71	91	58
73						
74	79	54	66	65	79	51
75	66	50	65		62	63
76	69	49	70		65	60
77						
78	66	55	72	59	65	53
79	67	59		70	68	58
80		67			65	
81						
82	68	56	66	65	70	52
83						
84						
85	69	45	69	71	74	
86	64	68		73	88	57
87	60		77	69	63	59
88						
89	65	50	67	66	69	
90	53	49	56	67	92	
91						
92						
93						
94	78	59		68	66	53
95	66	57		65	6	56
96	63	54			67	
97	67	56	90	69	65	54
98	68	50	66		79	52
99	69	54	54		69	64
100	59	44	53	79	66	49
101						
102	69	47	64		69	54
103	66	64	89	69	65	64
104	65	48	67		77	51
105	68	52	93	67	95	
106	67	54	52		79	52
107						
108	69	54	69		69	54
109	69	52	69	69	77	54
110	65	55	61	66	90	50
111						
112	69	59	69	69	69	
113	66	64	67	66	88	69
114	78	61	68	78	71	64
115						
116	67	48	38	68	60	54

117	65	53	65	70	79	50
118	69	58	66		84	63
119	69	49	59	67	68	

Source: Office of the Principal, CCS Abonima

APPENDIX IV

2020 RESULT

S/N	MATHS	ENGLISH	MARKETING	ECONS	C. EDU	BIOLOGY
1	68	60	65	54	91	
2	79	66	69	50	65	
3	70	69	66	64	68	69
4	69	59	59		69	
5	65	64		57	66	68
6	67	55	68	54	67	
7	44	54	67	49	49	
8	54	59	65	44	66	58
9	59	50	70	50	75	64
10	64	59	54	52	65	65
11	78	67	79	65	89	79
12	69	64	69	51	67	59
13	57	60	67	69	65	64
14	70	55	64	59	69	68
15	63	69	68	55	66	66
16	69	54	69	56	79	
17	79	64	55	58	70	
18	65	67		64	73	78
19	62	50	60	52	67	
20	68	59	64	61	48	
21	57	55	67	49	69	
22	69	69	66		65	69
23	67	61	78	63	74	61
24	65	64	59	55	79	
25	79	69	68	59	69	67
26	77	65	60		79	69
27	69	49	67	48	65	
28	50	67	69	64	54	68
29	59	64	79	69	68	66
30	68	59	65	65	44	
31	65	69	70	67	94	79
32	69	59	79		79	79
33	69	54	64	59	69	69
34	64	44	69		64	69
35						
36	69	69	69	69	92	69
37	69	54	69	59	79	

38	65	60		55	63	66
39	67	57	74	61	66	
40	99	63	71	68	90	69
41	69	64		50	64	65
42	69	60	60	52	69	69
43	68	49	54	67	93	59
44	63	68	65	55	87	66
45	69	69	69	54	79	69
46	65	61	65		67	
47	67	54	67	58	65	78
48	77	63		60	66	73
49	64	59	54	52	65	
50	73	67	60	64	58	68
51	67	63	66	69	82	64
52	70	55	77	61	65	68
53	69	60	54	54	70	65
54	68	66	70	50	78	67
55	65	64		58	70	66
56						
57	69	66	69	62	73	69
58	67	65	67	60	75	
59	68	59	65	63	80	69
60	79	67	69	67	69	68
61	65	50	73	50	65	
62	69	54	75	59	67	
63	66	64	69	55	59	66
64	67	59	65	67	70	68
65	79	69	72	58	69	66
66	68	58	68	62	64	
67	65	67	93	64	79	65
68	54	66	65	54	73	
69	69	64	69	50	89	69
70	59	59	64	49	69	69
71	67	63	73	51	78	
72	64	69	65	61	88	66
73	62	54	69	49	56	
74	65	65	55	56	74	67
75	64	50	79	54	65	
76	69	69		64	94	69
77	54	64	69	54	54	
78	69	60	65	43	69	67
79	64	61	76	60	86	64
80	79	59	68	50	79	
81	65	59	64	71	62	68
82	75	65	67	63	67	79
83	68	62		52	59	75
84	61	63		53	90	69
85	69	69	79	59	78	69

86	59	57	70	62	69	
87	66	65	77		74	67
88						
89	68	57		49	65	65
90	64	62	69	51	60	60
91	69	63	92	56	69	
92	65	60	66	59	65	
93	67	54		49	64	68
94	69	64	77	69	69	69
95	49	67	69	60	60	59
96	68	53	64	67	65	66
97	65	64	67	56	93	68
98	66	59	96	50	65	65
99	59	60	69	59	69	
100	70	57	67	51	91	67
101	65	64	71	56	69	69
102	56	50	77	50	65	65
103	70	59	63	54	68	66
104	53	64	69	52	79	
105	66	55	65	64	67	68
106	59	57	67	49	66	
107	64	58	69	59	70	60
108	73	69	66	66	64	
109	65	65	65	69	77	67
110	51	60	61	65	68	
111	66	58	79	59	89	71
112	69	58	79	59	89	71
113	67	65	64	50	63	65
114	72	56	67	51	78	69
115	66	63	65	63	70	
116	60	55	60	54	75	67
117	68	49	85	69	66	
118	69	69	69	60	68	
119	67	60	90	63	90	
120	64	66	65	55	69	
121	58	54	67	54	65	61
122	57	48	62	39	49	
123	95	67	60	65	93	76
124	79	66	69	59	73	
125	70	64	65	64	69	
126	79	64	67	69	55	63
127	65	60	59	61	67	69
128	64	61	55		69	64
129	69	69	69	54	85	
130	67	65	59		69	66
131	60	59	57		38	59
132	68	63	68	59	60	65
133	66	67	67	52	68	

134	54	49	56	49	49	
135	67	63	65	51	54	
136	69	61	69	59	59	68
137	65	62	66	53	65	66
138	92	69	79	65	89	70
139	67	65	69		84	
140	90	66		69	70	73
141	59	64	50	44	44	
142	50	63	64	47	40	
143	68	48	57	53	52	
144	66	61	60	55	68	67
145	59	64	68	40	54	
146	64	50	65	50	41	
147	60	54	69	54	66	
148	69	56	64	60	51	
149	50	60	67	64	53	60
150	62	53	59	51	59	
151	69	64	74	45	60	67
152	65	60	70	64	60	74
153	66	59		59	49	69
154	67	63	79	59	54	65
155	69	69		54		
156	66	61	69	50	59	70
157	68	62		51	64	66
158	54	55	73	62	45	
159	50	63	71	56	50	
160	63	66	59		55	69
161	59	68	65	54	61	75
162	67	57	68	52	65	79
163	54	61	54	50	54	65
164	57	64	64	59	69	78
165	69	55	67	55	66	67
166	65	63	91	51	84	68
167	68	62		49	62	66
168	49	50	64	44	68	
169	66	64	79	53	65	69
170	70	60	65	50	79	65
171	60	62	58	49	69	
172	54	63	69	54	66	
173	69	54	66	52	68	
174	59	68	78	55	90	66
175	65	65	77	53	65	68
176	54	59		50	69	
177	59	62	74	53	66	
178	64	57	50	56	73	69
179	90	66	69	67	76	
180	94	69	75	50	67	76
181	89	67	68	54	65	65

182	69	65	70	51	68	67
183	60	68	67	52	67	64
184	63	68		50	70	66
185	66	59	69	69	79	
186	59	64	65	65	66	79
187	60	60		44	68	65
188	55	61	68	64	39	
189	64	64		54	59	68
190	62	60		51	50	
191	60	63	66	59	55	67
192	63	61	69	56	68	70
193	59	65	90	55	65	69
194	65	55	73	49	69	
195	69	54	66	45	38	
196	54	68	67	52	58	66
197	66	64	79	50	67	
198	69	66	90	54	70	
199	67	60	75	59	66	64

Source: Office of the Principal, CCS Abonima

APPENDIX V

Chi-Square Test for the Data Collected

2018 RESULT

Descriptive Statistics

	N	Mean	Std. Deviation	Minimum	Maximum
MATHS	110	61.1273	11.68871	39.00	99.00
ENGLISH	110	55.0818	7.77921	39.00	97.00
MARKETING	105	63.7619	7.65547	49.00	93.00
ECONS	104	60.7115	6.08782	48.00	80.00
C.EDU	109	76.1009	10.81832	53.00	98.00
BIOLOGY	78	53.6538	6.95781	38.00	68.00

Test Statistics

	MATHS	ENGLISH	MARKETING	ECONS	C.EDU	BIOLOGY
Chi-Square	88.400[a]	88.545[b]	97.829[c]	40.192[d]	60.239[e]	75.179[f]
df	31	29	30	22	32	28
Asymp. Sig.	.000	.000	.000	.010	.002	.000

a. 32 cells (100.0%) have expected frequencies less than 5. The minimum expected cell frequency is 3.4.

b. 30 cells (100.0%) have expected frequencies less than 5. The minimum expected cell frequency is 3.7.

c. 31 cells (100.0%) have expected frequencies less than 5. The minimum expected cell frequency is 3.4.

d. 23 cells (100.0%) have expected frequencies less than 5. The minimum expected cell frequency is 4.5.

e. 33 cells (100.0%) have expected frequencies less than 5. The minimum expected cell frequency is 3.3.

f. 29 cells (100.0%) have expected frequencies less than 5. The minimum expected cell frequency is 2.7.

2019 RESULT

Descriptive Statistics

	N	Mean	Std. Deviation	Minimum	Maximum
MATHS	86	67.5349	4.25011	53.00	79.00
ENGLISH	86	54.2093	5.44300	38.00	68.00
MARKETING	81	64.7284	11.47172	38.00	97.00
ECONS	69	70.5217	9.10496	57.00	97.00
C.EDU	85	73.5176	13.76643	6.00	98.00
BIOLOGY	65	55.7077	4.87251	45.00	69.00

Test Statistics

	MATHS	ENGLISH	MARKETING	ECONS	C.EDU	BIOLOGY
Chi-Square	99.581[a]	90.512[b]	69.407[c]	57.391[d]	68.424[e]	47.200[f]
df	13	21	30	18	26	16
Asymp. Sig.	.000	.000	.000	.000	.000	.000

a. 0 cells (0.0%) have expected frequencies less than 5. The minimum expected cell frequency is 6.1.

b. 22 cells (100.0%) have expected frequencies less than 5. The minimum expected cell frequency is 3.9.

c. 31 cells (100.0%) have expected frequencies less than 5. The minimum expected cell frequency is 2.6.

d. 19 cells (100.0%) have expected frequencies less than 5. The minimum expected cell frequency is 3.6.

e. 27 cells (100.0%) have expected frequencies less than 5. The minimum expected cell frequency is 3.1.

f. 17 cells (100.0%) have expected frequencies less than 5. The minimum expected cell frequency is 3.8.

2020 RESULT

Descriptive Statistics

	N	Mean	Std. Deviation	Minimum	Maximum
MATHS	196	65.9541	8.16201	44.00	99.00
ENGLISH	196	61.1020	5.47346	44.00	69.00
MARKETING	175	68.1943	7.86941	50.00	96.00
ECONS	184	56.3696	6.73417	39.00	71.00
C.EDU	195	68.0256	11.57692	38.00	94.00
BIOLOGY	123	67.7805	4.47678	58.00	79.00

Test Statistics

	MATHS	ENGLISH	MARKETING	ECONS	C.EDU	BIOLOGY

Chi-Square	306.714[a]	82.571[b]	259.280[c]	133.109[d]	290.169[e]	132.650[f]
df	33	20	32	28	47	18
Asymp. Sig.	.000	.000	.000	.000	.000	.000

a. 0 cells (0.0%) have expected frequencies less than 5. The minimum expected cell frequency is 5.8.

b. 0 cells (0.0%) have expected frequencies less than 5. The minimum expected cell frequency is 9.3.

c. 0 cells (0.0%) have expected frequencies less than 5. The minimum expected cell frequency is 5.3.

d. 0 cells (0.0%) have expected frequencies less than 5. The minimum expected cell frequency is 6.3.

e. 48 cells (100.0%) have expected frequencies less than 5. The minimum expected cell frequency is 4.1.

f. 0 cells (0.0%) have expected frequencies less than 5. The minimum expected cell frequency is 6.5.

REGRESSION ANALYSIS OF SOME SERVICE EXPENDITURES ON THE NIGERIA GROSS DOMESTIC PRODUCT (GDP)

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Abstract

Between 1981 and 2021, this research work looked at the test for equality of regression models employing some service expenditures on the Nigerian GDP. This study has five particular objectives that were created and used. Relevant related literatures were reviewed based on the factors in the objectives. This study used secondary data acquired from the National Bureau of Statistics and the Central Bank of Nigeria statistical bulletin, which included 46 activity sectors. The data was analyzed using multiple regression models. MINITAB 16 and Microsoft Excel 2010 were used for all calculations. The series plot results for each service expenditures indicate an upward trend for the whole plot, implying a linear link between expenditures and Nigeria's total GDP. To find the significant parameters and test for model equality, the Durbin method of estimating a multiple regression model was used; this revealed that all of the models' parameters estimates were not significant at 5%, which appears biased. Model (1), the Military Government Period Model, was found to be the "best" of the three regression models, with the highest R^2 and Adj. R^2 values of 99.8%, the smallest AIC and BIC values of 170.356 and 174.134, and the highest F-value of 4494.669 determined. According to the findings, the government should devote a smaller amount of its budget to recurrent spending and focus more on capital spending, such as agriculture, education, and health, as it is the primary driver of economic growth.

Keywords: *Regression Analysis; Regression Model, Service Expenditures; Gross Domestic Product; Modelling*

1. Introduction

For the economy to function, governments must ensure that systems for contract enforcement, protection of lives and property, development of key infrastructure, and social amenities are in place. According to Mitchel (2005), if these fundamental government tasks are not funded, economic activity will be very low or non-existent. As a result, the importance of government budgetary allocations is highlighted. As a result, the government must be able to spend some money on growth-oriented programs. As a result, the role of government in an economy's growth process is frequently disputed in terms of size. The debate has centered on whether or not increased government spending is compatible with the government's goal of promoting faster economic growth.

The growing trend of government firms being privatized demonstrates that the private sector can deliver services more efficiently (higher quality at a lower cost). Evidence of a negative

relationship between government spending and economic growth is presented by Folster and Henrekson (2001), Bassanini et al. (2001), and Chandra (2004). It's also worth noting that when government spending isn't allocated efficiently to growth and development-oriented initiatives, it might lead to inflation and, as a result, growth stifling.

High government spending has a negative impact on economic growth, according to Mitchel (2005), because of costs associated with funding sources, financing of growth-destructive activities such as participation in international organizations (IMF, OECD, etc.) that advocate growth-retarding policies, and subsidizing economically unsustainable decisions such as welfare and unemployment benefits or insurance programs that discourage enterprise and private savings. Economic growth is fueled by the private sector's creative destruction or disruptive innovation. When it comes to funding sources, taxes, for example, have a negative impact on the willingness to labor or produce, whereas borrowing reduces private sector participation in the economy while also raising interest rates and thus inflation.

According to Gupta (1989), the relationship between economic growth and government spending is dependent on how government spending is defined. This theory is supported by Hansson and Henrekson (1994), who found that government consumption spending slows economic growth whereas education spending accelerates it.

Despite the fact that government spending in Nigeria has increased in recent years, there are still public outcries about deteriorating infrastructure. Furthermore, despite its importance for policy decisions, only a few empirical studies have examined the impact of government spending on economic growth holistically. More importantly, determining the influence of public expenditure on economic growth is a method to accelerate Nigeria's economy's growth in its goal to become one of the world's largest economies by 2020.

The Gross Domestic Product (GDP) is one of the most common metrics used to assess a country's economic health. It's also used to figure out what a person's standard of living is in a given economy. Gross Domestic Product, on the other hand, can be defined as the market value of all officially recognized final goods and services produced within a country during a specific time period. This means that instead of simply adding up the quantities of products and services, Gross Domestic Product takes into account the market value of each one. The Gross Domestic Product (GDP) is crucial in an economy since it is used to determine whether a country's economy is growing faster or slower. It's also used to compare the size of different economies around the world. The Gross Domestic Product (GDP) is once again used to compare the relative growth rates of countries around the world. The Federal Reserve in the United States, for example, uses it as one of the indications to determine whether the economy needs to be controlled or stimulated.

Consumption, investment, government expenditure, gross export, and gross import are the components of Gross Domestic Product calculated using the expenditure method. GDP = C + I + G + (X - M). The Value Added (or Production) technique and the Income (or By Type) approach are two more ways to calculate the Gross Domestic Product. To avoid duplicate counting, which could lead to the reporting of an erroneous figure of GDP, neither of the three methodologies includes intermediate items, but solely "new" products (final goods) and services when computing GDP. There are two types of GDP: real GDP and nominal GDP. Where Real GDP is the estimate of a country's economic production minus the influence of inflation, and Nominal GDP is the estimate that does not include price changes.

The term "regression evaluation" refers to a mathematical procedure for determining which variables can have an impact. The significance of regression analysis comes in the fact that it is a powerful statistical tool that allows a company to investigate the relationship between two or more variables. Shalabh (1997) investigated the features of the ratio technique in the presence of measurement mistakes for estimating the population. Regression analysis has numerous advantages: The regression forecasting technique is used for forecasting and determining the causal link between variables, as the name implies. The benefits of linear regression, which is the technique for modeling the value of one variable at the fee(s) of one, are an essential linked, practically similar, principle. Understanding the value of regression analysis, the advantages of linear regression, the advantages of regression evaluation, and the regression approach to forecasting can help a small business, or any business, gain a better understanding of the variables (or elements) that may affect its success in the weeks, months, and years ahead.

The significance of regression analysis is that it is all about data: information in the form of numbers and figures that define your business. The advantages of regression analysis are that it allows you to crunch the facts to assist you make better decisions for your business now and in the future. The regression technique to forecasting entails examining the relationships between data elements, allowing you to make predictions:

- Estimate revenue in the short and long term.
- Recognize the various stages of stock.
- Recognize supply and demand.
- Examine and recognize the impact of unusual variables on all of these factors.
- Companies might utilize regression analysis to figure out things like:
- Why has the number of customer service calls decreased in the last 12 months, or even in the last month?
- Predict how sales will look in the next six months.
- Deciding whether to choose one ad over another.
- Whether or not to expand the company or develop and market a new product.

The advantage of regression analysis is that it can be used to recognize all types of styles seen in records. These new insights are frequently quite useful in determining what can make a difference in your commercial enterprise. Regression analysis, then, is a crucial aspect in business since it is a statistical tool that allows businesses and their managers to make better-informed decisions based on hard data.

It's critical to understand that a regression analysis is essentially a statistical problem. Many facts-based standards have been adopted by businesses since they can be useful in assisting a company in deciding a variety of crucial issues and then making informed, well-researched decisions based on a variety of facts. According to Merriam-Webster, statistics is simply factual statistics (together with measurements or records) used as a foundation for reasoning, discussion, or calculation. Regression analysis makes use of data, particularly two or more variables, to provide some suggestions about where future information factors might be. The benefit of regression analysis is that it allows agencies to glimpse into the future using statistical calculations. The regression technique of forecasting allows groups to apply exact tactics in order for those forecasts, coupled with future income, future labor or supply needs, or possibly future challenging scenarios, to produce meaningful statistics.

In situations where complex sample strategies are used, regression analysis is frequently used in the analysis of survey data. The regression estimation technique, according to Okafor (2002), uses auxiliary facts to improve estimations of population metrics such as the mean and total. He also claimed that when the regression line of y on x does not pass through the origin but produces an intercept along the y-axis, regression estimation is employed to estimate the population.

On the expenditure side of the budget, the Nigerian economy has grown from a million Naira to a billion Naira and is projected to grow to a trillion Naira in the next decade. This is unsurprising if the economy is in a state of surplus or equilibrium in terms of balance of payment data. Even better if there are infrastructures to improve commerce with the system or social facilities to increase the welfare of the economy's ordinary citizen. Despite the fact that none of these are present, we always have a very high predicted spend. This shows that something is wrong, either with the way the government expands its budget or with the methods and methods by which it has traditionally been estimated.

For a resource- and cash-rich country (Nigeria) with nearly 70% of its population living in relative poverty, whose infrastructures are in a state of decay, whose health, education, agriculture, and other growth-promoting and welfare-enhancing institutions are in a state of near-collapse, whose roads (most of them) have become death traps due to their deplorable conditions, whose power sector is moribund, whose unemployment rates are near-unacceptably high. Despite these issues, the government has continued to raise its spending. As a result, one would anticipate Nigeria to attain a comparable level of economic growth, but this has not been the case (Chirwa & Odhiambo, 2016).

This research aims to model a set of expenditures on Nigeria's GDP. The majority of research on a particular set of expenditures have focused on describing the origins, causes, scale, and long-term sustainability of the government spending problem, as well on the country's overall GDP. Ajayi and Oke (2012), as well as Adesola (2012), have focused on the influence of a series of expenditures on economic growth (Nigeria GDP) (2009). Furthermore, studies in Nigeria, Europe, and other developed economies have measured government spending using recurrent expenditure, debt servicing costs, security spending, and education spending, but none have tested the magnitude of the impact that some series of expenditures could have on Nigeria GDP growth.

The problems highlighted above have been there over the years despite various works done by researchers and authors on the field of study. It will be unwise for the researcher of this work to also base the problems of this study on the above stated problems. On this premise, the researcher chooses to look at the problem of data obsolescence, which refers to the last time the research was conducted, the geographical problem, which refers to areas other researches have not covered and which variables among some series of expenditures on the Nigeria GDP component have not been tested, as well as the methodological problem, which informs the study's gaps and also serves as the foundation that piques the researcher's interest. All of these, in one way or another, pose an issue, and they are the motivation for the researcher to conduct this study, which aims to identify the services that have an impact on Nigeria's GDP.

Aim and Objectives of the Study

The goal of this study is to compare the effects of various service expenditures on Nigerian GDP from 1981 to 1999 (military government) and 2000 to 2021 (civil government). There are two types of government: military government and civil government. The study's specific goals are as follows:

1. Obtain the series plots of the various services on the Nigeria GDP.
2. Estimate the descriptive statistics of the series.
3. Compute the model formula and estimation of the model parameters (β_0, β_1, β_2 and β_3) for both models.
4. Obtain the residuals sum of square (RSS), mean sum of square (MSS), R-square value (R^2), adjusted R-square value (Adj. R^2), AIC and BIC of the Models.
5. Test for model equality and Check for significance variables (i.e. compare the result of the two regression methods used).

Scope of the study

The research focuses on simulating some series expenditures on Nigeria's GDP between 1981 and 2021. The research focuses on a multiple linear regression model with only four series of service expenditures (public administration, agricultural spending, health expenditure, and education expenditure) and the growth rate of Nigeria's GDP for over 41 years. The researcher used secondary data from the National Bureau of Statistics and the Central Bank of Nigeria statistical bulletin to focus on the gross domestic product by spending and income - yearly (' Billion). There are 46 activity categories in the implicit fee deflator desk of the rebased GDP data with new classifications; formerly, there were 33 activity sectors. A yearly statistics data of a periodic range of 41 years 1981 – 2021 in Nigeria was utilized to determine an adequate regression model to checkmate if any association existed between the various series expenditure and Nigeria GDP.

2. Literature Review

Khalaf (2013) pointed out that different types of estimators have been proposed as alternatives to the Ordinary Least Squares (OLS) estimator for the estimation of regression coefficients in the presence of multicollinearity in his research paper "A Comparison between Biased and Unbiased Estimators in Ordinary Least Squares Regression." Multicollinearity is known to make statistical inference difficult and may even substantially distort the inference in the general linear regression model, Y = X + e. Ridge regression defines a class of estimators of indexed by a scalar parameter k, as seen above. Simulation approaches are used to evaluate two methods of determining k in terms of Mean Square Error (MSE). Other ridge-type estimators that have been assessed elsewhere are compared. The estimated MSE of the suggested estimators was found to be lower than other ridge parameter estimators and the OLS estimator. The sample size and number of regressions are the other variables that were chosen to be varied. They create models with 25, 50, 100, and 150 observations, as well as two to four explanatory variables. The simulation experiments show that increasing the number of regressors and utilizing non-normal pseudo random numbers to create I e results in a larger estimated MSE, while increasing the sample size results in a lower estimated MSE.

Using the Johansen cointegration approach as a framework of study, Babatunde and Adefabi (2005) investigated the long-run Causal connection between Education and economic growth in Nigeria. According to the findings of the cointegrating technique, there is a long-run relationship between enrolments in elementary and secondary education, as well as the average years of schooling, and output per worker. The study found that a well-educated workforce has a positive and significant impact on economic growth through factor accumulation and overall factor productivity evolution.

In his study, Omotor (2004) looked at the factors that influence federal government spending in the education sector. His research reveals that Nigeria's education spending is in a state of flux, which reflects the state of the government's finances. According to the regression results, government money was the only significant factor of education spending. To change the sector's unstable tendency, the study suggests diversifying the sources of support for education.

In Nigeria, the conventional least squares method is used (OLS) Maku (2009) used time series data to explore the relationship between government spending and economic growth in Nigeria over the previous three decades, using the Ram (1986) model and regression real GDP on private investment and human capital investment. The Error Correction Model (ECM) was utilized to test for the presence of stationary in the variables using the Augmented Dicker Fuller (ADF) unit root test and the cointegration test to establish the long-run link among variables. During the evaluation period, empirical findings revealed that public and private sectors had no major impact on economic growth.

Using time series data from 1975 to 2004, Olorunfemi (2008) investigated the direction and strength of the relationship between public investment and economic growth in Nigeria, finding that public expenditure had a positive impact on economic growth and that there was no link between gross fixed capital formation and GDP. He said that disaggregated study revealed that only 37.1 percent of government spending is spent on capital projects, while 62.9 percent is spent on current projects.

In Nigeria, Shabana et al. (2017) looked into the relationship between government spending, health, and security spending, as well as economic growth. The data was analyzed using the vector error correction model and ordinary least square regression. For the years 2012 to 2015, the study reveals that government expenditure on health, security, and development projects has a short and long term impact on economic growth in South Africa.

Olopade and Olepade (2010) investigate the effects of fiscal and monetary policy on economic growth and development. The goal of their research was to figure out which aspects of government spending contribute to growth and development, which ones don't, and which ones should be removed or reduced to the bare minimum. The study incorporates a statistical framework that includes trends analysis and simple regression, as well as economic models and statistical methodologies. They discover no substantial association between the majority of expenditure components and economic development.

Obi and Obi (2020) investigated the influence of government spending on education in Nigeria. They said that Nigeria has spent a lot of money over the years to enhance the labor force's educational attainment and productivity, but that the country is still experiencing diminishing real production and poor economic growth. The article examines the impact of

education spending on economic growth as a strategy of accomplishing Nigeria's desired socioeconomic reform. Data from 1981 to 2012 is included in the study. The link between GDP and recurrent education expenditure was investigated using Johansen's co-integration analysis and ordinary least square (OLS) econometric approaches. The findings show that, while there is a positive association between education spending and economic growth, there is no long-run relationship across the study period. According to the report, this conundrum is caused by "labor market distortions, workforce redundancy, industrial disputes, and employment discontinuities, as well as leakages in Nigerian society, such as brain drain." To summarize, the findings of the study reveal that the educational sector has not been as productive as projected. The poor quality of graduates, rising occurrences of cultism in schools, and high dropout rates are all proof of this. The paper also recommends that the education system be improved by making better use of public resources through effective governance, accountability, and openness. Additionally, policymakers should make attempts to develop rules that will limit, conserve, and defend the plight of educational capital in other countries.

3. Materials and Methods

3.1 Research Design: This study used a cross-sectional research design, with the goal of modeling various series expenditures on the Nigerian GDP. For this study, the multiple regression model (OLS) methodology was employed, which included two regression models, Military Government and Civil Government model spending on the Nigeria GDP.

3.2 Nature and Source of Study Data: For this study, the researcher used secondary data from the National Bureau of Statistics and the Central Bank of Nigeria statistics bulletin. There are 46 activity categories in the implicit fee deflator desk of the rebased GDP data with new classifications; formerly, there were 33 activity sectors. A yearly statistics data of a periodic range of 41 years 1981 – 2021 in Nigeria was utilized to determine an adequate regression model to checkmate if any association existed between the various series expenditure and Nigeria GDP.

The parameters that make up the models are obtained using the following programs, including MINITAB 16 and Microsoft Excel 2010. The researcher used Microsoft Excel 2010 and MINITAB 16 to help with data analysis. The parameters for multiple regression models (OLS) and multiple interactive regression models were estimated using Microsoft Excel 2010 and MINITAB 16.

3.3 Method of Data Analysis

3.3.1 Model Specification

Suppose we have regression models of the form;

$$y = f(x)$$

$$y = \beta_0 + \beta_1 x + \varepsilon_i = (x'x)^{-1} x'y \quad (3.1)$$

Equation (3.1) is a simple linear regression model and

$$y = f(x_1, x_2, x_3, x_4)$$

$$y_i = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \ldots + \beta_k x_k + \varepsilon_i \quad (3.2)$$

$$= (x'x)^{-1} x'y$$

where,

y_i = Response (Nigerian GDP)

x_i = k^{th} predictor (some series expenditures) or independent variables

β_k = k^{th} population regression coefficient

Then, X_i is the matrix of the explanatory variables and is of the form:

$$X = \begin{bmatrix} 1 & x_{11} & x_{12} & x_{13} & \ldots\ x_{1k} \\ 1 & x_{21} & x_{22} & x_{23} & \ldots\ x_{2k} \\ 1 & x_{31} & x_{32} & x_{33} & x\ldots_{3k} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & x_{n1} & x_{n2} & x_{n3} & \ldots\ x_{nk} \end{bmatrix} \quad (3.7)$$

Y and β are of the form:

$$Y = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ \vdots \\ y_n \end{bmatrix}, \qquad \beta = \begin{bmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \\ \vdots \\ \beta_n \end{bmatrix}$$

Where

$$x'x = \begin{bmatrix} n & \Sigma x_1 & \Sigma x_2 & \Sigma x_3 & \Sigma x_4 \\ \Sigma x_1 & \Sigma x_1^2 & \Sigma x_1 x_2 & \Sigma x_1 x_3 & \Sigma x_1 x_4 \\ \Sigma x_2 & \Sigma x_1 x_2 & \Sigma x_2^2 & \Sigma x_2 x_3 & \Sigma x_2 x_4 \\ \Sigma x_3 & \Sigma x_1 x_3 & \Sigma x_2 x_3 & \Sigma x_3^2 & \Sigma x_3 x_4 \\ \Sigma x_4 & \Sigma x_1 x_4 & \Sigma x_2 x_4 & \Sigma x_3 x_4 & \Sigma x_4^2 \end{bmatrix} \quad (3.8)$$

$$x'y = \begin{bmatrix} \Sigma y \\ \Sigma x_1 y \\ \Sigma x_2 y \\ \Sigma x_3 y \\ . \\ . \\ . \\ \Sigma x_k y \end{bmatrix} \quad (3.9)$$

The matrix form of the model is giving by:

$$\begin{bmatrix} \Sigma y \\ \Sigma x_1 y \\ \Sigma x_2 y \\ \Sigma x_3 y \\ . \\ . \\ . \\ \Sigma x_k y \end{bmatrix} = \begin{bmatrix} n & \Sigma x_1 & \Sigma x_2 & \Sigma x_3 & \dots \Sigma x_k \\ \Sigma x_1 & \Sigma x_1^2 & \Sigma x_1 x_2 & \Sigma x_1 x_3 & \dots \Sigma x_1 x_k \\ \Sigma x_2 & \Sigma x_1 x_2 & \Sigma x_2^2 & \Sigma x_2 x_3 & \Sigma x_2 x_k \\ \Sigma x_3 & \Sigma x_1 x_3 & \Sigma x_2 x_3 & \Sigma x_3^2 & \Sigma x_3 x_k \\ \Sigma x_4 & \Sigma x_1 x_4 & \Sigma x_2 x_4 & \Sigma x_3 x_4 & \Sigma x_4^2 \end{bmatrix}^{-1} \begin{bmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \\ \beta_3 \\ . \\ . \\ . \\ \beta_4 \end{bmatrix}$$

$$\begin{bmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \\ \beta_3 \\ \beta_4 \end{bmatrix} = (x'x)^{-1}x'y = \begin{bmatrix} n & \Sigma x_1 & \Sigma x_2 & \Sigma x_3 & \Sigma x_4 \\ \Sigma x_1 & \Sigma x_1^2 & \Sigma x_1 x_2 & \Sigma x_1 x_3 & \Sigma x_1 x_4 \\ \Sigma x_2 & \Sigma x_1 x_2 & \Sigma x_2^2 & \Sigma x_2 x_3 & \Sigma x_2 x_4 \\ \Sigma x_3 & \Sigma x_1 x_3 & \Sigma x_2 x_3 & \Sigma x_3^2 & \Sigma x_3 x_4 \\ \Sigma x_4 & \Sigma x_1 x_4 & \Sigma x_2 x_4 & \Sigma x_3 x_4 & \Sigma x_4^2 \end{bmatrix}^{-1} \begin{bmatrix} \Sigma y \\ \Sigma x_1 y \\ \Sigma x_2 y \\ \Sigma x_3 y \\ \Sigma x_4 y \end{bmatrix}$$

and $$v(\hat{\beta}_{iv}) = MSE(x'x)^{-1}x'y \tag{3.10}$$

If *n* is odd, then the middle observation can be deleted. Using this approach, the estimators are constant but likely to have large variance which is the limitation of this Method.

3.4 Model Selection Criteria

The model choice criterion is used to determine the optimal manufacturing feature. The excellent model is the one that minimizes the criterion. Several criteria for selecting various models have been developed in recent years, and it takes the form of residual sum of squares errors (SSE) compounded by a penalty factor that relies on the model's complexity. Some of these criteria are mentioned further down:

3.4.1 Akaike Information Criteria (AIC)

Akaike (1974) devised a method known as Akaike Information Criteria. The format of this data is as follows:

$$AIC = n\ln\left[\frac{SSE}{n}\right] + 2(k) \tag{3.11}$$

Where;

- N = Sample size
- K = Number of parameter and
- SSE = Sum of square error.

3.4.2 SCHWARZ Bayesian Information Criterion (BIC)

Craven and Wahba (1978) proposed the SCHWARZ (BIC) criteria, which is now widely used. This method's format is as follows:

$$BIC = n \ln\left[\frac{SSE}{n}\right] + k \ln(n) \tag{3.12}$$

If there are at least eight observations, the charge for SCHWARZ (BIC) will also be reduced (Ramanathan, 1995).

3.4.3 Coefficient of Determination (R^2)

R^2 is one of the most important statistical parameters for decision-making and statistical judgments. It's a method for determining the proportion of one or more variables' outcomes that outnumber the others. This procedure takes the following form::

$$R^2 = \frac{SSR}{SST} \tag{3.13}$$

Where; SSR= Sum of square Residual

SST= Sum of Square Total

3.4.4 Sum of Squares (SS)

The total of distances squared. SS The entire variation in the data is the total variation in the data. The portion of the variation explained by the model is referred to as SS Regression, whereas the fraction not explained by the model and attributed to error is referred to as SS Error. The calculations are completed:

Sources of variation	**Sum of squares (SS)**
SS Regression	$(\hat{y}_i - \bar{y})^2$
SS Error	$(y_i - \hat{y}_i)^2$
SS Total	$(y_i - \bar{y}_i)^2$

3.4.5 Degrees of freedom (DF)

To calculate the sum of squares, the number of independent pieces of information including the answer data is needed. The degrees of freedom for each model component are as follows:

Sources of variation	**DF**

Regression	P
Error	n - p -1
Total	n - 1

Notation

n = number of observations

p = number of terms in the model

3.4.6 MS Regression

The formula for mean square regression is:

$$MS_{\text{Re gression}} = \frac{SS_{\text{Re gression}}}{DF_{\text{Re gression}}} = \frac{\sum (\hat{y}_i - \bar{y})^2}{P}$$

3.4.7 MS Error

Mean square error, which is the variance around the fitted regression line. MS Error = s^2. The formula is:

$$MSE = \frac{SS_{Error}}{DF_{Error}} = \frac{\sum (y_i - \hat{y}_i)^2}{n - p - 1}$$

3.4.8 MS Total

The formula for mean square total is:

$$MS_{Total} = \frac{SS_{Total}}{DF_{Total}} = \frac{\sum (y_i - \bar{y})^2}{n - 1}$$

3.4.9 F-value (F)

At least one of the coefficients is not equal to zero if the estimated F-value is bigger than the F-value from the F-distribution. The p-value is calculated using the F-value. The formula for calculating the F-value is as follows:

$$\frac{\text{MS Regression}}{\text{MS Error}}$$

3.4.10 p-value (P)

Used in hypothesis tests to determine if a null hypothesis should be rejected or not. If the null hypothesis is true, the p-value is the probability of getting a test statistic that is at least as extreme as the actual computed value. The 0.05 cut-off number for the p-value is often employed. You reject the null hypothesis if the estimated p-value of a test statistic is less than 0.05, for example.

4. Results and Discussion

The results for series plots of various services on the Nigeria GDP, descriptive statistics of the series, estimation of the model parameters (β_0, β_1, β_2 and β_3), residuals sum of square (RSS), mean sum of square (MSS), R-square value (R^2), adjusted R-square value (Adj. R^2), AIC and BIC of the Models, and discussion of findings are all covered in this section. However, significant variables were also checked (by comparing the results of the two regression methods utilized).

4.2.1 Series Plots of the Various Services on the Nigeria GDP

The yearly average GDP by expenditures and series plots (expenditures on Gross Domestic Product) were examined in this part to see whether there were any correlations, trend components, or seasonality effects, if they were present in the data sets.

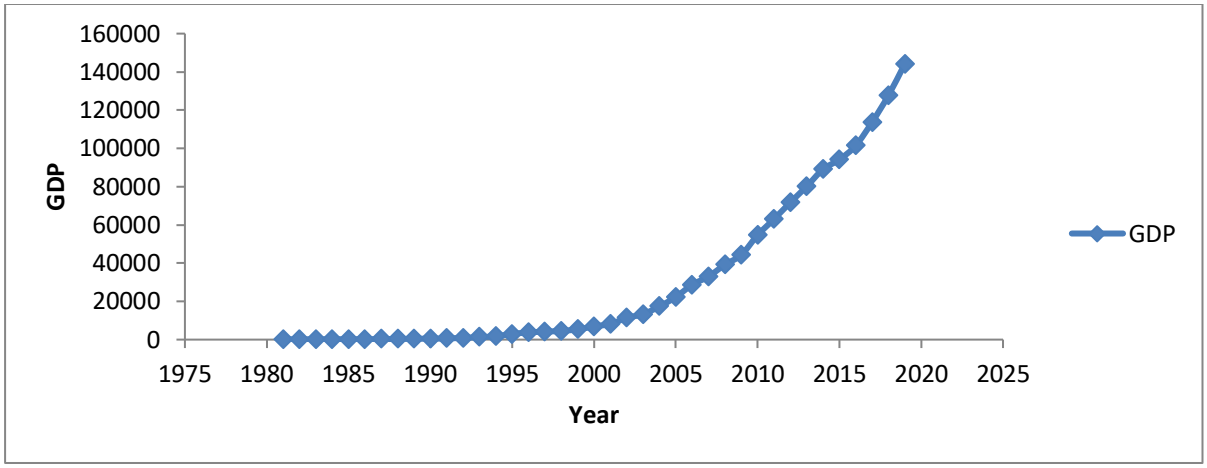

Figure 1: Series Plot of the general GDP of Nigeria (Yearly)

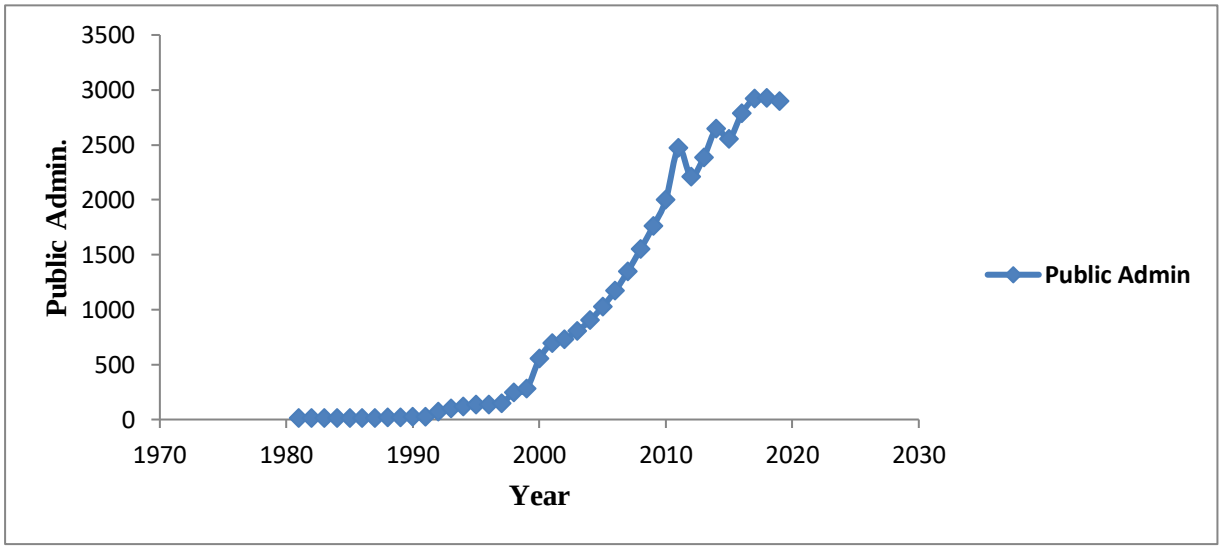

Figure 2: Series Plot of the Expenditure on Public Administration (Yearly)

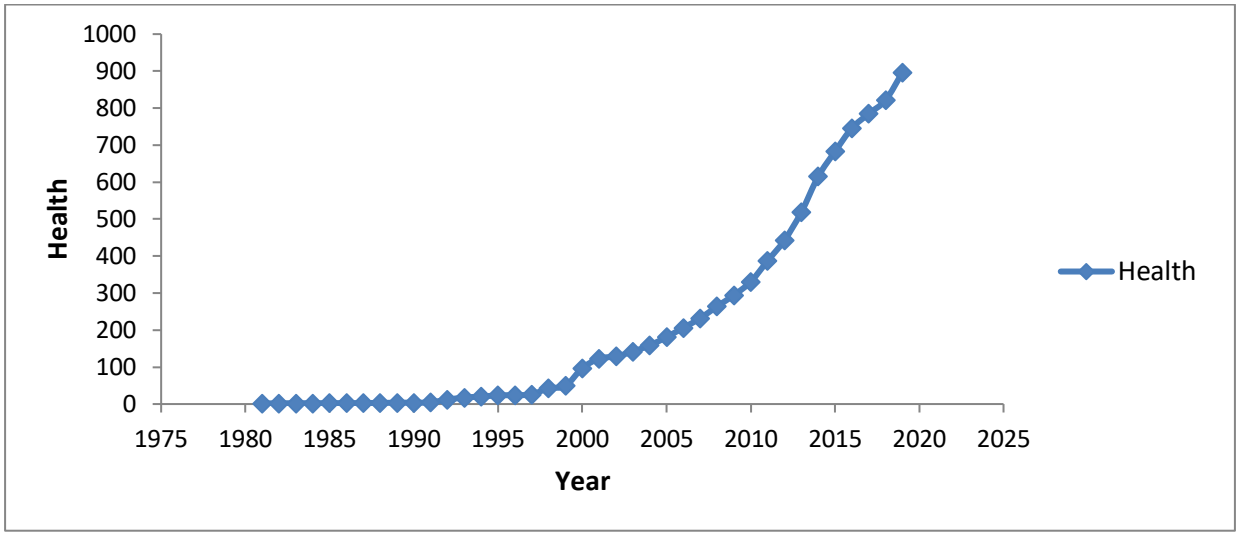

Figure 3: Series Plot of the Expenditure on Health (Yearly)

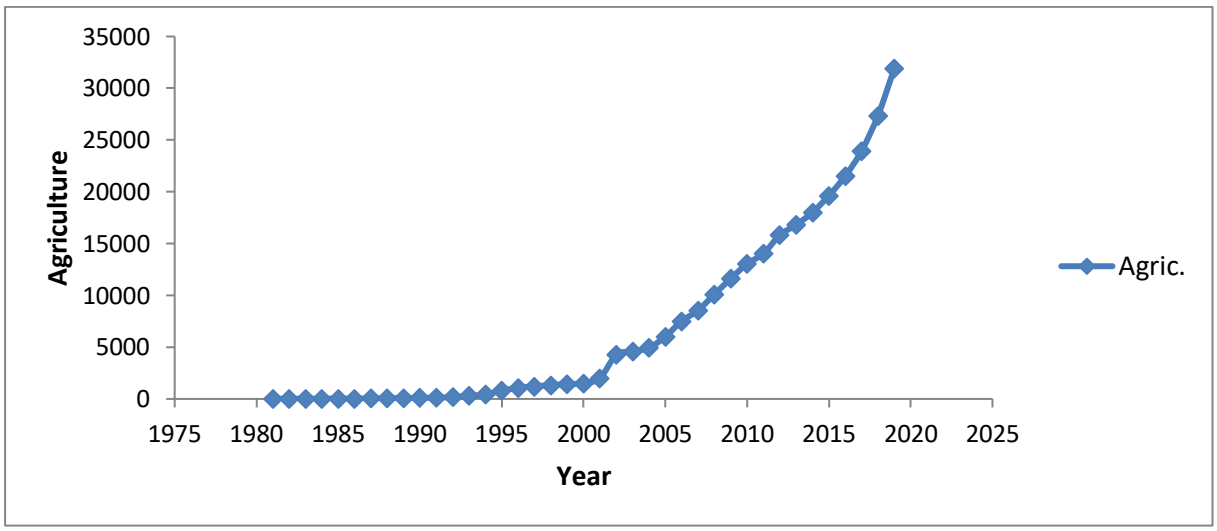

Figure 4: Series Plot of the Expenditure on Agriculture (Yearly)

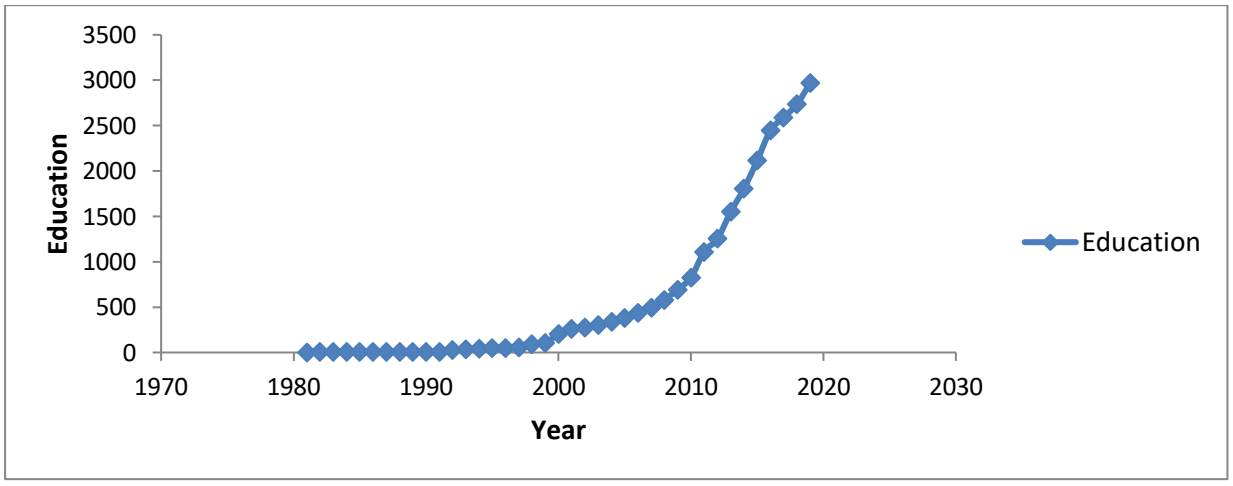

Figure 5: Series Plot of the Expenditure on Education (Yearly)

Figure 1 depicts the annual expenditure behavior of Nigeria's GDP, with the highest rate in 2019 and the lowest rate in 1981. However, the annual GDP statistics show an increase from 1981 to 2019 (or a swing upward). Figure 2 depicts the yearly expenditure behavior of public administration, with the highest peak in 2018 and the lowest expenditure rate in 1981. The

annual public administration series, on the other hand, shows a gain from the start (or swing higher); 1981 to 2010, and a varied increase and fall from 2011 to 2019.

Figure 3 depicts the yearly expenditure behavior of Health, with the highest rate in 2019 and the lowest rate in 1981. However, the annual health data show an increase from 1981 to 2019 (or a tilt upward). Figure 4 depicts the yearly expenditure behavior of Agriculture, with the highest rate in 2019 and the lowest rate in 1981. However, the annual agricultural series shows an increase from 1981 to 2019 (or a swing upward). Figure 5 depicts the annual expenditure behavior of education, with the highest rate in 2019 and the lowest rate in 1981. However, from 1981 to 2019, the annual schooling series shows a gain (or swing upward).

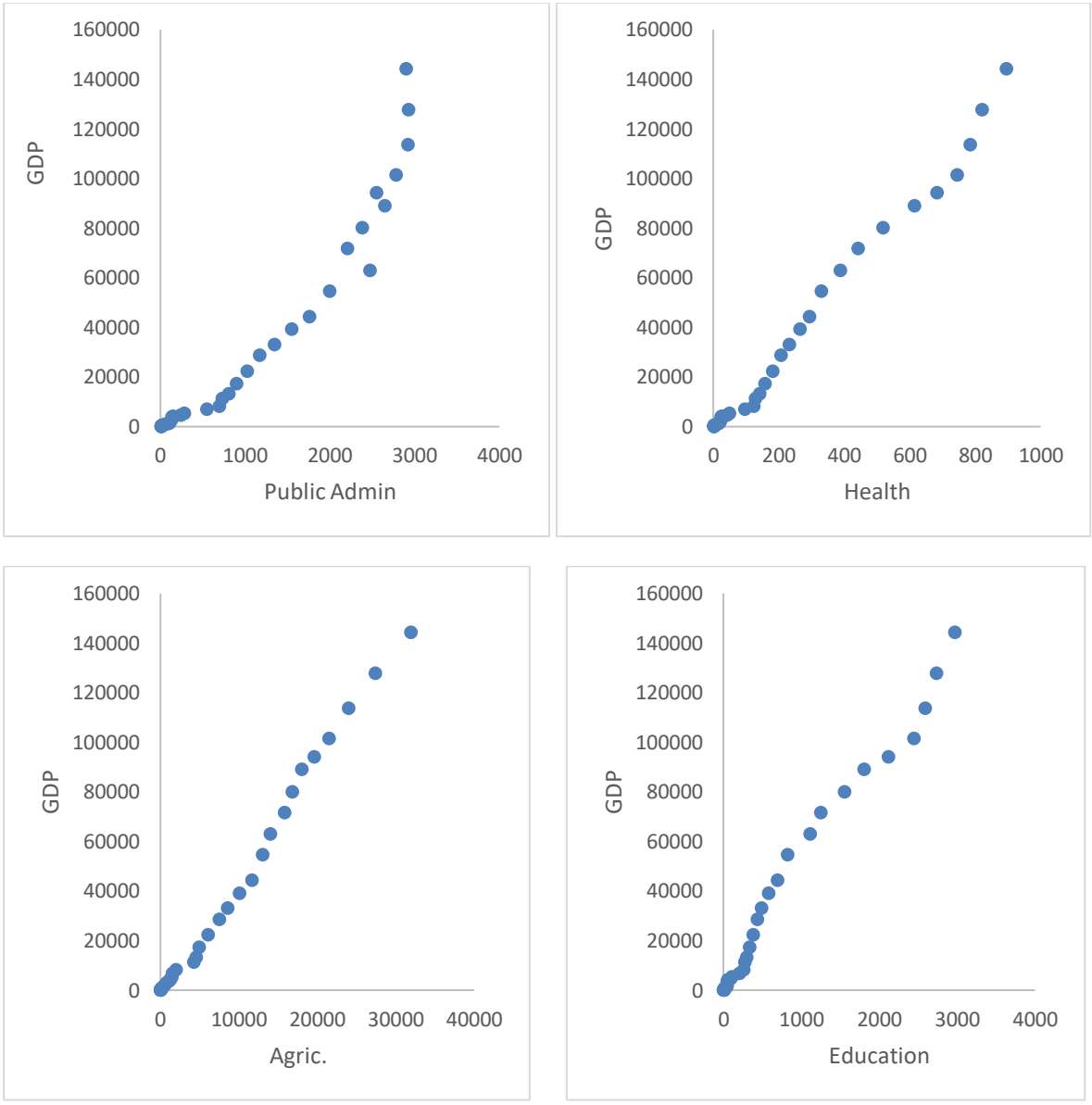

Figure 6: Scatter Plot of the Expenditures against Total GDP of Nigeria

The entire figure in Figure 6 demonstrates an upward trend. These findings point to a linear link between expenditures and Nigeria's total GDP. To select the significant parameters and test for model equality, the Durbin method of estimating a multiple regression model was used. Following that, the data was split into two models: military government and civil government.

4.2.2 Descriptive Statistics of the Various Services on the Nigeria GDP

This section focuses on descriptive statistics of the various services on the Nigeria GDP; Public Administration, Health, Agriculture, Education and general GDP.

Table 1: **Descriptive Statistics of the Various Services on the Nigeria GDP**

Variable	Mean	SE Mean	St.Dev	Minim.	Q1	Median	Q3	Maximum.	Skewness	Kurtosis
Health (x_1)	212.8	43.7	273.0	1.6	3.9	97.2	331.0	896.2	1.29	0.42
Agric. (x_2)	6939	1427	8911	17	107	1508	13049	31904	1.26	0.61
Edu (x_3)	612	142	886	3	8	206	827	2969	1.55	1.17
PA (x_4)	967	172	1072	9	22	551	1998	2926	0.74	-1.05
GDP (y)	30560	6670	41655	145	500	6897	54612	144210	1.34	0.66

Table 1 shows the average of the various service series of expenditure on the Nigeria GDP, with projected values of 212.8 for Health, 6939 for Agriculture, 612 for Education, 967 for Public Administration, and 30560 for Total Nigeria GDP.

4.2.3 Regression Analysis

The researcher will divide the data into two parts in this section: values for various service expenditures on the Nigeria GDP from 1981 to 1999, and values for various service expenditures on the Nigeria GDP from 1981 to 1999. (2000-2021). The first regression model (Military Government period model) was built using data from 1981 to 1999, whereas the second regression model was built using data from 2000 to 2021. (Civil Government period model). A pooled regression model was created once more. All of the model parameters, accuracy assessments, model equality tests, and significance variable checks were completed here.

4.2.3.1 Regression Model (1)

n_1=19, Σx_1=246.13, Σx_2=7368.334, Σx_3=521.77, Σx_4=1395.91, Σy_1=27726.604, $\Sigma x_1 x_2$ =221931.512, $\Sigma x_1 x_3$ =14873.0264, $\Sigma x_1 x_4$ =39790.1959, $\Sigma x_2 x_3$=470485.7088, $\Sigma x_2 x_4$ =1258702.02, $\Sigma x_3 x_4$ =84352.6054, Σx_1^2=7015.7957, Σx_2^2=7438922.79, Σx_3^2=31529.8405, Σx_4^2 =225670.7285, $\Sigma x_1 y_1$=802420.2088, $\Sigma x_2 y_1$ =26594428.45, $\Sigma x_3 y_1$=1701091.642, $\Sigma x_4 y_1$ =4550976.63.

$$x'x = \begin{bmatrix} 19 & 246.13 & 7368.334 & 521.77 & 1395.91 \\ 246.13 & 7015.7957 & 221931.512 & 14873.0264 & 39790.1959 \\ 7368.334 & 221931.512 & 7438922.79 & 470485.7088 & 1258702.02 \\ 521.77 & 14873.0264 & 470485.7088 & 31529.8405 & 84352.6054 \\ 1395.91 & 39790.1959 & 1258702.02 & 84352.6054 & 225670.7285 \end{bmatrix}$$

$$x'y_1 = \begin{bmatrix} 27726.604 \\ 802420.2088 \\ 26594428.45 \\ 1701091.642 \\ 4550976.63 \end{bmatrix}$$

,

$$x'y_1 = \begin{bmatrix} 27726.604 \\ 802420.2088 \\ 26594428.45 \\ 1701091.642 \\ 4550976.63 \end{bmatrix} = \begin{bmatrix} 19 & 246.13 & 7368.334 & 521.77 & 1395.91 \\ 246.13 & 7015.7957 & 221931.512 & 14873.0264 & 39790.1959 \\ 7368.334 & 221931.512 & 7438922.79 & 470485.7088 & 1258702.02 \\ 521.77 & 14873.0264 & 470485.7088 & 31529.8405 & 84352.6054 \\ 1395.91 & 39790.1959 & 1258702.02 & 84352.6054 & 225670.7285 \end{bmatrix}^{-1} \begin{bmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \\ \beta_3 \\ \beta_4 \end{bmatrix}$$

$$\begin{bmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \\ \beta_3 \\ \beta_4 \end{bmatrix} = (x'x)^{-1}x'y_1 = \begin{bmatrix} 19 & 246.13 & 7368.334 & 521.77 & 1395.91 \\ 246.13 & 7015.7957 & 221931.512 & 14873.0264 & 39790.1959 \\ 7368.334 & 221931.512 & 7438922.79 & 470485.7088 & 1258702.02 \\ 521.77 & 14873.0264 & 470485.7088 & 31529.8405 & 84352.6054 \\ 1395.91 & 39790.1959 & 1258702.02 & 84352.6054 & 225670.7285 \end{bmatrix}^{-1} \begin{bmatrix} 27726.604 \\ 802420.2088 \\ 26594428.45 \\ 1701091.642 \\ 4550976.63 \end{bmatrix}$$

$$\begin{bmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \\ \beta_3 \\ \beta_4 \end{bmatrix} = (x'x)^{-1}x'y_1 = \begin{bmatrix} 0.100853 & -1.11727 & 9.33E-05 & 1.288408 & -0.28574 \\ -1.11727 & 7244.442 & 0.042527 & -1852.44 & -585.152 \\ 9.33E-05 & 0.042527 & 2.74E-06 & -0.01358 & -0.00244 \\ 1.288408 & -1852.44 & -0.01358 & 6530.53 & -2114.33 \\ -0.28574 & -585.152 & -0.00244 & -2114.33 & 893.4977 \end{bmatrix} \begin{bmatrix} 27726.604 \\ 802420.2088 \\ 26594428.45 \\ 1701091.642 \\ 4550976.63 \end{bmatrix}$$

$$\begin{bmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \\ \beta_3 \\ \beta_4 \end{bmatrix} = (x'x)^{-1}x'y_1 = \begin{bmatrix} 76.80086222 \\ 5059.821971 \\ 3.00599524 \\ -8913.12769 \\ 2442.383008 \end{bmatrix}$$

Table 2: Descriptive Statistics of the Model (1)

Model (1)	**Parameter ± SE**	**t-test**	**P**	**Remark**
β_0	78.842 ± 24.772	3.183	0.006**	Significant
β_1	5059.82 ± 21.122	0.385	0.706	Not Significant
β_2	3.006 ± 0.123	24.110	0.000**	Significant
β_3	-8913.128 ± -134.770	-1.277	0.221	Not Significant
β_4	2442.383 ± 0.752	4.118	0.001**	Significant

Footnote: **= Significant at p< 0.05

The required estimated model is

$$\hat{y}_1 = 76.801 + 5059.822x_1 + 3.006x_2 - 8913.128x_3 + 2442.383x_4 \quad (4.1)$$

For health expenditure, agriculture expenditure, education expenditure, public administration expenditure, and total Nigeria GDP, table 2 summarized parameter estimates, t-test, p-value, and standard error of the Durbin method, accordingly. Similarly, the findings of the acquired parameters revealed that not all of the parameters are significant at 5%.

4.2.3.2 Regression Model (2)

n_2=20, Σx_1=8053.23, Σx_2=263246.15, Σx_3=23365.71, Σx_4=36328.1, Σy_2=1164094.166, $\Sigma x_1 x_2$=148992731.5, $\Sigma x_1 x_3$=14196905.08, $\Sigma x_1 x_4$=18729818.38, $\Sigma x_2 x_3$=459389440.7, $\Sigma x_2 x_4$ =612103558.7, $\Sigma x_3 x_4$=56745354.58, Σx_1^2=4591304.371, Σx_2^2=4887544274, Σx_3^2 =44400972.35, Σx_4^2=79939238.91, $\Sigma x_1 y_2$=682668795.6, $\Sigma x_2 y_2$=22300003276, $\Sigma x_3 y_3$ =2116591702, $\Sigma x_4 y_2$=2779839684.

$$x'x = \begin{bmatrix} 20 & 8053.23 & 263246.2 & 23365.71 & 36328. \\ 8053.23 & 4591304 & 1.49E+08 & 14196905 & 18729818.4 \\ 263246.2 & 1.49E+08 & 4.89E+09 & 4.59E+08 & 612103559 \\ 23365.71 & 14196905 & 4.59E+08 & 44400972 & 56745354.6 \\ 36328.1 & 18729818 & 6.12E+08 & 56745355 & 79939238.9 \end{bmatrix}$$

$$x'y_1 = \begin{bmatrix} 1164094.166 \\ 682668795.6 \\ 22300003276 \\ 2116591702 \\ 2779839684 \end{bmatrix}$$

,

$$x'y_1 = \begin{bmatrix} 1164094.166 \\ 682668795.6 \\ 22300003276 \\ 2116591702 \\ 2779839684 \end{bmatrix} = \begin{bmatrix} 20 & 8053.23 & 263246.2 & 23365.71 & 36328. \\ 8053.23 & 4591304 & 1.49E+08 & 14196905 & 18729818.4 \\ 263246.2 & 1.49E+08 & 4.89E+09 & 4.59E+08 & 612103559 \\ 23365.71 & 14196905 & 4.59E+08 & 44400972 & 56745354.6 \\ 36328.1 & 18729818 & 6.12E+08 & 56745355 & 79939238.9 \end{bmatrix}^{-1} \begin{bmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \\ \beta_3 \\ \beta_4 \end{bmatrix}$$

$$\begin{bmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \\ \beta_3 \\ \beta_4 \end{bmatrix} = (x'x)^{-1}x'y_1 = \begin{bmatrix} 20 & 8053.23 & 263246.2 & 23365.71 & 36328. \\ 8053.23 & 4591304 & 1.49E+08 & 14196905 & 18729818.4 \\ 263246.2 & 1.49E+08 & 4.89E+09 & 4.59E+08 & 612103559 \\ 23365.71 & 14196905 & 4.59E+08 & 44400972 & 56745354.6 \\ 36328.1 & 18729818 & 6.12E+08 & 56745355 & 79939238.9 \end{bmatrix}^{-1} \begin{bmatrix} 1164094.166 \\ 682668795.6 \\ 22300003276 \\ 2116591702 \\ 2779839684 \end{bmatrix}$$

$$\begin{bmatrix}\beta_0\\ \beta_1\\ \beta_2\\ \beta_3\\ \beta_4\end{bmatrix} = (x'x)^{-1}x'y_1 = \begin{bmatrix}0.922117 & -0.01252 & 3.82E-05 & 0.003051 & 5.6406E-05\\ -0.01252 & 0.000358 & -1E-06 & -8E-05 & -1.395E-05\\ 3.82E-05 & -1E-06 & 2.28E-08 & 1.06E-07 & -3.007E-08\\ 0.003051 & -8E-05 & 1.06E-07 & 1.88E-05 & 3.0956E-06\\ 5.64E-05 & -1.4E-05 & -3E-08 & 3.1E-06 & 1.2891E-06\end{bmatrix}\begin{bmatrix}1164094.166\\ 682668795.6\\ 22300003276\\ 2116591702\\ 2779839684\end{bmatrix}$$

$$\begin{bmatrix}\beta_0\\ \beta_1\\ \beta_2\\ \beta_3\\ \beta_4\end{bmatrix} = (x'x)^{-1}x'y_1 = \begin{bmatrix}-8038.840431\\ 34.92868969\\ 2.821846089\\ 5.371642797\\ 4.823534303\end{bmatrix}$$

Table 3: Descriptive Statistics of the Model (2)

Model (2)	Parameter $\pm$ SE	t-test	P	Remark
β_0	-8038.840 ± 2078.533	-3.868	0.002**	Significant
β_1	34.929 ± 40.956	0.853	0.407	Not Significant
β_2	2.822 ± 0.327	8.642	0.000**	Significant
β_3	5.372 ± 9.387	0.572	0.576	Not Significant
β_4	4.824 ± 2.458	1.963	0.069**	Not Significant

Footnote: **= Significant at p< 0.05

The required estimated model is

$$\hat{y}_2 = -8038.840 + 34.928x_1 + 2.8218x_2 + 5.3716x_3 + 4.8235x_4 \quad (4.2)$$

For health expenditure, agriculture expenditure, education expenditure, public administration expenditure, and total Nigeria GDP, table 3 summarized parameter estimates, t-test, p-value, and standard error of the Durbin method, accordingly. Similarly, the findings of the acquired parameters revealed that not all of the parameters are significant at 5%.

4.2.3.3 Pooled Regression Model

n_p = 39, Σx_1=8299.36, Σx_2=270614.484, Σx_3=23887.48, Σx_4=37724.01, Σy_p=1191820.77, Σx_1x_2=149214663, Σx_1x_3=14211778.1, Σx_1x_4=18769608.58, Σx_2x_3=459859926.4, Σx_2x_4 =613362260.7, Σx_3x_4=56829707.19, Σx_1^2=4598320.167, Σx_2^2=4894983197, Σx_3^2 =44432502.19, Σx_4^2=80164909.64, Σx_1y_p=683471215.8, Σx_2y_p=22326597704, Σx_3y_p =2118292794, Σx_4y_p=2784390661.

$$x'x = \begin{bmatrix} 39 & 8299.36 & 270614.5 & 23887.48 & 37724.01 \\ 8299.36 & 4598320 & 1.49E+08 & 14211778 & 18769609 \\ 270614.5 & 1.49E+08 & 4.89E+09 & 4.6E+08 & 6.13E+08 \\ 23887.48 & 14211778 & 4.6E+08 & 44432502 & 56829707 \\ 37724.01 & 18769609 & 6.13E+08 & 56829707 & 80164910 \end{bmatrix},$$

$$x'y_p = \begin{bmatrix} 683471215.8 \\ 22326597704 \\ 2118292794 \\ 2784390661 \\ 4550976.63 \end{bmatrix}$$

$$x'y_p = \begin{bmatrix} 683471215.8 \\ 22326597704 \\ 2118292794 \\ 2784390661 \\ 4550976.63 \end{bmatrix} = \begin{bmatrix} 39 & 8299.36 & 270614.5 & 23887.48 & 37724.01 \\ 8299.36 & 4598320 & 1.49E+08 & 14211778 & 18769609 \\ 270614.5 & 1.49E+08 & 4.89E+09 & 4.6E+08 & 6.13E+08 \\ 23887.48 & 14211778 & 4.6E+08 & 44432502 & 56829707 \\ 37724.01 & 18769609 & 6.13E+08 & 56829707 & 80164910 \end{bmatrix}^{-1} \begin{bmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \\ \beta_3 \\ \beta_4 \end{bmatrix}$$

$$\begin{bmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \\ \beta_3 \\ \beta_4 \end{bmatrix} = (x'x)^{-1}x'y_1 = \begin{bmatrix} -550.213 \\ -65.739 \\ 3.123 \\ 29.959 \\ 5.253 \end{bmatrix}$$

Table 4: Descriptive Statistics of the Pooled Model

Pooled Model	Parameter ± SE	t-test	P	Remark
β_0	-550.213 ± 475.377	-1.157	0.255	Not Significant
β_1	-65.739 ± 28.458	-2.310	0.027**	Significant
β_2	3.123 ± 0.291	10.739	0.000**	Significant
β_3	29.959 ± 6.179	4.849	0.000**	Significant
β_4	5.253 ± 2.264	2.320	0.026**	Significant

Footnote: **= Significant at p< 0.05

The required estimated model is

$$\hat{y}_p = -550.213 - 65.739x_1 + 3.123x_2 + 29.959x_3 + 5.253x_4 \quad (4.3)$$

For health expenditure, agriculture expenditure, education expenditure, public administration expenditure, and total Nigeria GDP, table 4 summarized parameter estimates, t-test, p-value,

and standard error of the Durbin method, respectively. Similarly, the obtained parameter results revealed that all parameters are significant at 5% except the constant term, which is not significant.

4.2.3.4 Computation of the Model Selection Criteria

4.2.3.4.1 Akaike Information Criteria (AIC) for the Three Models

$$AIC_1 = 19\ln\left[\frac{97683.941}{19}\right] + 2(4) = 170.356$$

$$AIC_2 = 20\ln\left[\frac{70277996.699}{20}\right] + 2(4) = 309.445$$

$$AIC_{pooled} = 39\ln\left[\frac{135770143.894}{39}\right] + 2(4) = 595.454$$

4.2.3.4.2 SCHWARZ Bayesian Information Criterion (BIC) for the Three Models

$$BIC_1 = 19\ln\left[\frac{97683.941}{19}\right] + 4\ln(19) = 174.134$$

$$BIC_2 = 20\ln\left[\frac{70277996.699}{20}\right] + 4\ln(20) = 313.428$$

$$BIC_{pooled} = 39\ln\left[\frac{135770143.894}{39}\right] + 4\ln(39) = 602.108$$

4.2.5 Comparison of the three Identified Regression Models

We compared the three identified regression models for the various service expenditures on the Nigeria GDP to determine the model that has more effect on total GDP of Nigeria in Table 5.

Table 5: Regression Analysis Summary of the Parameter Estimates

MODELS	PARAMETER ESTIMATES β_0 (p-value) t-test	β_1 (p-value) t-test	β_2 (p-value) t-test	β_3 (p-value) t-test	β_4 (p-value) t-test	MSE	R^2	R^2(ad	AIC	BIC	F
Model (1)	**78.842** ± **24.772** **(0.006**)** **3.183**	5059.82 ± 21.122 (0.706) 0.385	**3.006** ± **0.123** **(0.000**)** **24.110**	8913.128 ± - 134.77 (0.221) -1.277	**2442.383** ± **0.752** **(0.001**)** **4.118**	**6105.246**	**99.8%**	**99.8%**	**170. 356**	**174. 134**	**4494 .669**
Model (2)	**-8038.840** ± **2078.533** **(0.002**)** **-3.868**	34.929 ± 40.956 (0.407) 0.853	**2.822** ± **0.327** **(0.000**)** **8.642**	5.372 ± 9.387 (0.576) 0.572	4.824 ± 2.458 (0.069) 1.963	4685199.780	99.8%	99.7%	309. 445	313. 428	1837 .510
Poole d Model	-550.213 ± 475.377 (0.255) -1.157	**-65.739** ± **28.458** **(0.027**)** **-2.310**	**3.123** ± **0.291** **(0.000**)** **10.739**	**29.959** ± **6.179** **(0.000**)** **4.849**	**5.253** ± **2.264** **(0.026**)** **2.320**	3993239.526	99.8%	99.8%	595. 454	602. 108	4119 .503

Footnote: **= Significant at p< 0.05

A summary regression analysis of parameter estimates, AIC, BIC, MSE, standard error, t-test, p-values, and F-values for the models is shown in Table 5 for (model 1, 2 and the pooled model). It's worth noting that all of the model parameters' estimations aren't significant at 5%, which appears to be skewed. The Military Government Period Model, which has the highest R-square and R-square adjusted values with 99.8%, the smallest AIC and BIC values of 170.356 and 174.134, and the highest F-value of 4494.669 calculated, is the "best" model among the three regression models in Table 5. As a result, model (1) of estimation is the best, as it has the most impact on Nigeria's total GDP.

As a result, the three variables, Agriculture spending (X_2), Public Administration expenditure (X_4) and other factors not included in the model, which is represented by the constant parameter β_0. However, only Agriculture expenditure (X_2) and the constant parameter β_0 are significant for Military Government period model. For the pooled regression model, all the estimated parameters performance well and were also significant and adequate except the constant parameter β_0.

5. Conclusion

Regression analysis was used to create a series plot for the separate service expenditures and their combined plot, which revealed an upward trend for the overall plot and implied a linear relationship between the expenditures and Nigeria's total GDP. To select the significant parameters and test for model equality, the Durbin method of estimating a multiple regression model was used. Following that, the data was split into two models: Military Government and Civil Government. All of the model parameter estimates were not significant at 5%, which appears to be skewed. Agriculture expenditure (X_2), Public Administration expenditure (X_4), and other factors not included in the model, which is represented by the constant parameter β_0, were the only three variables that were significant.

The Military Government Period Model, which has the highest R-square and R-square adjusted values with 99.8%, the smallest AIC and BIC values of 170.356 and 174.134, and the highest F-value of 4494.669 calculated, was revealed to be the "best" model among the three regression models in Table 5. As a result, model (1) of estimation is the best, as it has the most impact on Nigeria's total GDP.

6. Recommendations

This study therefore recommended based on the findings that:

1. The government should make an effort to expand its health-care spending in order to reach out to citizens in rural areas. People living in rural areas will be in good health as a result of the increase in her health expenditure, allowing them to continue their everyday activities of fishing and farming. On the other hand, they should help provide free health services such as prenatal care, maternity care, and children aged 0 to 5, among others. It will improve the health of rural residents while also helping the government meet its welfare goals.
2. Government should devote a smaller amount of its budget to recurrent spending and focus more on capital spending, such as agriculture, education, and health, as it is the primary driver of economic growth.

3. The Federal Government of Nigeria should raise its spending on highway projects (Public Administration) since this will provide the necessary infrastructure to boost private and public sector productivity, facilitate the distribution of raw and finished goods, and boost economic growth.
4. Capital and recurrent service expenditures should be allocated primarily toward productive economic activity. This will promote economic activity and, potentially, counteract the negative impact on economic growth.

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COMPARATIVE STUDY OF NUMERICAL APPROXIMATION SCHEMES FOR LAPLACIAN OPERATOR IN POLAR MESH SYSTEM ON 9-POINTS STENCIL INCLUDING MIXED PARTIAL DERIVATIVE BY FINITE DIFFERENCE METHOD

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Abstract:

In this resech paper we have discussed on two different types of discretization schmes for laplacian operator which carries nine points stencil including two pair of lines symmeical and asymmetrical from planned molecule.it is well organized and modicfied five point scheme which has been devloped on finite difference method.FDM is a method which is being used for decretization of of PDES as well as ODES.In this paper we will use two different types of scheme which are devloped by FDM on polar coordinate system and in last we will discuss on error analysis,staibility and graphically behaviour of both scheme.

Keywords: *Finite differences method*

Introduction:

The discret operator of isotropic laplacian equation has a dynamic role in several fundamental problems for computational simmulation modeling.The accuracy of FDM scheme is associated to the increases of stencils and comprehand to aper order in intentional directon.The proposed molecule of FDM technique is explained on isotropic that shows the procedure about the discrtization of laplacian on polar coordinate system and which is being carried out on the explicit technique of FDM. Intimate the scientific outcomes,the performances of polar coordinate system is reflected to peg away in his anistropy. Partial differential equation is being discrtized on appropriate substitue of polar gird system with boundary value condition.The constancy,illustration and equilibrium consideration will be conceded out for proposed scheme.

Importance of Laplacian operator

There are several schemes present which have been used for the derivation of differentiation and numerical results but discretization of laplacian operator keeps dynamic role. The laplacian operator is denoted by $\nabla^2 p$ .where u shows the property of operator which is non similar infield space which characterizes Cartesian coordinate system and keeps so many physical properties and also used for solving PDES and ODES for laplacian equation just like diffusion model of heat,wave spreading, gradient curl, divergence [1].For solution of physical and mathematical models and task the laplacian operator is very significant among the above scale. Laplacian operator contributes in many fields just like classical, quantum wave instead of this it is also effected on computational or mathematical models [2].There are huge application of laplacian operator in engineering for examples electricity ,fluid flow dynamics and consistent heat condition and so many other [3]. The discrete laplacian operator has operated several different projects of geometric dispensation, editing of mesh, interpolation of shape and reconstruction of surface are the best examples [4]. The linear time independent (BVC) in an essential class of problems which is being constituted by Laplacian and Poisson equations. There are many classical and absolute gird systems where the laplacian operator is dissociable. For linear ordinary differential equation with boundary value condition the numerical techniques are often applied. Modern numerical and mathematical techniques are used for solution of problems of of higher order partial and ordinary differential approximation. Finite difference approximation have three significant types in engineering fields just like finite difference method, finite element method and finite volume method and the qualities of these methods are solving the PDES and ODES [5].

For the estimation of gradient function the finite difference technique is being used in several mathematical and computational fields as in the sense of brink gird in the processing of image which is in considerable in numerical derivation especially for first derivative. For different structure of flow in the field of scientific fluid dynamics the finite difference techniques is very valuable [6].FDM is broadly used for the access to perform with FT (Fourier transform) to carried out the BVP in the girds of FDM which are used for large amount in the problems especially in nonlinear equations. [7] in addition it is being used for second order partial differential equation in term of time independent.[8-12].

Methodology.

Scheme 1: Derivation has been taken from Tailor series.

$$\nabla^2 P = \frac{\partial^2 P}{\partial r^2} + \frac{1}{r}\frac{\partial P}{\partial r} + \frac{1}{r^2}\frac{\partial^2 P}{\partial \theta^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots(1)$$

$$\frac{\partial^2 P}{\partial r^2} = \frac{1}{4(\Delta r)^2}\left[P_{(i+1,j+1)} + P_{(i-1,j+1)} + P_{(i+1,j-1)} + P_{(i-1,j-1)} + 2P_{(i+1,j)} + 2P_{(i-1,j)} - 8P_{(i,j)}\right] \ldots\ldots\ldots(2)$$

$$\frac{\partial^2 P}{\partial \theta^2} = \frac{1}{4(\Delta \theta)^2}\left[P_{(i+1,j+1)} + P_{(i-1,j+1)} + P_{(i+1,j-1)} + P_{(i-1,j-1)} + 2P_{(i,j-1)} + 2P_{(i,j+1)} - 8P_{(i,j)}\right] \ldots\ldots\ldots\ldots..(3)$$

$\alpha = \left[\dfrac{r^2.\Delta r^2.\Delta\theta^2}{2(\Delta r^2 + \Delta\theta^2.r^2)}\right]$ is a weighting factor.

$$\nabla^2 P_{(i,j)} = [\alpha].\left[\begin{array}{l} \dfrac{1}{4(\Delta r)^2}\left[P_{(i+1,j+1)} + P_{(i+1,j-1)} + P_{(i-1,j+1)} + P_{(i-1,j-1)} + 2P_{(i+1,j)} + 2P_{(i-1,j)}\right] + \\ \dfrac{1}{r}\dfrac{1}{8(\Delta r)}\left[P_{(i+1,j+1)} + P_{(i+1,j-1)} - P_{(i-1,j+1)} - P_{(i-1,j-1)} + 2P_{(i+1,j)} - 2P_{(i-1,j)}\right] + \\ \dfrac{1}{r^2}\dfrac{1}{4(\Delta\theta)^2}\left[P_{(i+1,j+1)} + P_{(i-1,j+1)} + P_{(i+1,j-1)} + P_{(i-1,j-1)} + 2P_{(i,j-1)} + 2P_{(i,j+1)}\right] \end{array}\right] - P_{(i,j)}$$

……(4)

Scheme 2: Derivation has been taken from Tailor series.

$$\frac{\partial^2 P}{\partial r^2} = \frac{1}{3(\Delta r)^2}\left[\begin{array}{l} P_{(i+1,j+1)} - 2P_{(i,j+1)} + P_{(i-1,j+1)} + P_{(i+1,j)} - 2P_{(i,j)} \\ +P_{(i-1,j)} + P_{(i+1,j-1)} - 2P_{(i,j-1)} + P_{(i-1,j-1)} \end{array}\right] \ldots\ldots\ldots\ldots\ldots\ldots\ldots(5)$$

$$\frac{\partial^2 P}{\partial \theta^2} = \frac{1}{3(\Delta \theta)^2}\left[\begin{array}{l} P_{(i+1,j+1)} - 2P_{(i+1,j)} + P_{(i+1,j-1)} + P_{(i,j+1)} - 2P_{(i,j)} \\ +P_{(i,j-1)} + P_{(i-1,j+1)} - 2P_{(i-1,j)} + P_{(i-1,j-1)} \end{array}\right] \ldots\ldots\ldots\ldots\ldots(6)$$

$$\alpha = \left[\frac{3r^2.\Delta r^2.\Delta\theta^2}{2(\Delta r^2 + \Delta\theta^2.r^2)} \right] \text{is a weighting Factor}$$

$$\nabla^2\omega_{(i,j)} = [\alpha].\left[\begin{array}{l} \frac{1}{3(\Delta r)^2}\left[P_{(i+1,j+1)} - 2P_{(i,j+1)} + P_{(i-1,j+1)} + P_{(i+1,j)} - 2P_{(i,j)} + P_{(i-1,j)} + P_{(i+1,j-1)} - 2P_{(i,j-1)} + 2P_{(i-1,j-1)}\right] + \\ \frac{1}{r}\frac{1}{6(\Delta r)}\left[P_{(i+1,j+1)} + P_{(i+1,j-1)} - P_{(i-1,j+1)} - P_{(i-1,j-1)} + P_{(i+1,j)} - P_{(i-1,j)}\right] + \\ \frac{1}{r^2}\frac{1}{3(\Delta\theta)^2}\left[P_{(i+1,j+1)} - 2P_{(i,+1j)} + P_{(i+1,j-1)} + P_{(i,j+1)} - 2P_{(i,j)} + P_{(i,j-1)} + P_{(i-1,j+1)} - 2P_{(i-1,j)} + P_{(i-1,j-1)}\right] \end{array} \right] - P_{(i,j)}$$

Results with Example:

For the discretization scheme following notations are adopted. $r_a = 3$ Fixed in computation

$p_{i,j} = r^3 sin\theta \quad and\ r_a = i(\Delta r) \quad i = 0,1,2,3 \ldots\ldots n$ And

$\theta_j = j(\Delta\theta) \qquad j = 0,1,2,3,\ldots,m$

In the polar mesh system following values of the constants are chosen $\Delta r = 0.1$, $\Delta\theta = \pi/180 radians$

		Result of Scheme 1			Result of Scheme 2		
r	**Radian**	**Exact Value**	**Approximate value**	**Error**	**Exact Value**	**Approximate value**	**Error**
3.1	0.017453293	0.009074857	0.009089024	1.42E-05	0.009074857	0.009089024	-1.42E-05
3.1	0.034906585	0.01814971	0.018178048	-2.83E-05	0.018149711	0.018178048	-2.83E-05
3.1	0.052359878	0.05235988	0.027267067	-4.25E-05	0.052359878	0.027224567	-4.25E-05
3.1	0.06981317	0.03629941	0.036356088	-5.67E-05	0.06981317	0.036299415	-5.67E-05
3.1	0.08726646	0.04537427	0.04544510	-7.08E-	0.08726646	0.04537426	-7.08E-05

	3		3	05	3	7	
3.2	0.017453293	0.0099817	0.009996327	-1.46E-05	0.017453293	0.009981704	-1.46E-05
3.2	0.034906585	0.01996341	0.019992653	-2.92E-05	0.034906585	0.019963406	-2.92E-05
3.2	0.052359878	0.02994511	0.029988976	-4.39E-05	0.052359878	0.029945109	-4.39E-05
3.2	0.06981317	0.03992681	0.039985303	-5.85E-05	0.06981317	0.039926808	-5.85E-05
3.2	0.087266463	0.0499085	0.04998162	-7.31E-05	0.087266463	0.049908504	-7.31E-05
3.3	0.017453293	0.01094704	0.010962118	-1.51E-05	0.017453293	0.010947037	-1.51E-05
3.3	0.034906585	0.02189441	0.021924233	-3.02E-05	0.034906585	0.021894071	-3.02E-05
3.3	0.052359878	0.03284111	0.032886349	-4.52E-05	0.032841105	0.032886349	-4.5244E-05
3.3	0.06981317	0.04378814	0.043848459	-6.03E-05	0.043788135	0.043848459	-6.0324E-05
3.3	0.087266463	0.05473516	0.054810569	-7.54E-05	0.054735161	0.054810569	-7.5407E-05

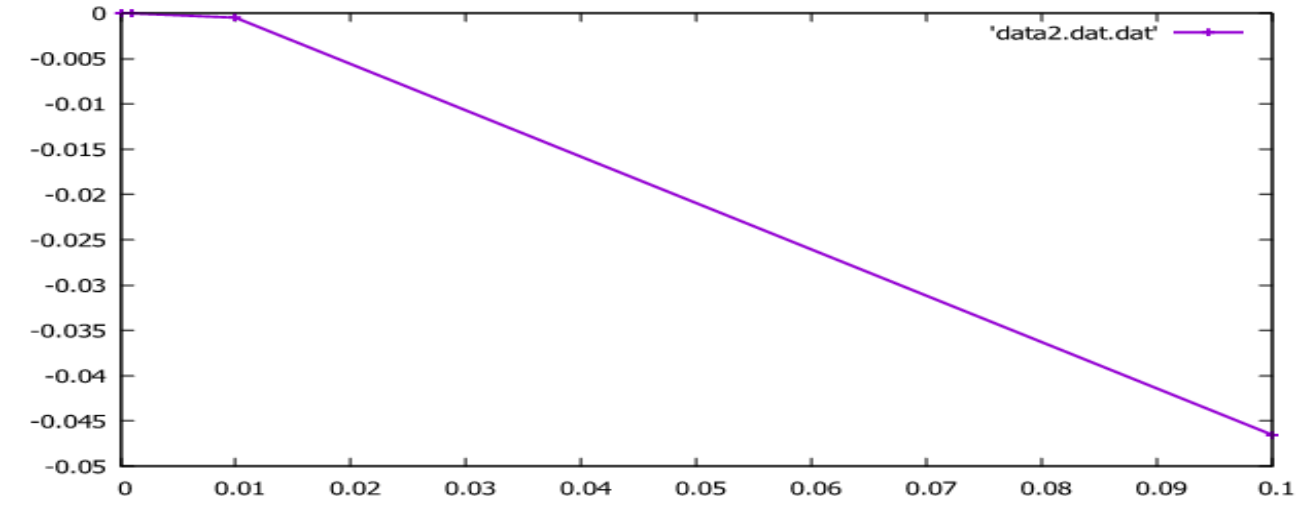
Results of Developed Scheme

ERROR ANALYSIS OF SCHEMES.

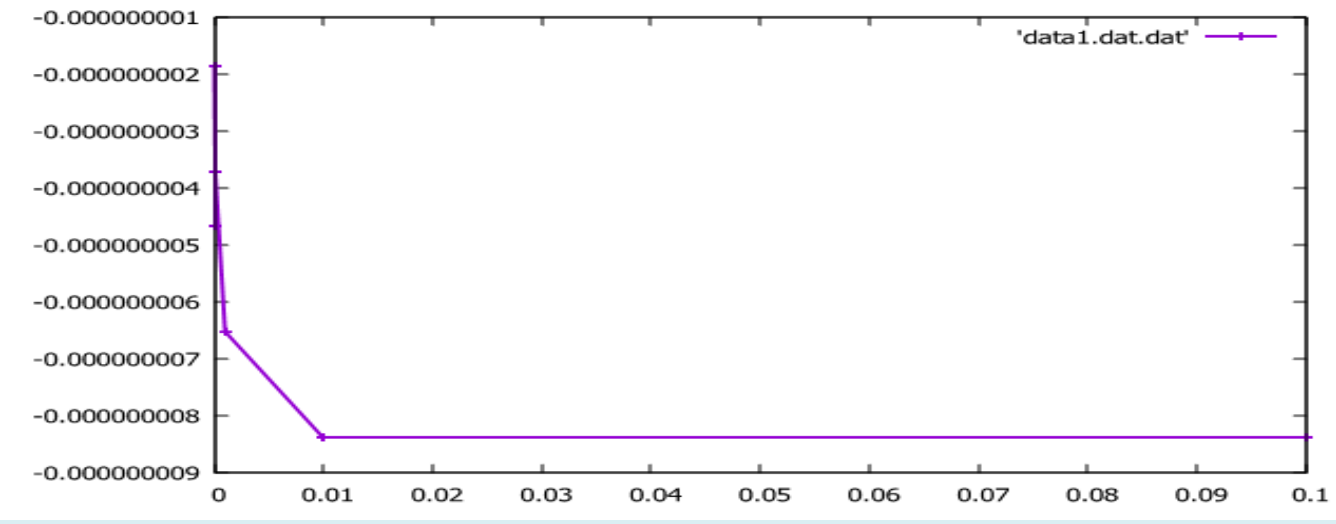
Error of Scheme 1		Error of Scheme 2	
Change in Δr	*ChangeinError*	*Changein Δr*	*ChangeinError*
0.000001	-5.7220459E-06	0.000001	-1.86264515E-09
0.00001	0	0.00001	-1.86264515E-09
0.0001	-3.8146973E-06	0.0001	-3.72529030E-09
0.001	-1.1440918E-05	0.001	-6.51922258E-09
0.01	-4.7492981E-04	0.01	-8.38190032E-09
0.1	-4.6508789E-02	0.1	-8.38190032E-09

Error Analysis by Graphically

Error of Scheme 1



Error of Scheme 2



Stability Results of Proposed Schemes

Scheme 1:

The estimation consequences have achieved by plummeting Δr beside error. This is taken from the suggested structure of given molecule in the discretization of Polar coordinate system. Here for observation we have reserved some changed standards of Δr which easily visualize in above figure 1 and the indicated figure shows the error decreases linearly and quickly among the standards of Δr 0.1 near to 0.00998302.So the decreasing value from 0.00998302, error converge very fast to 0.

Scheme 2:

The scientific outcomes has been found out with the help of CDS method which is being applied on discretization of laplacian operator on nine point stencil on PCS. These results have found out by reducing Δr beside error. Here the pictorial form of selected values of Δr and his error specification can be seen in figure 2 and in this figure you can easily analyzed the standards of outcomes where the error is falling lineally and hastily among the value of Δr (0.01 to 0.000966277). The convergence criteria of this scheme is very quickly to be zero because of his decreasing value Δr up to 0.00966277. The scheme 2 shows the poor stable scheme due to his more fast convergence to zero in the discretization of laplacian operator.

Review on Consequences of Schemes.

Scheme 1:

The error of this scheme is find out by the investigation on discretization of laplacian operator which is in contradiction of several reduces values of Δr and his scheme's outcomes can be visualized in table 1 and pictorial data is mentioned in figure 1.Here consequences indicates that whenever the error approaches 0 as Δr approaches 0. This can be exposed and this is being established on computational scheme of discretization of laplacian operator which is adequately unwavering.

Scheme 2:

In this scheme same kind of strategies have applied for observation of planned proposed scheme which is discretised on nine point stencil on polar coordinate system. In this scheme by the investigation, error consequences have acquired against the different outcomes of Δr which in shifted in table 2 and his molecule data can be observed in figure 2. In the results indicates that Δr will be near about zero if error will approaches zero. So for the same guidelines and applied step we can say that the crested proposed computational scheme of discretization of laplacian equation is appropriately steady and accurate.

Discussion on the Schemes

Scheme 1:

The consequences of this scheme, which is being discretized on 9 point stencil on PCS has been gained by implementing the CDS and FORTRAN language code with addition of open Dx Software.All the outcomes of discretization are developed and listed in table 1. The computational outcomes shows the accuracy and stability of scheme 1 and in these outcomes of isotropic isolated laplacian operator, the accuracy and stability will be counted more than 6 fraction points.

Scheme 2:

In this planned scheme 2, which is discretized on nine point stencil of laplacian operator in PCS. Here we have developed another CDS and FORTRAN language code to development of the some computational, scientific and graphical consequences of scheme 2. After implementing the code, the gained results of planned scheme has compared with the analytic consequences of proposed scheme 2 for checking the error analysis and stability purposes. Which can be visualized in table 2.The computational outcomes of this discretized laplacian operator displays suitable precision. In the above observation of the discretization of proposed scheme 2 the precisionvestiges more than nine unit points and the error is lesser the first scheme.

Future Work

i. For the evaluation of discretization of laplacian operator in PCS on 13,17 & 25 stencils, the approximation of Finite difference method will be applied including PDES with mixed derivatives.

ii. Work on the Literature of pdes and mixed pdes is needed by implementing the finite difference method on laplacian operator, on not only cylindrical but spherical coordinate system on 13,17 & 25 stencils as well as.

Examples

i. Solve $U_{r,\theta} = r^3 cos\theta.$

ii. Solve $U_{r,\theta} = r^3 tan\theta.$

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SOLVING A CLASS OF TWO-DIMENSIONAL NONLINEAR VOLTERRA INTEGRAL EQUATIONS AS A GENERALIZED PROBLEM OF MOMENTS

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Abstract

It will be shown that solve an equation two-dimensional Volterra nonlinear can be solved numerically applying the techniques of inverse generalized moments problem in two steps writing the Volterra's equation as a Klein-Gordon equation of the form $w_{tt} - w_{xx} = H(x,t)$, which $H(x,t)$ it is unknown. In a first step, $H(x,t)$ is numerically approximate, and in a second step we numerically approximate the solution of Klein-Gordon equation using the $H(x,t)$ previously approximated.The method is illustrated with examples.

Keywords*: Klein-Gordon, nonlinear Volterra integral equations, generalized moment problem, inverse problem.*

INTRODUCTION

We want to find $w(x,t)$ such that

$$w(x,t) - \int_0^t \int_0^x K(x,t,y,z,w(y,z))dydz = f(x,t) \quad (x,y) \in D$$

Where $w(x,t)$ is unknown function and the functions $f(x,t)$ known continuous about a D domain where $D = \{(x,t); x > 0\ , t > 0\}$ and $K(x,t,y,z,w)$ known continuous about a E domain where $E = \{(x,t,y,z); 0 \leq y \leq x,\ 0 \leq z \leq t\ ,\ x > 0\ , t > 0, -\infty < w < \infty\}$.

Also f and K are r times continuously differentiable over D and E respectively with r = 2. In that case the solution w will be r = 2 times continuously differentiable over D. The underlying space is $L^2(D)$

Note that

$$w(0,t) = f(0,t) \quad , \quad w(x,0) = f(x,0) \qquad t \geq 0 \ , \ x \geq 0$$

$$w_x(0,t) \quad , \quad w_t(x,0) \quad t \geq 0 \ , \ x \geq 0$$ they are known

Integral equations is a special topic in Applied Mathematics, as they constitute an important tool to model many problems in fields such as engineering, astrophysics, chemistry, quantum mechanics and many other fields. They are also applied in initial condition and boundary value problems for partial differential equations.

With so many applications, integral equations have been extensively studied. For example in [1] is investigated a collocation method for the approximate solution of Hammerstein integral equations in two dimensions. In [2] a numerical technique based on the Sinc collocation method is presented for the solution of two-dimensional Volterra integral equations of first and second kinds. The Sinc function properties are provided and the global convergence analysis is obtained to guarantee the efficiently of our method. In [3] a class of two-dimensional linear and nonlinear Volterra integral equations is solved by means of an analytic technique, namely the Homotopy analysis method (HAM). In [4] a numerical iterative algorithm based on combination of the successive approximations method and the quadrature formula for solving two-dimensional nonlinear Volterra integral equations is proposed. This algorithm uses a trapezoidal quadrature rule for Lipschitzian functions applied at each iterative step.

In paper [5], we develop two-step collocation (2-SC) methods to solve two-dimensional nonlinear Volterra integral equations (2D-NVIEs) of the second kind. Here we convert a 2D-NVIE of the second kind to a one-dimensional case, and then we solve the resulting equation numerically by two-step collocation methods.

In [6] the approximate solutions for two different type of two-dimensional nonlinear integral equations: two-dimensional nonlinear Volterra-Fredholm integral equations and the nonlinear mixed Volterra-Fredholm integral equations are obtained using the Laguerre wavelet method. To do this, these two-dimensional nonlinear integral equations are transformed into a system of nonlinear algebraic equations in matrix form.

In [7] se proponen new theorems of the reduced differential transform method (RDTM) for solving a class of two-dimensional linear and nonlinear Volterra integral equations (VIEs) of the second kind.

In [8] the rational Haar wavelet method has been used to solve the two-dimensional Volterra integral equations. Numerical solutions and the rate of convergence, are presented.

In [9] Using fixed-point techniques and Faber Schauder systems in Banach spaces, is obtained an approximation of the solution of two-dimensional nonlinear Volterra, Fredholm and mixed Volterra-Fredholm integral equations.

The objective of this work is to show that we can solve the problem using the techniques of inverse moments problem. We focus the study on the numerical approximation.

The interest is not to compare with the existing methods, but to present a different method to my novel criteria.

The generalized moments problem [10,11,12], is to find a function $f(x)$ about a domain $\Omega \subset R^d$ that satisfies the sequence of equations

$$\mu_i = \int_\Omega g_i(x) f(x) dx \quad i \epsilon N - - - - - - - - - - - - \quad (1)$$

where N is the set of the natural numbers, $(g_i(x))$ is a given sequence of functions in $L^2(\Omega)$ linearly independent known and the succession of real numbers $\{\mu_i\}_{i \epsilon N}$ are known data.

The moments problem is an ill-conditioned problem in the sense that there may be no solution and if there is no continuous dependence on the given data [10,11,12]. There are several methods to build regularized solutions. One of them is the truncated expansion method [10].

This method is to approximate (1) with the finite moments problem

$$\mu_i = \int_\Omega g_i(x) f(x) dx \quad i = 1,2, \dots, n. - - - - - - - - \quad (2)$$

where it is considered as approximate solution of to $p_n(x) = \sum_{i=0}^{n} \lambda_i \phi_i(x)$, and the functions $\{\phi_i(x)\}_{i=1,..,n}$ result of orthonormalize $\langle g_1, g_2, \dots, g_n \rangle$ being λ_i the coefficients based on the data μ_i. In the subspace generated by $\langle g_1, g_2, \dots, g_n \rangle$ the solution is stable. If $n \epsilon N$ is chosen in an appropriate way then the solution of (2) it approaches the solution of the problem (1).

In the case where the data μ_i are inaccurate the convergence theorems should be applied and error estimates for the regularized solution (pages 19 - 30 of [10]).

ARTICLE ORGANIZATION

To find $w(x,t)$

$$w(x,t) - \int_0^t \int_0^x K(x,t,y,z,w(y,z)) dy dz = f(x,t) \quad (x,y) \epsilon D$$

Where $w(x,t)$ is unknown function and the function $f(x,t)$ known continuous about a D domain where $D = \{(x,t); x > 0, t > 0\}$ and $K(x,t,y,z,w)$ known continuous about a E

domain where $E = \{(x,t,y,z); 0 \le y \le x,\ 0 \le z \le t\ ,\ x > 0\ , t > 0, -\infty < w < \infty\}$

$$w(0,t) = f(0,t) \quad , \quad w(x,0) = f(x,0) \qquad t \ge 0 \quad x \ge 0$$

$$w_x(0,t) \quad , \quad w_t(x,0) \quad t \ge 0 \qquad x \ge 0$$ they are known

We will do it in two steps. The next section describes the first step. The section that follows explains the second step. Then it is explained how the generalized moment problem is solved with the truncated expansion method. Finally the numerical example and the conclusions.

FIRST STEP

We consider

$$w(x,t) - \int_0^t \int_0^x K(x,t,y,z,w(y,z))dydz = f(x,t) \quad ----- \quad (3)$$

We differentiate (3) with respect to *t* twice:

$$w_{tt}(x,t) = \left(\int_0^t \int_0^x K(x,t,y,z,w(y,z))dydz\right)_{tt} + f_{tt}(x,t) = R(x,t)$$

$$w_{tt}(x,t) - w_{xx}(x,t) = R(x,t) - w_{xx}(x,t) = H(x,t)$$

the conditions are:

$$w(0,t) = f(0,t) \quad , \quad w(x,0) = f(x,0) \qquad t \ge 0 \quad x \ge 0$$

Note that

$$w_t(x,t) = \int_0^t \int_0^x K_t(x,t,y,z,w(y,z))dydz + \int_0^x K(x,t,y,t,w(y,t))dy + f_t\,(x,t)$$

Therefore

$$w_t(x,0) = \int_0^x K(x\,,0,y,0,w(y,0))dy + f_t(x,0)$$

Analogously

$$w_x(0,t) = \int_0^t K(0\,,t,0,z,w(0,t))dz + f_x(0,t)$$

That is $w_x(0,t)\quad t \geq 0\quad ,\quad w_t(x,0)\qquad \boldsymbol{x} \geq 0$ they are known

We take as an auxiliary function

$$u(m,r,x,t) = e^{-mx}e^{-rt}$$

Note that $u_{xx} = m^2u$ and $u_{tt} = r^2u.$

We consider

$$w_{xx}(x,t) - w_{tt}(x,t) \ = -H(x,t)$$

We define the vector field

$$F^* = \big(F_1(w), F_2(w)\big) = (w_x, -w_t)$$

Since $div(F^*) = -H(x,t)$ we have to:

$$\iint_D u\, div(F^*)dA = \iint_D u\,\big(-H(x,t)\big)dA$$

In addition, as $udiv(F^*) = div(uF^*) - F^*.\nabla u,$ so

$$\iint_D u\, div(F^*)dA = \iint_D div(uF^*)dA - \iint_D F^*.\nabla u dA$$

Where $\nabla u = (u_x, u_t).$

And

$$\iint_D F^* . \nabla u dA = \iint_D (F_1 u_x + F_2 u_t) dA$$

Integrating by parts with respect to x:

$$\iint_D F_1 \, u_x dA = \int_0^\infty \int_0^\infty F_1 \, u_x dx dt =$$

$$= \int_0^\infty \big(-w(a_1, t) u_x(m, r, a_1, t)\big) dt - \iint_D w u_{xx} \, dA =$$

$$\int_0^\infty \big(-w(0, t) u_x(m, r, 0, t)\big) dt - \iint_D w \, (m)^2 u dA$$

Analogously

$$\iint_D F_2 \, u_t dA = \int_0^\infty \int_0^\infty F_2 \, u_t dx dt = \int_0^\infty \big(-w(x, 0) u_t(m, r, x, 0)\big) dt - \int_0^\infty \int_0^\infty w(r)^2 u dx dt$$

then

$$\iint_D F^* . \nabla u dA = \int_0^\infty \big(-w(0, t) u_x(m, r, 0, t)\big) dt -$$

$$- \int_0^\infty \big(-w(x, 0) u_t(m, r, x, 0)\big) dt - \iint_D w \, u(m^2 - r^2) dA = A(m, r)$$

On the other hand,

$$\int_C (u F^*) . n ds =$$

$$= \int_0^\infty u(m, r, x, 0) w_t(x, 0) dx - \int_0^\infty u(m, r, 0, t) w_x(0, t) dt = G(m, r)$$

$$\therefore \iint u \big(-H(x, t)\big) dA = G(m, r) - A(m, r) + \iint_D w \, u(m^2 - r^2) dA$$

So if we do $r = m$:

$$\iint u\left(-H(x,t)\right)dA = G(m,m) - A(m,m)$$

That is

$$\iint u\left(H(x,t)\right)dA = -G(m,m) + A(m,m) = \phi(m)$$

To solve this integral equation we give integer values to m, $m = 0,1,2,\dots n$

Then

$$\int_{a_1}^{\infty} H(x,t)R_m(x,t)dx = \phi(m) = \mu_m - - - - - - - - (4)$$

We interpret (4) as a generalized moments problem.

$p_{1n}(x,t)$ is the numerical approximation found with the truncated expansion method for $H(x,t)$, with $R_m(x,t) = u(m,m,x,t) = e^{-mx}\,e^{-mt}$ $m = 0,1,2,\dots n$ where n is conveniently chosen.

In section 4 the truncated expansion method will be explained in detail and a theorem will be given that explains what is the accuracy of the approximation found by this method.

APPROACH TO $w(x,t)$ - SECOND STEP

To find an approximation of $w(x,t)$ a similar approach to the previous one is made where $H(x,t)$ is replaced by $p_{1n}(x)$ and we do not consider $r = m$.

We take the auxiliary function $u(m,r,x,t) = e^{-(m+1)x}e^{-(r+1)t}$.

Note that $u_{xx} = (m+1)^2u$ and $u_{tt} = (r+1)^2u$.

We define the vector field $F^* = \left(F_1(w), F_2(w)\right) = (w_x, -w_t)$

Since $div(F^*) = -H(x,t)$ we have to:

$$\iint_D u\,div(F^*)dA = \iint_D u\left(-H(x,t)\right)dA$$

In addition, as $udiv(F^*) = div(uF^*) - F^*.\nabla u$, so:

$$\iint_D u\,div(F^*)dA = \iint_D div(uF^*)dA - \iint_D F^*.\nabla u dA$$

Thus

$$\therefore \iint u\left(-H(x,t)\right)dA = G(m,r) - A(m,r) + \iint_D w\,u((m+1)^2-(r+1)^2)dA$$

Then

$$\iint_D w\,u((m+1)^2-(r+1)^2)dA = -G(m,r) + A(m,r) + \iint_D u\,H(x,t)dA$$

where $G(m,r)\ y\ A(m,r)$ they are like before.

We replace $H(x,t)$ by $p_{1n}(x)$ and then

$$\iint_D w(x,t)H_{mr}(x,t)dA = \frac{-G(m,r) + A(m,r) + \iint_D u\,p_{1n}(x)\,dA}{((m+1)^2-(r+1)^2)} = \phi(m,r)$$
$$= \mu_{mr} - - - -(5)$$

where

$$H_{m,r}(x) = u(m,r,x,t)$$

We can consider (5) as a two-dimensional generalized moment problem if we discretize giving m and r non-negative integer values $m = 0,1,2,\dots n_1$; $r = 0,1,2,\dots,n_2$, where n_1 and n_2 are conveniently chosen.

An approximation $p_{2n}(x,t)$ is found by the truncated expansion method for $w(x,t)$ where $n = n_1 \times n_2$.

SOLUTION OF THE GENERALIZED MOMENTS PROBLEM

We can apply the detailed truncated expansion method in [12] and generalized in [13] and [14] to find an approximation $p_n(x,t)$ for the corresponding finite problem with $i = 0,1,2,\dots,n$ where n is the number of moments μ_i. We consider the basis $\phi_i(x,t)\quad i = 0,1,2,\dots,n$ obtained by applying the Gram-Schmidt orthonormalization process on $H_i(x,t)\quad i = 0,1,2,\dots,n$.

We approximate the solution $w(x,t)$ with [12] and generalized in [13] y [14]:

$$p_n(x,t) = \sum_{i=0}^{n} \lambda_i \phi_i(x,t)$$

where

$$\lambda_i = \sum_{j=0}^{i} C_{ij}\,\mu_j \quad i = 0,1,2,\dots,n$$

And the coefficients C_{ij} verify

$$C_{ij} = \left(\sum_{k=j}^{i-1} (-1) \frac{\langle H_i(x,t) | \phi_k(x,t) \rangle}{\|\phi_k(x,t)\|^2} C_{kj} \right) . \|\phi_i(x,t)\|^{-1} \quad 1 < i \leq n \,; 1 \leq j < i \,.$$

The terms of the diagonal are $\|\phi_i(x,t)\|^{-1}$ $i = 0,1,\dots,n$.

The proof of the following theorem is in [14,15].

In [15] the demonstration is made for b_2 finite. If $b_2 = \infty$ instead of taking the Legendre polynomials we take the Laguerre polynomials. En [16] the demonstration is made for the one-dimensional case.

This Theorem gives a measure about the accuracy of the approximation.

Theorem

We considerer $b_1 = b_2 = \infty$.

Sea $\{\mu_i\}_{i=0}^{n}$ be a set of real numbers and suppose that $f(x,t)\, \epsilon\, L^2((a_1,b_1) \times (a_2,b_2))$ for two positive numbers ε and M verify:

$$\sum_{i=0}^{n} \left| \iint_E H_i\,(x,t) f(x,t) dt dx - \mu_i \right|^2 \leq \varepsilon^2 \quad .$$

$$\iint_E \left(x\, {f_x}^2 + t\, {f_t}^2 \right) Exp[x+t] dt dx \leq M^2 \text{ ------- (6).}$$

And it must be fulfilled that

$$t^i f(x,t) \to 0 \quad \text{si} \quad t \to \infty \quad \text{para todo} \quad i \in N$$

then

$$\int_{a_1}^{b_1}\int_{a_2}^{b_2}|f(x,t)|^2\,dtdx \le min_i\left\{\|C^TC\|\varepsilon^2+\frac{1}{8(n+1)^2}M^2; i=0,1,\dots,n\right\}$$

where C it is a triangular matrix with elements $C_{ij}\quad (1<i\le n;\ 1\le j<i)$

and

$$\int_{a_1}^{b_1}\int_{a_2}^{b_2}|f(x,t)-p_n(x,t)|^2\,dtdx \le \|C^TC\|\varepsilon^2+\frac{1}{8(n+1)^2}M^2.$$

If b_2 it is not infinite then (6) change by

$$\int_{a_2}^{b_2}\int_{a_1}^{b_1}((b_1-a_1)^2f_x^2+(b_2-a_2)^2f_t^2)dx\,dt \le M^2$$

NUMERICAL EXAMPLES

Example 1

We consider the equation

$w(x,t)-\int_0^t\int_0^x e^{-(x+y)}\sqrt{w(y,z)}dydz = e^{-t-\frac{x^2}{20}}-2e^{10-\frac{t}{2}-x}(-1+e^{\frac{t}{2}})\sqrt{10\pi}\,(Erf(\sqrt{10})+Erf(\frac{-20+x}{2\sqrt{10}}))$

whose solution is: $w(x,t)=e^{-t-\frac{x^2}{20}}$

we take n=5 moments and is approaching $p_{1n}(x,t)\approx H(x,t)$ where the accuracy is

$$\int_0^\infty\int_0^\infty e^{-x-t}\,(p_{1n}(x,t)-H(x,t))^2dtdx = 0.170473$$

We consider this norm since the basis is $e^{-mx}\ e^{-mt} \quad m = 1,2,\ldots 5.$

In the Fig.1 the graphics of: $p_{15}(x,t)$ (magenta color) and $H(x,t)$ (light blue color) are superimposed.

we take $n = 6$ moments and is approaching $w(x,t)$ where the accuracy is

$$\int_0^\infty \int_0^\infty e^{-2x-3t}\left(p_{26}(x,t) - w(x,t)\right)^2 dtdx = 0.00899724$$

We consider this norm since the basis is $\{e^{-3t-2x}, e^{-4t-2x}, e^{-4t-3x}, e^{-5t-3x}, e^{-5t-4x}, e^{-6t-4x}\}$.

In the Fig. 2 the graphics of:$p_{26}(x)$(magenta color) and $w(x,t)$ (light blue color) are superimposed.

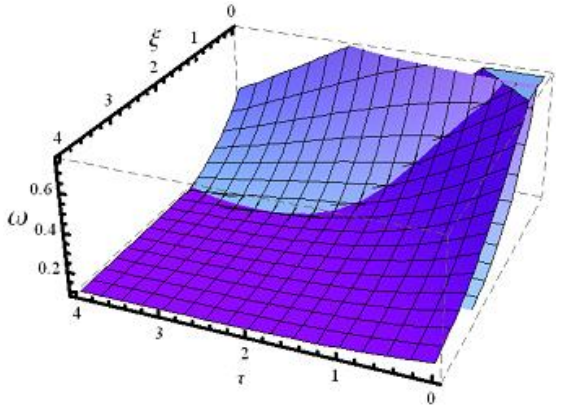

Fig. 1: $p_{15}(x,t)$ and $H(x,t)$ example 1

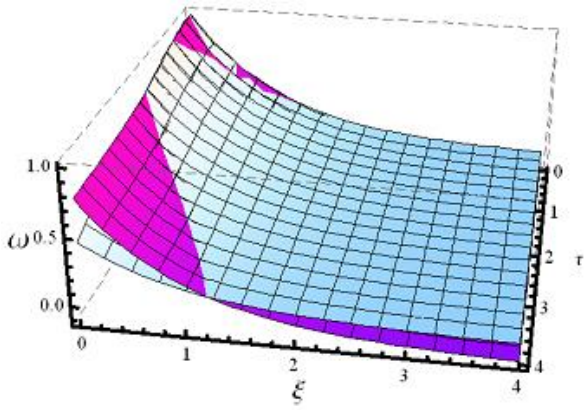

Fig. 2: $w(x,t)$ and $p_{26}(x,t)$ example 1

Example 2

We consider the equation

$$w(x,t) - \int_0^t \int_0^x (sin(x)+2)e^{-(x-y)}\sqrt{w(y,z)}dydz = \frac{e^{-x-t}}{1+x}$$
$$- 2\sqrt{2}\left[-2e^{-x}(1-e^{-\frac{t}{2}})DawsonF\left(\frac{1}{\sqrt{2}}\right)\right.$$
$$\left. - 2\sqrt{1+x}\left(\sqrt{\frac{e^{-x-t}}{1+x}} - \sqrt{\frac{e^{-x}}{1+x}}\right)DawsonF\left(\sqrt{\frac{1+x}{2}}\right)\right](2+sin(x))$$

whose solution is: $w(x,t) = \frac{e^{-(t+x)}}{1+x}$

we take n=5 moments and is approaching $p_{1n}(x,t) \approx H(x,t)$ where the accuracy is

$$\int_0^\infty \int_0^\infty e^{-x-t}\,(p_{1n}(x,t) - H(x,t))^2 dtdx = 0.237494$$

We consider this norm since the basis is $e^{-mx}\,e^{-mt} \quad m = 1,2,\ldots 5.$

In the Fig.1 the graphics of: $p_{15}(x,t)$ (magenta color) and $H(x,t)$ (light blue color) are superimposed.

we take $n = 6$ moments and is approaching $w(x,t)$ where the accuracy is

$$\int_0^\infty \int_0^\infty e^{-2x-3t}\big(p_{26}(x,t) - w(x,t)\big)^2\, dtdx = 0.00838925$$

We consider this norm since the basis is $\{e^{-3t-2x}, e^{-4t-2x}, e^{-4t-3x}, e^{-5t-3x}, e^{-5t-4x}, e^{-6t-4x}\}$.

In the Fig. 4 the graphics of: $p_{26}(x)$ (magenta color) and $\varphi(x)$ (light blue color) are superimposed.

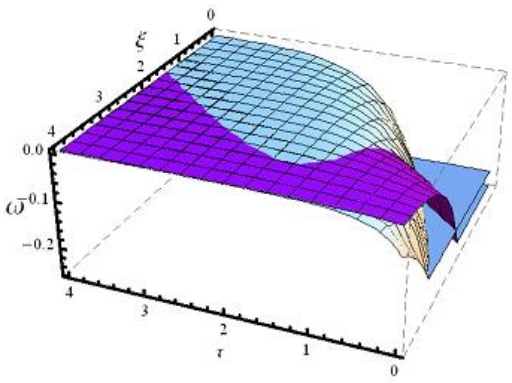

Fig. 3: $p_{15}(x)$ and $H(x,t)$ example 2

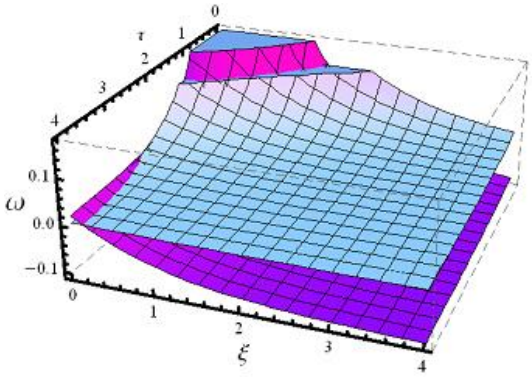

Fig. 4:$w(x,t)$ and $p_{26}(x,t)$ example 2

CONCLUSION

To find $w(x,t)$

$$w(x,t) - \int_0^t \int_0^x K(x,t,y,z,w(y,z))dydz = f(x,t) \quad (x,y) \in D$$

where $w(x,t)$is unknown function and the functions $f(x,t)$ and $K(x,t,y,z,w)$ known continuous about a D domain and E domain respectively with $D = \{(x,t); x > 0\ , t > 0\}$ and $E = \{(x,t,y,z); 0 \le y \le x,\ 0 \le z \le t\ ,\ x > 0\ , t > 0, -\infty < w < \infty\}$ we will do it in two steps.

We differentiate with respect to *t* twice and consider the equation in partial derivatives of second order

$$w_{xx}(x,t) - w_{tt}(x,t) = -H(x,t)$$

with $H(x,t)$ unknown.

1. In a first step we approximate $H(x,t)$ with $p_{1n}(x)$ solving the integral equation

$$\int_{a_1}^{\infty} H(x,t)R_m(x,t)dx = \phi(m) = \mu_m$$

which we interpret as a generalized moments problem and $R_m(x,t) = u(m,m,x,t) = e^{-mx}\, e^{-mt} \quad m = 0,1,2,\dots n$ where n is conveniently chosen.

2. To find an approximation of $w(x,t)$ we consider:

$$\iint_D w(x,t) H_{mr}(x,t)dA = \frac{-G(m,r) + A(m,r) + \iint_D u\, p_{1n}(x)dA}{((m+1)^2 - (r+1)^2)} = \phi(m,r) = \mu_{mr}$$

where $H_{m,r}(x) = u(m,r,x,t)$. We can consider it as a two-dimensional generalized moment problem if we discretize giving *m* and *r* non-negative integer values $m\ and\ r$. An approximation $p_{2n}(x,t)$ is found by the truncated expansion method for $w(x,t)$ where $n = n_1 \times n_2$.

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AN ASSESSMENT STUDY OF SOME ASIAN COUNTRIES ON THE BASIS OF PHYSICAL QUALITY OF LIFE INDICATORS

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Abstract:

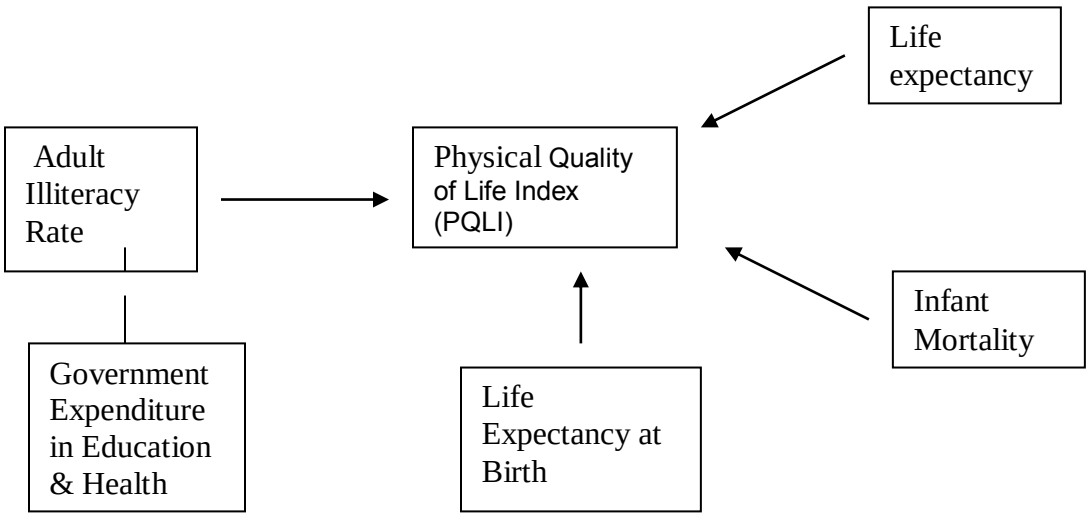
The physical quality-of-life index (PQLI) is an attempt to measure the quality of life or well-being of a country. The value is a single number derived from basic literacy rate, infant mortality, and life expectancy at age one. Quality of life is an umbrella concept that refers to all aspects of a person's life, including physical health; psychological well-being; social well-being; financial well-being; some approaches to quality of life emphasizes the social and psychological aspects of life, and contrast quality of life with quality of care. The growing attention to quality of life and the desire to minimize the negative effects of disease and health care on this quality reflects the highest of public health aspirations. Adult literacy has also risen from 46-70%. Developing world today is healthier, better fed and batter educated. But program has been far from even. The no. of people living in poverty continues to grow. There is almost no improvement IMR &. in education. In poor countries South Asia's IMR today are about the lame in early 1970. So I think there exist some relation between socioeconomic conditions &.health status. In a country, WHO has. listed these two indicators 1.Health Indicators and 2.Physical Quality of

Life Indicator. Among those here I want to consider only the following as social indicators:Y: 1. IMR 2. Adult illiteracy & X: 1. Public exp. on education. % GNP 2. Public expo on health % GDP.I have taken a linear combination of expenditure on education % GNP (Y1) & expenditure on health % GDP (Y2) as U to represent a economic (welfare) condition of a country against this I have taken a linear combination of IMR per 1000 (X1) and adult illiteracy % rate (X2) as V to represent PQL of a country. After using Canonical correlation I have reached a conclusion that the increase of expenditure on health and education purpose implies decrease of IMR & illiteracy.In 1st Quadrant contains only Saudi Arabia whose expenditure on health and education is high but illiteracy rate also high (exceptional one).And 2nd Quadrant contains those countries whose expenditure on health and education is high illiteracy rate and IMR is low. The 4th Quadrant contains those countries whose expenditure is lower and illiteracy rate is higher comparatively. Here Iraq Pakisthan , Bangaledesh and Nepal lie.There is no country in 3rd quadrant whose expenditure is less and illiteracy is also low. After concrete analysis and from the graph we can rank the 14 Asian Countries according to their better Physical Quality of Life as follows:1. Japan 2. Korea Republic 3. Mongolia 4. Malayasia 5. Srilanka 6. Viatnam 7. China 8. Iran 9. India 10. Iraq 11. Pakisthan 12. Bangaladesh 13. Nepal 14. Saudi Arabia

Keywords: *Physical Quality of Life, Health Indicators, physical quality-of-life index, IMR, GNP,GDP.*

INTRODUCTION:

The Overseas Development Council (then under the leadership of Jim Grant) developed and publicized a measure of (physical) quality of life many years ago. It combines literacy rate, infant mortality rate, and life expectancy, using scales from the lowest to the highest values in the global system. It weights the three scales equally. The literacy rate is, in turn, a function of the per capita spending levels on education, estimated cross-sectionally.



The physical quality-of-life index (PQLI) is an attempt to measure the quality of life or well-being of a country. The value is a single number derived from basic literacy rate, infant mortality, and life expectancy at age one.

The term "quality of life" has different meanings to different people. For some researchers and clinicians, quality of life means almost anything beyond information about death and death rates. For others quality of life is an umbrella concept that refers to all aspects of a person's life, including physical health; psychological well-being; social well-being; financial well-being; some approaches to quality of life emphasize the social and psychological aspects of life, and contrast quality of life with quality of care.

By the same token, characteristics of a person, such as income, health status, mental health status, disease profiles, educational level, and housing situation can be summed to create an overall quality-of-life measure.

The growing attention is on the quality of life and the desire to minimize the negative effects of disease and health care. Quality reflects the highest of public health aspirations.

According to WHO report (1976) it was recognized that in both developed and developing countries the standard of health services the public expected was not being provided. Against this background the 30^{th} world health assembly resolves in May 1977 that the main social target of Govt. and W.H.O. is to provide them to lead a socially and economically productive life by the year 2000.

On the verge of the 21st century, dramatic, political, social and economic changes have overtaken the world. Living standards have risen over the past 25 years. Despite an increase in population from 2.9 billion people in 1970 to 4.8 billion in 1996 per capita income growth in developing countries has arranged about 1.3% a year. IMR have fallen from I.04 per thousand l.b in 1970 to 1975 to 59 in 1996. On average, life expectancy has risen by 4 month each year since 1970.

Adult literacy has also risen from 46-70%. Developing world today is healthier, better fed and batter educated. The no. of people living in poverty continue to grow. There is almost no improvement IMR &. in education. In poor countries South Asia's IMR today are about the lame in early 1970. So I think there exists some relation between socioeconomic conditions. &. health status in a country.

WHO has listed these two indicators as follows:

Health Indicators are:

1. Public expenditure on education. % GNP
2. Public expenditure on education. % GDP
3. Expenditure On social activity
4. Income distribution.
5. Popu. Growth rate
6. Work condition.
7. GNP or GDP

Physical Quality of Life Indicator:

1. IMR
2. Adult illiteracy rate
3. Human poverty Index
4. MMR
5. Nutrition status of children
6. Life expectancy at birth
7. Low birth weight

Table-1 Showing characteristics of development indicators

Indicators	IMR(per 1000)	Adult illiteracy % rate	Expenditure on Education % GNP	Expenditure on Health % GDP
Country	Y1	Y2	X1	X2
Japan	11	1	3.8	7.2
China	1722	18.5	2.3	3.8
Korea Rep.	14	2	3.7	5.4
India	3671	48	3.5	5.6

Malaysia	13	16.5	5.3	2.5
Saudi Arabia	33	37.2	5.5	3.1
Srilanka	11	9.8	3.1	1.9
Nepal	145	72.5	2.9	5
Mongolia	8	17.1	5.6	6.7
Bangladesh	537	61.9	2.3	2.4
Pakistan	819	62.2	3	3.5
Vietnam	157	6.3	2.7	5.2
Iraq	1	42	3.5	1.5
Iran	189	21.4	4	4.8
Data Source: World Development Indicator & Human Development Report.				
AVG =	522.214	29.7429	3.65714	4.18571

Literacy Rate:-The percentage of people who can with understanding both Read & write a short, simple statement on their everyday life. Literacy rate 111 a sound Indicator of social development.

METHODOLOGY:

In this problem we define Y1,Y2 & X1,X2 as

Y1= children dying before age 1. (IMR)

Y2=adult illiteracy rate.

XL=. Public expenditure on education. % GNP

X2= Public expenditure on health % of GDP

Here I observe Yl, Y2 & X1, X2 are Correlated.

To find a correlation between Y= (Yl, Y2) &X= (X1, X2) we can use canonical correlation.

Here we take a linear combination of Y1, Y2 &. X1 , X2 as

V=L1Y1+L2Y2 & U= K1X1+K2X2 (say)

We choose L1, L2 & K1, K2 s.t Correlation. Coefficient between U & V maximum and this Correlation. Coefficient is the Canonical Correlation between U & V.

To find L1, L2 & K1, K2 we do the following:

1. Find the variance covariance matrix of X & Y and represent it by Q.

2. We do partition Q = (Q11 Q12

Q21 Q22)

A=inv(Q11)Q12 inv(Q22)Q21 &

B=inv(Q22)Q21 inv(Q11)Q12

3. Find Eigen values of A & B corresponding Eigen vectors. Let the Eigen value of

A are λ1> λ2> λ3 then Eigen vector corresponding to λ1 say a will be our k

Then the Eigen value of B am μ1> μ2> μ3 then Eigen vector corresponding to μ say b will be our l .Root of maximum (λ1, μ1) is the Correlation between U & V.

We calculate λ1or μ1 & corresponding eigen vector by Power Method.

1. Set k=1 let Y 0= any orthonormal vector.

2. Zk=A Y(k-l) .

3. Find norm or Zk ie. Cardinality (Zk)

4. Yk=Zk /norm (Zk)

5. If = (approx.) Y(k-1), then norm (Zk) is the largest Eigen value & Yk is the

Corresponding Eigen vector.

6. If Yk not equal to Y(k-l), increase k by 1 & repeat the procedure until

Yk=Y(k-1) .

RESULTS AND DISCUSSION:

Table-2 Showing Averages

Indicators	Yl	Y2	Xl	X2
AVG =	522.214	29.7429	3.65714	4.18571

$$Q = \begin{bmatrix} 13658742 & 91682.97 & -4289.37 & 3372.943 \\ 91682.97 & 7529.214 & -94.0643 & -168.831 \\ -4289.37 & -94.0643 & 15.97429 & 4.091429 \\ 3372.943 & -168.831 & 4.091429 & 41.21714 \end{bmatrix}$$

$$Ql1 = \begin{bmatrix} 13658742 & 91682.97 \\ 91682.97 & 7529.214 \end{bmatrix} \qquad Q22 = \begin{bmatrix} 15.97429 & 4.091429 \\ 4.091429 & 41.21714 \end{bmatrix}$$

$$Q21 = \begin{bmatrix} -4289.37 & -94.0643 \\ 3372.943 & -168.831 \end{bmatrix} \qquad Q12 = \begin{bmatrix} -4289.37 & 3372.943 \\ -94.0643 & -168.831 \end{bmatrix}$$

$$Al = \begin{bmatrix} -0.00025 & 0.000433 \\ -0.00944 & -0.02769 \end{bmatrix} \qquad A2 = \begin{bmatrix} -297.029 & -4.9656 \\ 111.3181 & -3.60322 \end{bmatrix}$$

$$INV(Q11) = \begin{bmatrix} .079708 & -0.9707 \\ -0.9707 & 0.000145 \end{bmatrix} \qquad INV(Q22) = \begin{bmatrix} .064234 & -.00638 \\ -0.00638 & .02480 \end{bmatrix}$$

$$Al = \begin{bmatrix} -0.00025 & 0.000433 \\ -0.00944 & -0.02769 \end{bmatrix} \qquad A2 = \begin{bmatrix} -297.029 & -4.9656 \\ 111.3181 & -3.60322 \end{bmatrix}$$

$$A = Al*A2 = \begin{bmatrix} 0.122638 & -0.00031 \\ -0.27865 & 0.146668 \end{bmatrix}$$

$$Bl = \begin{bmatrix} -297.029 & -4.9656 \\ 111.3181 & -3.60322 \end{bmatrix}$$

$$B2 = \begin{bmatrix} -0.00025 & 0.000433 \\ -0.00944 & -0.02769 \end{bmatrix}$$

$$B = \begin{bmatrix} 0.121335 & 0.008953 \\ 0.006114 & 0.147971 \end{bmatrix}$$

$$Z = A*Y0 \qquad Y0 = \begin{matrix} 1 \\ 0 \end{matrix}$$

NORM (Z) =CARDINALITY (Z) = CAR (Z) (say)

$$B = \begin{bmatrix} 0.121335 & 0.008953 \\ 0.006114 & 0.147971 \end{bmatrix}$$

$$Y0 = \begin{matrix} 1 \\ 0 \end{matrix}$$

$$Z1 = B*Y0 = \begin{matrix} 0.121335 \\ 0.006114 \end{matrix}$$

NORM(Zl)= 0 . 121489

Y1 = Z1/CAR (Z1) = [0.998733
0.050324]

Z1= [0.122638 CAR (Z1) = 0.304441
-0.27865]

Z2 = A*Y1= [0.049691 NORM (Z2) =0.251447
-0.24649]

Y2= Z2/CAR(Z2) = [4.197618
-0.98028]

Z3= A*Y2= [0.024544
-0.19884] NORM (Z3) = 0.20035

Y3= Z3/CAR (Z3)= [0.122507
-0.99247]

Z4= A*Y3= [0.015337 NORM (Z4) =0.180352
-0.1797]

Finally we get MAX(NORM(Zk)) =0.152696 where

K2= -0.99982 , K1=0.019206 and

L1= 0.31762 , L2= 0.948218

Therefore,

U = K1(Y1-MEAN(Yl))+K2(Y2-MEAN(Y2))

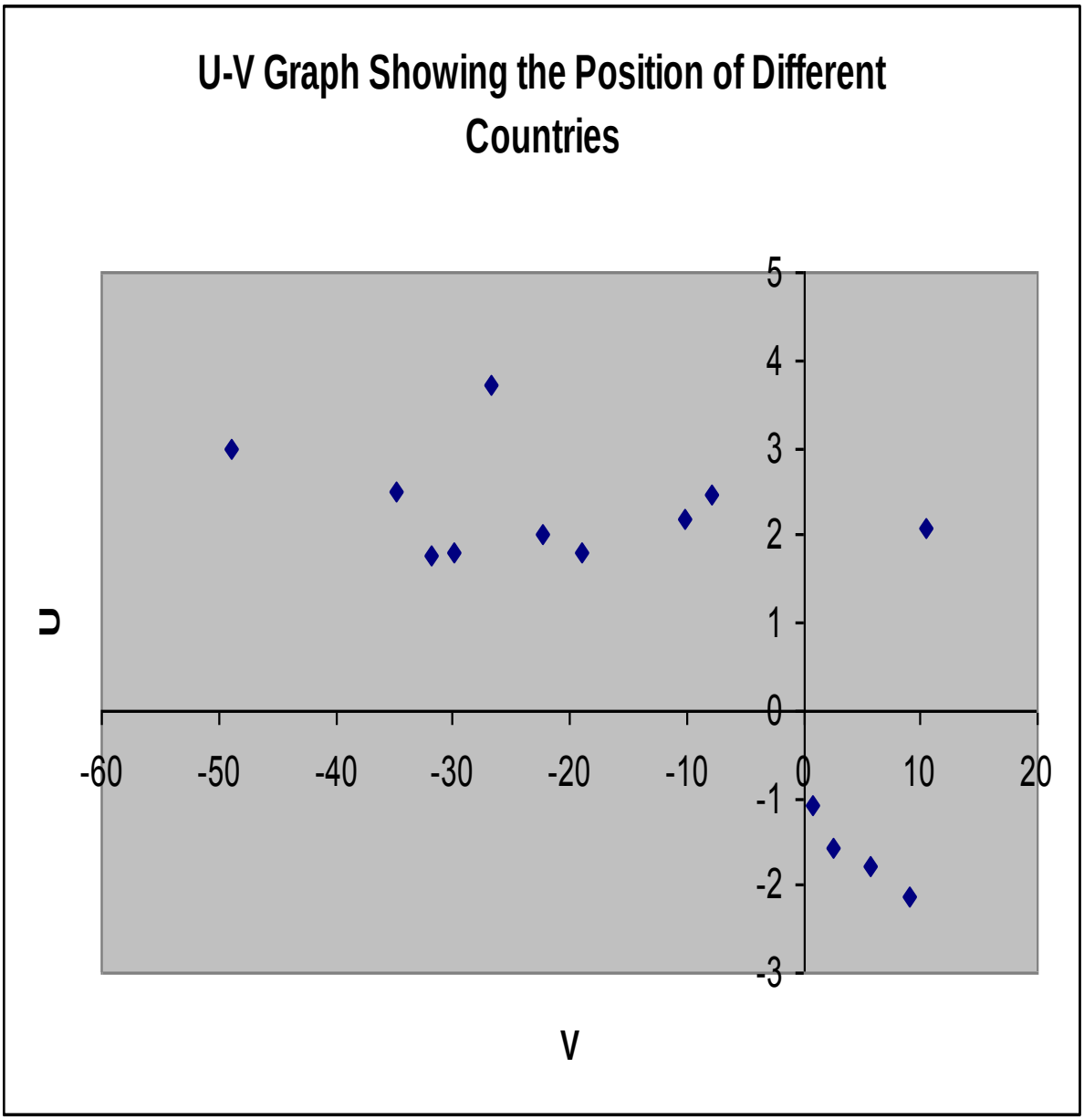
V = L1(X1-MEAN(X1))+L2(X2-MEAN(X2))

Table 3: Showing Different countries and their U, V values

Sl. No.	Country	u	v
1	Japan	-48.9842	2.9835
2	China	-18.9789	1.7917
3	Korea Rep.	-34.8732	2.4937
4	India	-7.8328	2.4517
5	Malaysia	-29.934	1.8013
6	Saudi Arabia	10.4806	2.0719
7	Sri Lanka	-26.7572	3.7243
8	Nepal	9.0143	-2.134
9	Mongolia	-31.8674	1.7513

10	Bangladesh	5.7981	-1.7832
11	Pakistan	2.5515	-1.5893
12	Vietnam	-22.3018	1.9983
13	Iraq	0.7958	-1.103
14	Iran	-10.1347	2.1735

Graph1 : Showing Position of different countries



CONCLUSIONS:

I have taken a linear combination of expenditure on education % GNP (Y1) & expenditure on health % GDP (Y2) as U to represent a economic (welfare) condition of a country against this I have taken a linear combination of IMR per 1000 (X1) and adult illiteracy % rate (X2) as V to represent PQL of a country. After using Canonical correlation I have reached a conclusion that the increase of expenditure on health and education purpose implies decrease of IMR & illiteracy.

In 1st Quadrant contains only Saudi Arabia whose expenditure on health and education is high but illiteracy rate also high (exceptional one).

And 2nd Quadrant contains those countries whose expenditure on health and education is high illiteracy rate and IMR is low.

The 4th Quadrant contains those countries whose expenditure is lower and illiteracy rate is higher comparatively. Here Iraq Pakisthan , Bangaledesh and Nepal lie.

There is no country in 3rd quadrant whose expenditure is less and illiteracy is also low.

After concrete analysis and from the graph we can rank the 14 Asian Countries according to their better Physical Quality of Life as follows:

1. Japan

2. Korea Republic

3. Mongolia

4. Malayasia

5. Srilanka

6. Viatnam

7. China

8. Iran

9. India

10. Iraq

11. Pakisthan

12. Bangaladesh

13. Nepal

14. Saudi Arabia

The ‘r‘value has come 0.397.

After using Canonical correlation I have reached a conclusion that the increase of expenditure on health and education purpose implies decrease of IMR & illiteracy.

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TECHNOLOGY INTEGRATION FOR MATHEMATICS EDUCATION IN A DEVELOPING COUNTRIES WITH A FOCUS ON UNITED KINGDOM.

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Abstract:

The present study focused on the Technology Integration for Mathematics Education in a Developing Countries. This paper classified the educational status between United Kingdom and Australian educational context. The main objective of this study to find out the educational outcomes in developing countries in present context. The methodology of the study is qualitative and quantitative methods are applied. The study is a mixed type involving interpretative, analytical study of documents, interview and study both primary & secondary sources, like books, university news, expert opinion, articles, journals, thesis and websites, etc.

Keywords: *Australian Education, Developing Countries, ICT, Mathematics Education, Technology Integration, United Kingdom.*

Introduction:

Technology has the potential to improve student learning and to provide opportunities in teaching that were not previously available (Gulati, 2008). Technological development worldwide has encouraged academics to integrate technology into education to support all forms of learning. The main highlight of teaching and learning experiences are based on interactive learning environments. Learning and communication technologies (ICT) are a fundamental piece of daily life, which include the teaching-studying process (Das, 2021a).The combination of the internet, websites and e- learning systems provide new directions for education, facilitating students' access to learning resources and their interaction with peers and teachers, allowing learning in their own time and space. It has enabled learners to access and interact with peers in ways not envisioned even a decade ago. Students in western classes have access a wide variety of software expanding the possibilities for learning with modern technologies showing their effectiveness in providing a new

approach to mathematics education. These statements are particularly applicable to students in developed nations, not so for students in some developing countries. For staff in developing nations faced with limited professional development, little use of technology and limited resources to purchase technology it is difficult to imagine what is possible (Gulati,2008). Research suggests that an e-learning system is one solution to bridge the education gap. However, there is an increasing and extensive awareness that the pedagogical and technical expertise of the teacher is extremely critical (Hennessy, Harrison &Wamakote, 2010).

The upsides of on line materials include get admission to without community programming establishment, simplicity of dispersion and fresh for engineers, and everlasting accessibility for customers so long as the internet is available (Das & Das, 2020).From the advancement of developed nations, this is aimed to find ways to bridge the educational gap, by skipping the long learning curves undertaken in western nations and introducing staff in developing countries to current best practices in education. This research investigates what is best practice in developed nation, and in particular mathematics education with a view to identifying ways to capitalize on lessons learned through educational developments that have been tried and tested. The outcome would be to deliver an advanced starting position for staff and students in the developing nations of UK who are beginning their exploration of what is possible.The countries like Netherland, United Kingdom and Malta have recognized the importance of technical support to help teachers to use ICT within the classroom.(Yang & Wang, 2012).Right now, there's a deficiency of exploration that caused the usage of automated degrees for learning science. Learning and communication technologies (ICT) are a fundamental piece of daily life, which include the teaching-studying process (Das, 2019a).

Purpose and Objective:

The purpose of this research is to identify ways to develop tertiary level mathematics and statistics capabilities in UK. The efficient introduction of technology into developing countries of UK is dependent upon the availability and accessibility of hardware, software and communications infrastructure. However, the real challenge for educationists is how to utilize the potential of ICT to complement the role of the teacher as an effective tool in supporting teaching and learning in a learner-centered environment, instead of within a more traditional pedagogy.

Institutional technology research in higher education is conducted for many reasons. For example, it may relate to the provision of access to higher education for people otherwise disenfranchised by traditional delivery system or on finding innovative approaches to employing technology to enhance the quality of teaching and learning. All instructional technology research focuses on questions of how people learn and perform, especially with respect as to how learning and performance are influenced, supported, or perhaps even caused by technology.

The introduction of technology to developing countries was undertaken with a view to improving mathematics learning and hence questions as to how students are taught and how

they learn mathematics and perform in ICT rich environments need to be explored. In order to investigate best practice and what is possible it was necessary to develop understanding of:

- The issues confronting nations in terms of integrating ICT into education;
- Theories of learning as they relate to the integration of technology into teaching and learning
- Best practices in using technology to create blended learning (face to face teaching combined with e-learning sites) such as;
- The infrastructure, available in developed nations to enable the adoption and adaptation of e-learning;
- The rationale such as the desired educational outcomes for the selection of technologies and practices used in modern online mathematics learning environments.
- The provision of, or the needs of staff for professional development in relation to technologies readily available, both generally and specific to mathematics.

The research objectives examined ways to improve mathematics education through the use of technology and to identify ways to introduce these technologies into the UK. In particular, the intent is to;

- Develop, trial and evaluate an effective template, or structure, for embedding mathematics learning support into e-learning site;
- Adapt best practice, in terms of technology use as developed in an Australian context, o meet the needs of the nations;
- Develop, implement and evaluate effective professional staff development processes for the use of technology in these developing countries.

Table 1.1. Research questions and issues in secondary level.

Overarching research question	
How can tertiary mathematics education in United Kingdom enhanced through the integration of technology into teaching and learning?	
In the Australian educational context	In developing countries

To examine best practices, it was important to firstly determine that the selected learning outcomes for the chosen subject were positive. This was undertaken through addressing the questions: How effective is the selected subject MATH subject in terms of impact on student's confidence or perceived competency? How effective are the offline resources and the online learning provided in terms of usefulness in helping students understand? How effective is the design of the e-learning site in terms of its provisions of attributes such as clarity or good access to materials?	To examine the possibility of introducing technology along with effective practices to mathematics students in developing country, the following questions was asked? What is the current state of access to technology in a mathematics department in a developing countries? Especially, what is the access to computer laboratories software, hardware and the internet? How and why are they used?
In the terms of the provision of mathematics education development what strategies, may be applicable to education in the developing country context?	Ho can professional development be used to introduce current technology to United Nations mathematics staff?

Analysis of This Study in Brief:

To answer the research questions involves the selection of a methodology through which data are gathered. These are several different research paradigms involving collection of quantitative or qualitative data or the combining of the two types of data. How data are collected and what type of data relates to the trustworthiness of those data and thereby the implications and findings drawn. Discussion and selection of the paradigm framing the study and the research methodologies can be described in these case study.

This Study in Brief:

Two case studies were the basis of this exploration to build knowledge of best practices leads to implementation of professional development in a developing countries (refer, Table 1.2). Through immersion in an Australian tertiary mathematics education context, the researchers investigated mathematics education seeking to identify best practices in a department involved some of these practices in context through the trial of a professional development initiative (refer, case study 3).

The first case study, the Redesign of e-learning site, examined that involved redesigning an e-learning site for a first year service subject that is a mathematics subject designed by science students, namely MATH151. The choice of subject is redesign which involved altering the structure of the pages and developing learning support resources, which included orientation and showing the process of solving mathematics problems. It also involved evaluating three successive groups of student to discover the effectiveness of learning designs used to combine learning resources, in terms of improved student learning outcomes.

Table 1.2. Case studies and the main methods of data collections.

AUSTRALIAN CONTEXT		
	Case study 1 Redesign of e-learning site of MATH151 at secondary level	Case study 2 Professional development for technology in mathematics education at secondary level
Objective	To explore students perceptions of the utility and efficacy of the subject e-learning site	To explore the technologies used in Australian mathematics classrooms with the view to considering which might have relevance in the secondary level.
Participants	Students at UOW	Mathematics Academic staff Mathematics secondary level students General Staff
Instruments	Survey for student in one subject(MATH151) in 2010,2011, 2012,	Students development in math department at secondary level.

Developing Countries Context:

Case study 3 Professional Development in secondary level	
Objective	To trial various technologies with staff students To identify the potential of professional development through determining the impact of the sessions on subsequent practice in one classroom
Participants	UK Mathematics Academic staff, Mathematics secondary level students and General staff
Instruments	Interview math staff at SQU Evaluation of training
Outcomes	Reflection on the adequacy of professional development

	theories Identification of factors impeding the achievement of optimal benefits from the technology applications in the learning

Analysis of Case Studies in Developing Countries:

Remaining in the Australian context in Case study 2,Professional development for technology in mathematics education, interviews were used to identify what technologies staff used, how they were used and the difficulties staff found in learning technology in terms of self-development and staff development.

The focus of the third case study, Professional Development in Secondary Level, involved introducing technologies to academic staff in developing countries.Introducing technologies to academic staff in developing countries, an analysis of issues regarding educational technology. The insight into issues that led into the development of a professional development package for staff for the adoption, implementation and evaluation of the professional development package.

Educational Theories of Learning:

As an organizing framework that brings an additional layer of understanding to concrete experience by implying relationship, consistency and a degree of predictability and testability'. Learning theories are commonly consulted in the instructional design process in many traditional educational settings.

The case studies explored in this article conducted at the tertiary level, involve adult participants, students, the learners, staff, and those engaging in professional development. Knowles(1990) argues that adult learners bring a great deal of experience to the learning environment and therefore the active participation of learners should be encouraged and acted upon when designing and implementing educational programs. Social constructivism emphasis the importance of social interactions in affecting the individual's generation of knowledge or facts about the world and this is important for adult learning. A deep discussions of the learning theories are as follows:-

1. **Behaviourism:**

Behaviourism is associated with stimulus and responses. The learner is trained to respond based on a stimulus. Behaviourists concentrate their efforts on observable learner behavior and reinforcement. Behaviourists believe that a student has learned something by observing their changes in behavioural responses. This is an objective approach where knowledge is perceived as facts that can be transmitted from teacher to student. These implications include:-

- Students should be informed of the explicit online lessons learning outcomes;

- Students must be tested to examine an individual student's achievement level and to provide feedback;
- The learning materials should be sequenced appropriately to promote learning; and
- Students must be provided with feedback so that they can examine how they are doing and take corrective actions if required.

Behaviourists' strategies are relevant to this study in two ways. Firstly, when designing materials for an e-learning site, issues of specifying objectives, feedback and reinforcement are apparent. Secondly, with regard to professional development, behaviourist theory is relevant when setting the goals of the training, practice and observation. However, some educators argue that not all learning is observable and there is more to learn than a change in behaviour.

2. Cognitivism:

Cognitivism opens up the black box of the mind, regarding the learner as an information processor. Good and Brophy defines learning from a cognitivist perspective as the acquisition or reorganization of cognitive structure through human processes and the storage of information. Cognitive theories focus on how learners process information with memory, motivation, abstraction, thinking, and reflection, all playing a significant part in learning. As such, cognitivist approaches have provided a foundation for learner- centred education. Graphic organizers and note talkingskills are the examples of cognitive theories that are of particular importance for teaching.

Several aspects of cognitivism are of importance when exploring online learning and hence of importance for the case study. The redesign of e- learning. For example, in cognitivism, the learner is seen to use different types of memory during learning. This means that the strategies implemented for online learning should allow students to perceive information in ways that facilitate transfer to working memory. Such strategies include proper location of information on the screen; appropriate use of colour graphics and size of text; the packing of the information; and the mode of the delivery such as, audio, visuals, animation and video.

3. Constructivism:

Vygotsky defined constructivist learning theory as the active construction of new knowledge based on a learner's previous experience. Constructivist theory suggests that students learn by actively constructing knowledge, comparing new information to previously learned information, thinking about and working though discrepancies and ultimately reaching new understandings. Constructivists see learners as being active rather than passive. Constructivism students work.

Constructivist learning theory with its focus on knowledge construction based on learners' previous experience is a good fit for e-learning. Koohang(2009) suggests a model based on

constuctivist learning theory for promoting learning in e-learning activities, learning assessment and the instructor's roles.

While both constuctivist and cognitivist learning theories provide ample pedagogical practices for designing instructional opportunities, their underlying principles do not often intersect and may leave something to be desired. In this study, a combination of aspects from existing learning theories, behaviourism, cognitivism, and constructivism, along with the principles of connectivism has been adopted.

Connectivism:

Connectivism is a learning theory for the digital age, that contrasts with behaviourism, cognitivism, and constructivism. The limitations of these earlier theories include their intrapersonal view of learning and their failure to address learning that is located within technology and organizations.

In connectivism the starting point for learning occurs when knowledge is activated through the process of a learner connecting to and feeding information into a learning community. Learning is seen as a network phenomenon, influenced, aided, and enhanced by socialization, technology, diversity, strength of ties, and context of occurrence. Connectivism involves opportunities for peer-to-peer learning networks where learning occurs in short bursts driven by the needs and interests of the learners and where flexible learning activities encourage interaction between instructors and students and among learners.

Table 1.3: Theories of Learning:

Theories	behaviourism	Cognitivism	Constructivism	Connectivism
How learning occurs	Black box-observable behavior main focus	Structured, computational	Social, meaning created by each learner(personal)	Distributed within a network, social, technologically enhanced, recognizing and interpreting patterns.
Influencing factors	Nature of reward, punishment	Existing schema, previous experiences	Engagement, participation, social, cultural	Diversity of network, strength of ties
Role of memory	Memory is the handwriting of repeated experiences	Encoding, storage, retrieval	Prior knowledge remixed to current context	Adaptive patterns, representative of current state.
How	Stimulus,	Duplicating	Socialization	Connecting to nodes

transfer occurs	response	knowledge constructs knower	of

In this theory there is much more intense focus, compared to other learning theories, on the networked and shared (or sharing) experiences(T schofen&Mackness, 2012).
Social media and emerging technologies are used to facilitate learning through providing opportunities for external learning situation that are not always available, or feasible, in face-to-face classrooms. Connectivism appears relevant to the delivery of the subjects in the Redesign of e-learning site study, as students were to connect to and use learning resources and to communicate via the e-learning site. It is also relevant in the study Technologies in Higher Education where academic staff use different technology for communication with each other and for self-professional development.

Improving Mathematics Education through Technology:

To ensure targeting of staff development, the survey improving mathematics education through the use of technology and interviews were used to identify which software and resources were already used, their needs, and the difficulties faced in integrating technology. This information provided the background context to ensure that professional development was orientated in a manner that suited staff. Secondly, the survey was used to identify difficulties facing lectures wishing to integrate technology into their teaching.

Table1.4. Evaluation of Professional Development in UK

Phases	Source of data for each stage for evaluation
Design	Literature review to identify issues in developing countries Case study 1 Redesign of e-learning site: identification of e-learning site practices in Australia focusing on design, resources, activities and supports students learning to inform design of subjects. Case Study 2 Professional development for technology in mathematics education. Identification of best practices, technologies used and techniques for professional development in Australia to inform professional development. Professional Development in UK: Identification through surveys of staff needs in relation to technology use in developing countries.
Development	Choice of topics determined from Case study 1 and 2 The preparation of professional development packages.
Implementation	Delivery of the professional development countries
Institutionalization	Professional Development in UK: Interviews and surveys of participants regarding the usefulness of the technology;

	One class developed to the point of implementation of the professional development program. Issues in transfer explored Request for and use of workshops.

Analysis of Evaluation Case Study: Redesign of e-learning:

The first case study provided an opportunity for immersion into the Australian educational context. It explored the Redesign of e-learning site with a view to improving student learning experiences, e-learning is already known to have influenced the field of teaching, training, and development(Manochehr,2006), with many schools using systems such as Moodle to provide communication and additional resources for students. There are claims that e-learning, or online learning as it is also known, can improve student learning if it is purposely planned. However, there are also concerns that there is insufficient evidence to show that e-learning is an effective delivery mechanism. Furthermore, there are many new technologies in e-learning environments, such as blogs, wikis, podcasting, interactive pads and smart boards, videos resources and video conferences, Tablets PC, Moddle/ e-learning site and social websites, which have been used to support students learning. Even though, academic staff have been willing to open up to the use of these technologies in their classrooms, the question to address is whether these technologies are effective in improving learning.

Evaluation is important in this article where a redesign of e-learning site is undertaken as part of the exploration of best practices. One challenge facing evaluators of technology-based programs is to design studies that can provide the feedback needed to enhance their design or to provide evidence as to their effectiveness in answering some of the pressing issues facing teaching and learning with technology (Owston, 2008).

Conclusion:

There was a premature end to professional development. This intended site for development to implement the professional development program. The disruption to the planned implementation of a professional development program in a stark reality for many developing nations. This study confirmed what is reported in the mathematics education. Development in many countries is interrupted political events, such as political instability, resulting in a loss of continuity and development being set back. Plans to return to implement a professional development program were put on hold.

The strongest recommendation from this case study regarding the embedding of technology into mathematics education develops a policy for technology use along with a plan for implementing it within the curriculum. A requirement to use technology should motivate the use of technology, in many cases existing technology and software, while facilitating the use of freely available software. Lectures believe that they would integrate more technology if the shortage of equipment, such as document projectors in lecture room, were addressed. To use

technology lectures also need professional development and more technical support so that they can become comfortable with the integration of technology into the classroom.

An examination of how professional development was conducted showed that these practices were similar to the workshops followed by one-on-one facilitation to achieve specific learning outcomes are used in developing countries. Opportunities to access funding through small grants and large grants were similar. For example, staff may apply for small internal grants (Educational Development Strategic Funds) and extremely through the Office of Learning and Teaching, which acts to promote excellence in higher education learning and teaching. To support and sponsor research activities and projects at all levels. Staff members have several sources of funds and grants available such as Research Council. However, the scope of funding to motivate developments was, and almost certainly will remain, and more limited.

In this case study staff desire for more recent forms of technology such as a document camera and the interactive whiteboards. This might imply that despite their poor accessibility, these staff considered these tools would be valuable addition for the teaching of Mathematics. There are many potential variations on the professional development of teaching staff. These range from teaching practice guides or funding to attend conferences, to formal teaching programs. Workshops that would be of benefit to staff include:

i. Combining open learning to provide across curriculum learning support
ii. Rudimentary html for web page design
iii. LaTeX documents for design resources

The purpose of Mathematics education is to draw how tertiary mathematics education enhanced through the integration of technology and to triangulate upon answers to questions addressed with respect to the need for professional development for teachers.

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ROSELLE PLANT PIGMENTS AS NATURAL PHOTOSENSITIZERS FOR DYE SENSITIZED SOLAR CELLS: THE EFFECT OF TIN OXIDE BLOCKING LAYER ON THE PHOTOELECTRIC PROPERTIES.

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Abstract

Dye sensitized solar cells (DSSCs) assembled on titanium dioxide (TiO_2) synthesized with plant dye extracted from dried Roselle plant pigments were fabricated. The dye extract and active layers were characterized using a (UV 752) ultraviolet-visible-Near Infra-Red (UV-VIS-NI) spectrophotometer, at the wavelength interval of 250 nm to 1100 nm, while the Tauc model was used to obtain the optical band gap. The absorption coefficient and behaviour of the extinction coefficient was investigated. The current and voltage features of the dye sensitized solar cell fabricated with the synthesized TiO_2 Photo-anode were analysed. The optical band gap of the dye sensitized solar cells fabricated were 2.6eV, 2.8eV and 3.0 eV respectively, while the conversion efficiency of the dye sensitized solar cells was 1.34%, 1.32% and 2.9%.

Keywords: *Titanium dioxide, optical properties, dye sensitized solar cell, band gap, Hibiscus sabdariffa, absorption coefficient, extinction coefficient.*

1. INTRODUCTION

The issues on energy crisis and its sustainability is one global topic seeking great attention and potentials for research. Solar energy is anticipated to be a major player as a sustainable energy source, there is need of coming up with pragmatic techniques for conversion, storage and distribution of the energy from the sun towards the issue on energy predicament and sustainable use. Dye-sensitized Solar cells are photo-voltaic gadgets capable of transforming evident light from the sun into electricity, and in addition are centered on a permeable tinny film of an extensive band-gap semiconductor oxide altered by colorant (Chergui *et al*, 2015). The manufacturing cost of Dye-sensitized Solar cells, which are third-generation solar cells is much less than that of silicon solar cells which have succeeded in electricity renovation proficiencies

extending from 15% to 20%; nevertheless, applicable extraordinary assembling budget plus the habit of using harmful substances in the production of extremely refined silicon associated with the industrial procedure has stirred the exploration aimed at having alternative in the dye-sensitized solar cell which is a naturally friendly and a very cheap budget solar cell. Dye-sensitized solar cells have expanded weighty consideration since (Nazerruddin *et al*, 2001; O'Regan and Gratzel, 1991) reported an extraordinarily high conversion efficiency of nearly 10% using nanocrystalline mesoporous Titanium dioxide (TiO_2) film (O'Regan and Gratzel, 1991). Yet, these biological solar cells are still limited in low power conversion, proficiencies, and charge recombination is one of the principle explanations behind low current in dye-sensitized solar cells, so the performance of (DSSCs) is seriously decreased. A compact layer has been corroborated hypothetically and practically operational to wedge the electron recombination through the ancillary path. A compacted hindering deposit amongst the accompanying electron and the nano-crystalline Titanium dioxide (TiO_2) layer can viably forestall recombination at the cathode/electrolyte interface.

1.1 Solar Cell

To exploit the power of sunlight based energy and convert it to useful structures like power stays a vast task. Photovoltaic gadgets are the principle sunlight based verve change frameworks to gather sun based energy. These photovoltaic gadgets are known as sunlight based cells, they convert the striking beams of light from sun oriented emission toward electricity by means of the age plus ensuing assortment of elementary particle gap sets. Sun powered cells are arranged into three ages subject to their introduction and cost-sufficiency. The original of sun powered cells has generally higher effectiveness with costly creation costs. They are the old style instances of sun oriented cells and incorporate Silicon and Germanium. The business showcase is commanded by this age. Flimsy film sunlight based cells dependent on Cadmium Telluride (CdTe), Nanocrystalline Silicon, Amorphous silicon, and Copper indium gallium Selenide (CIGS) etc. mark the second age group cell. They have lower proficiency yet are a lot less expensive to create and utilize a less outer manufacture process. Third age group sun oriented devices comprise the categories of cells which weren't assembled during the first as well as second ages. A color sharpened (dye) sunlight based cell is a natural sun based cell of the third era.

Most third-age advances are not yet economically executed; however a lot of inquiry and a great deal regarding exploration is ongoing which are showing favourable prospect (Imran, 2013).

2. MATERIALS AND METHODS

2.1 Preparation of natural dye solution

Dried calyxes of Roselle plant were used as sensitizers of the dye sensitized solar cell fabricated. They were washed with distilled water, and the dye solution was prepared by boiling the roselle plant calyxes in 40cl of water to boiling temperature as seen in plate 2a and 2b respectively. The dye solution was allowed to cool down for 3hours and the cell was soaked overnight at room temperature in the dye extract, to allow the dye molecules to form covalent

bond to the surface of the TiO_2. It was then stored away from light (in the dark); after 12 hours, the cell was brought out and rinsed with water and dried at 60^0C.

Fig 1. Hibiscus sabdariffa calyxes in water at boiling temperature.

Fig 2. Hibiscus sabdariffa dye extract

2.2 DSSC Assembling

The photoanode was prepared first by etching the fluorine doped tin oxide (FTO) glass (Solaronics) using electro-etching method, followed with a deposition of a blocking layer. The blocking layer was deposited to FTO glass substrate from a precursor of total volume of 40ml, which has a concentration of 0.1m of titanium isopropoxide, 0.4m of acetylacetone and methanol using the spin coating method, and subsequently annealed to 450^0C for 30 minutes. The Titanium dioxide (TiO_2) (usually a paste) was deposited onto the fluorine doped tin oxide

(FTO) glass substrate by 'screen printing method'. On top of the glass substrates, a layer of (solaronix Ti-Nanoxide D/SP) screen printable paste was applied, outlining it on the active area. The sample was annealed gradually from 150^0C to 300^0C and then finally 450^0C. Once it is annealed to 150^0C, the samples were left to dwell for 10minutes and then annealed to 300^0C and left to dwell for another 10minutes and finally annealed to 450^0C. After annealing, the samples were allowed to cool down for 30minutes. On top of the fluorine doped tin oxide (FTO) glass substrates, a layer of Zirconium dioxide (Solaronix Zr-nanoxide Z/SP) screen printable paste was deposited using the screen printing method. This layer insulates the titanium dioxide mesoporous layer from the counter electrode. The sample was annealed to 400^0C for 30minutes and allowed to cool down for 30minutes. Propanol and cotton wool was used to clean the screen printing machine to rid it off any particle of the previous layer deposited. This layer becomes the negative terminal of the monolithic dye sensitized solar cells.

The counter electrode was prepared by screen printing a platinum (Pt) catalyst gel coating on the FTO glass substrate. A thin layer of platinum (Pt) was deposited on the glass substrates by screen printing (Solaronics platisol T/SP paste), which acts as a catalyst and annealed to 400^0C for 30 minutes. A layer of (Solaronix Elcocarb G/SP) screen printable paste was deposited using screen printing method on the active area. The elcocarb is just a conducting material and was printed to bridge the two conductive layers as it helps to transfer voltage to the back layer, and it becomes the positive terminal of the monolithic dye sensitized solar cells. The samples were then annealed to 400^0C for 30minutes, activating the elcocarb layer for working and then allowed to cool down to 100^0C.

The cell was soaked overnight at room temperature in the dye extract, for it to absorb into all the layers. It was then stored away from light (in the dark); after 12 hours, the cells were brought out and rinsed with water and dried at 60^0C.

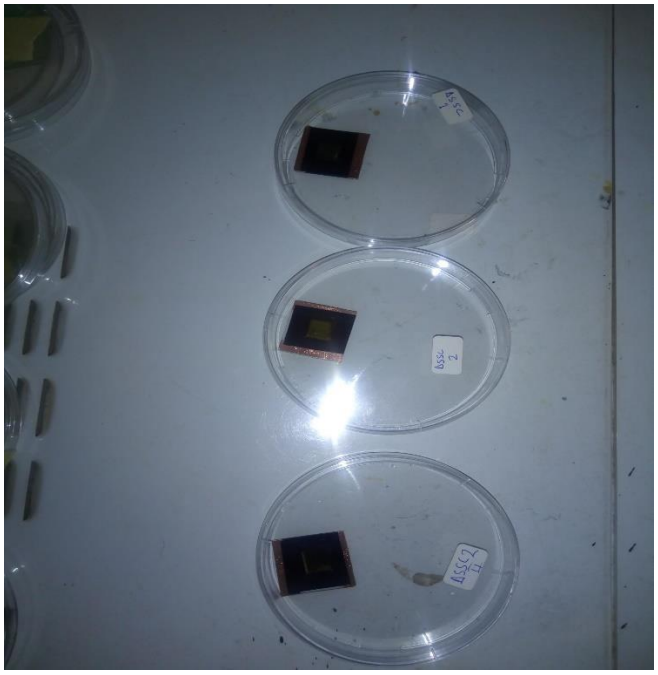

Fig 3. The Three samples of fabricated DSSCs

2.3 Optical Characterization

A (UV 752) ultraviolet-visible-Near Infra-Red (UV-VIS-NI) spectrophotometer U.K was used to carry out the optical study of the cells at the wavelength interval of 250 nm to 1100 nm. Absorbance values were obtained using the spectrophotometer and other optical properties like transmittance, reflectance, refractive index, absorption coefficient, extinction coefficient and energy band gap were evaluated using the following equations;

The transmittance (T) of the cells was evaluated using Equation (2.1) given by Lokhande *et al.* (2002).

$$T = 10^{-A} \tag{2.1}$$

Reflectance was obtained using equation (2.2) given by Lokhande *et al.* (2002).

$$R = 1 - (A + T) \tag{2.2}$$

Refractive index of the cells was calculated using Equation (2.3) as given by Ilenikhena, (2008) and Ohwofosirai *et al*. (2014).

$$\eta = \frac{(1+\sqrt{R})}{(1-\sqrt{\mathrm{R}})} \tag{2.3}$$

The absorption coefficient was calculated from absorbance spectra using the Equation (2.4) given by Suresh and Isha (2016) and Mursal (2018). Where *d* is the thickness and measured in micrometer (μm), and *A* is the absorbance.

$$\alpha = \frac{2.303A}{d} \tag{2.4}$$

The extinction coefficient was obtained using Equation (2.5) (Tezel *et al.*, 2017).

$$\mathrm{k} = \frac{\alpha\lambda}{4\pi} \tag{2.5}$$

Optical conductivity was estimated using Equation (2.6) as given by (Ohwofosirai *et al.*, 2014).

$$\sigma_o = \frac{\alpha\eta c}{4\pi} \tag{2.6}$$

Where c is the speed of light.

The energy band gap was estimated using Tauc's model given in Equation (2.7) as given by (Tezel *et al.*, 2017)

$$(\alpha hv)^n = \beta\left(hv - E_g\right) \tag{2.7}$$

Where α is the absorption coefficient, *h* is the Planck constant, v is the photon's frequency, *E*g is the band gap energy and β is a constant. The *n* factor depends on the nature of the electron transition and is equal to ½ or 2 for the direct and indirect transition band gaps respectively.

2.4 Solar Simulation Characterization

The IV characteristics curves of the fabricated DSSCs were measured under an illumination of 100 mW/cm^2 using a Newport solar simulator (model 94043A), and I.V acquired by a Keithley 2400 source meter. Cell surface area is 1cm x 1cm.

3. RESULTS AND DISCUSSION

3.1 Optical characterization

The three layers in the DSSC are dye (roselle plant), TiO_2 layer and synthesized TiO_2 layer having absorption peak at 475, 335, and 320 nm respectively as shown in fig 4. The roselle plant (hibiscus sabdariffa) is a natural dye with wider absorption peak at lower wavelength range of solar spectrum. The spectrum of the synthesized TiO_2 shows even a wider absorption band and is expected to harvest more solar energy. The absorption coefficient revealed to what extent into the layers of the DSSC, light of a particular wavelength can enter before absorption. Fig 4 shows that the absorption coefficient is not constant but strongly depends on wavelength.

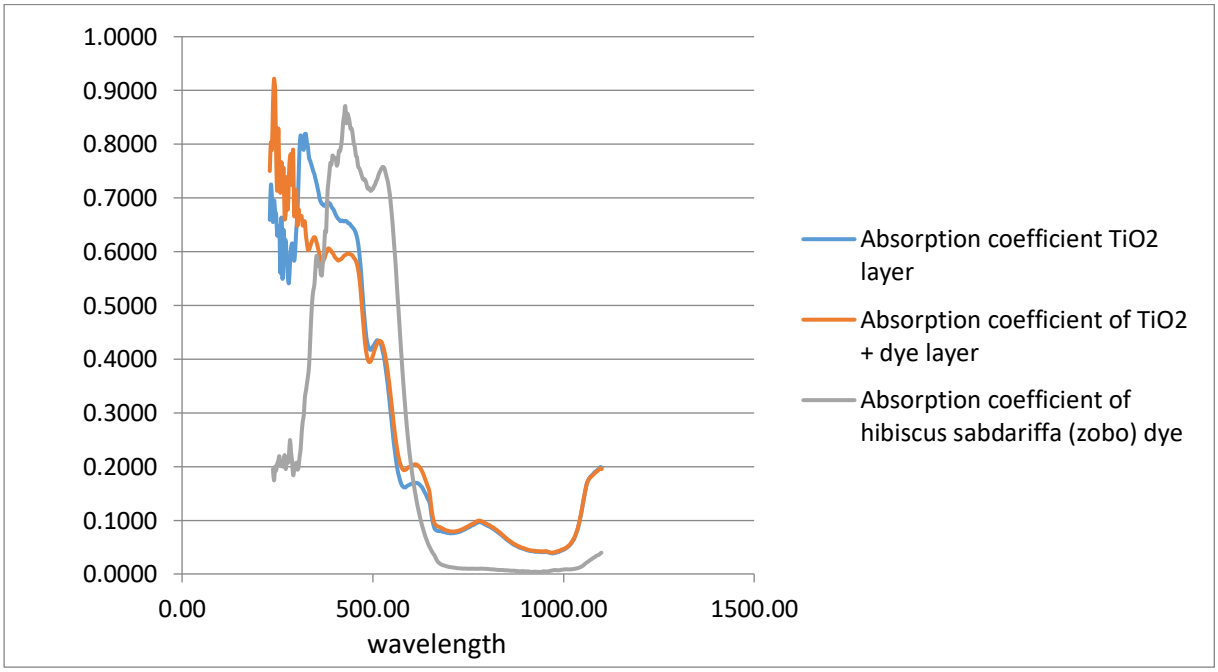

Fig. 4 Absorption coefficient of the different layers of the DSSC

The extinction coefficient of the dye, TiO2 layer and Synthesized TiO2 layer of the fabricated DSSC was investigated. The natural dye showed strong absorption in the visible and near infrared radiation wavelength. The synthesized TiO2 layer showed strong absorption in the infrared radiation spectrum as shown in Fig 5.

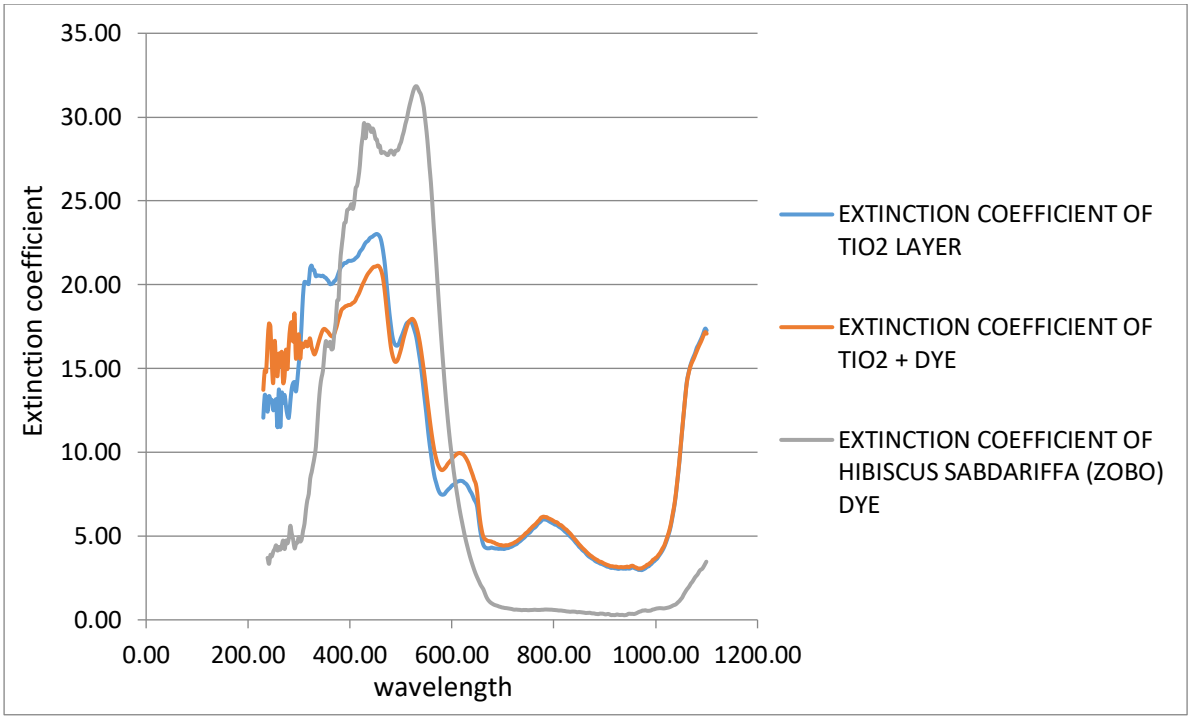

Fig. 5 Extinction coefficient of the different layers of the DSSC

3.2 Photovoltaic properties

Fig 6, Fig 7, and Fig 8 respectively shows the current voltage curves of the various DSSCs fabricated. Cell sample 1, 2, and 3 showed similar value of open circuit voltage, however cell 2 showed the highest value of short circuit current whereas cell 1 and 3 showed lower values in their short circuit current.

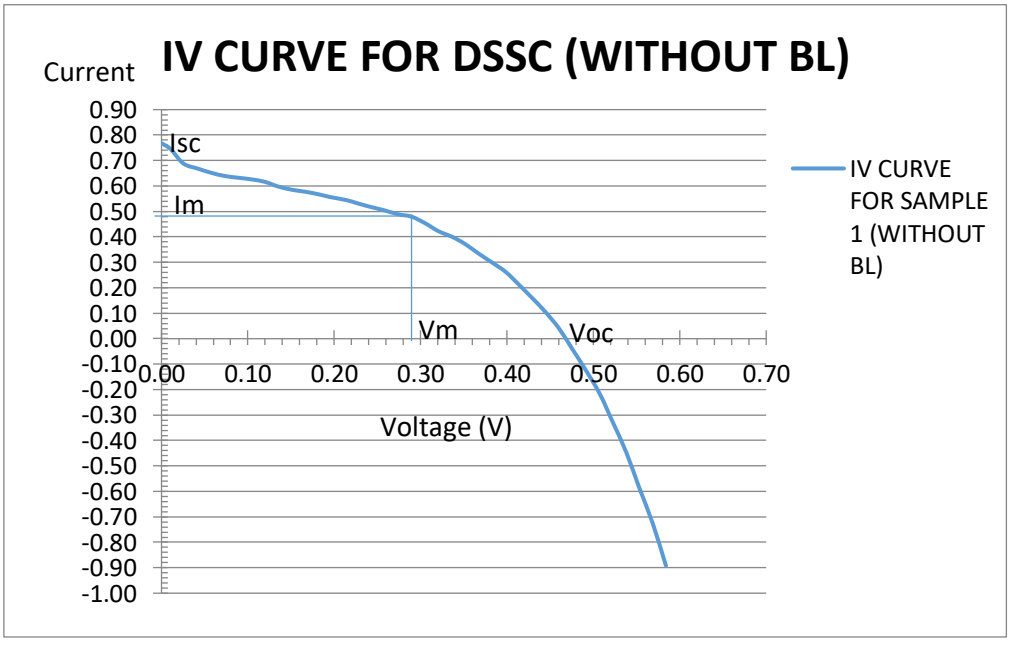

Fig 6. I/V curve for Sample 1 (DSSC without blocking layer)

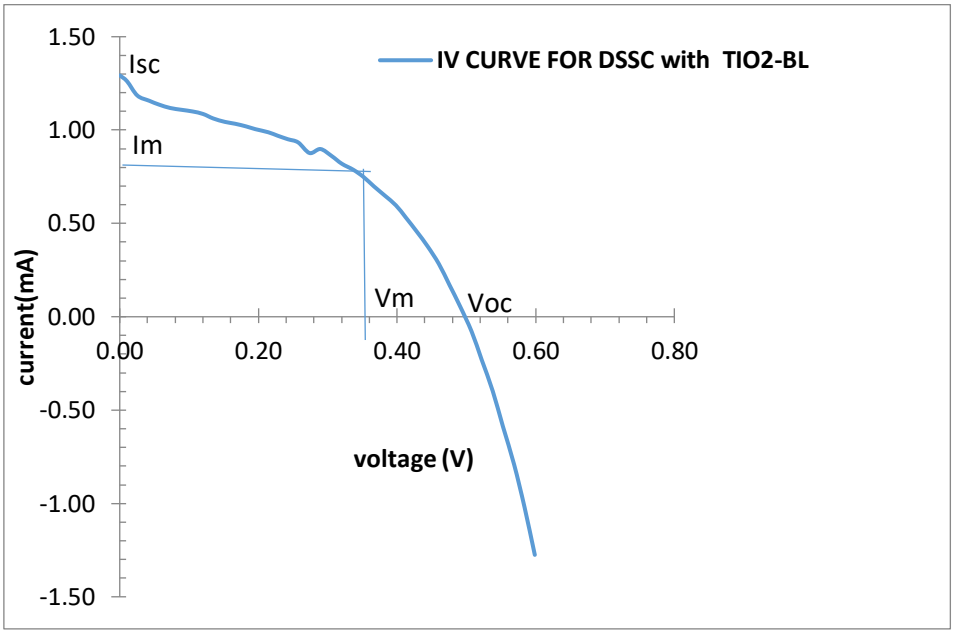

Fig 7. I/V curve for Sample 2 (DSSC with TIO_2 blocking layer)

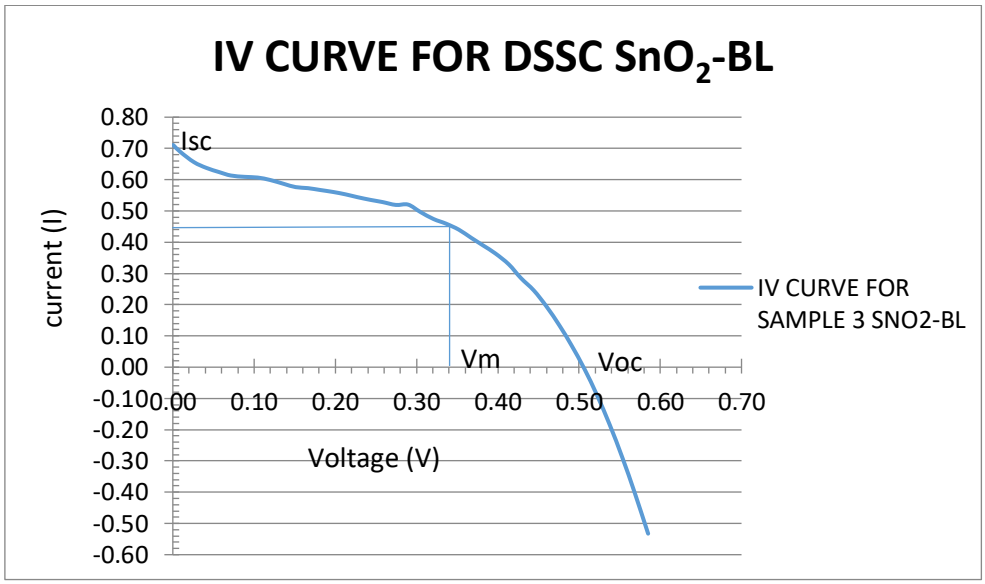

Fig 8. I/V curve for sample 3 (DSSC with SNO_2 blocking layer)

These values were measured under the illumination of 100 mW/cm^2. To investigate the DSSC performance, short circuit current (Isc), open-circuit voltage (Voc), maximum power point, fill factor (FF), and conversion efficiency (ŋ) were determined. Short circuit current (Isc) and open-circuit voltage, Im and Vm were determined from the IV curve interception with y- and x-axes respectively. Using these parameters, fill factor (FF) and conversion efficiency (ή) were calculated. All cell parameters of the fabricated DSSCs are presented in Table 1.

Table 1: Photovoltaic parameters of the Dye-sensitized solar cells (DSSCs) sensitized by Hibiscus sabdariffa.

TiO_2 DSSC samples (cell)	Voc (V)	Isc (mA/cm^2)	Vm (V)	Im (mA/cm^2)	FF	ή %
1.(without blocking layer)	0.47	0.78	0.29	0.48	0.38	1.34
2.(TiO_2 blocking layer)	0.50	1.3	0.36	0.8	0.44	2.9
3.(SNO_2 blocking layer)	0.50	0.70	0.35	0.45	0.37	1.32

CONCLUSION

Finally, three samples of dye-sensitized solar cells were fabricated and their optical properties such as absorption coefficient and extinction coefficient were determined using a (UV 752) ultraviolet-visible near infra-red (UV-VIS-NI) spectrophotometer. The photovoltaic properties of the fabricated DSSCs were ascertained using a Newpot solar simulator (Model 94043A) and I.V acquired by a Keithley 2400 source meter. Cell surface area was 1cm X 1cm, and short circuit current (Isc) and open-circuit voltage (Voc) values were obtained using Keithley 2400 source meter and other solar cell properties like fill factor (FF) and efficiency were also evaluated. The results presented showed an improved power conversion on cell 2 fabricated with a TiO2 sensitized with dye and having titanium dioxide as blocking layer with an efficiency of about 2.9%.

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EFFECTS OF TEACHING GEOMETRY USING MOBILE APP INSTRUCTIONAL TECHNIQUE ON STUDENTS INTEREST IN BAYELSA STATE

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Abstract

The study investigated the effect of teaching geometry using mobile application instructional technique on senior secondary students' interest in Geometry in Bayelsa State. A quasi-experimental, non-equivalent control group design was used in the study. All the 506 SS2 mathematics students in the three federal unity schools formed the population of the study. One hundred and seventy nine (179) students from two schools formed the sample of the study. Intact classes were assigned by flipping of coin to either experimental or control group; and separately taught by their regular mathematics teachers who had earlier been trained for the purpose. All the groups were pre and post-tested. Geometry Interest Inventory (GII) was used as instruments for both the control and experimental groups. Three research questions and three hypotheses guided the study. The mean and standard deviation were used to answer the research questions, while Analysis of Covariance (ANCOVA) was used in testing the hypotheses at .05 levels of significance. The result showed that the use of mobile application instructional technique as a medium of teaching improves students' interest in geometry. Also there was no significant effect between teaching method, gender and choice of discipline on students' interest in Geometry. The study recommended that the use of mobile application should be integrated into the teaching and learning of mathematics generally.

Keywords: *Mobile App Instructional Technique, Interest, Geometry*

Introduction

Mathematics in general is linked with the development of any nation in the world. Mathematics as a discipline opens and shuts more doors for men and women than any other content area. Whether it is in business, science, engineering or technology, it is tremendously important that a person be well-armed with mathematics if they are going to have options in their lives (Charles-Owaba & Omeodu, 2018). Mathematics represents the superb and sublime product of reason as well as the upper limits of what one hopes to attain in all rational domain. Charles-Owaba and Omeodu, (2018) citing Adenegan (2012) described mathematics as a model of thinking, for developing scientific structure, for drawing conclusions and for problem-solving. The supremacy of mathematics over other subjects is extolled by the National Policy on Education (NPE, 2014), where it stated that mathematics should be made a core subject in the primary and secondary

education levels. The policy strongly emphasized an effective teaching and learning of subject through the use of variety of strategies and this has consistently generated interest amongst scholars over the years.

Geometry is one of the most important branches of mathematics and it is concerned with the properties and relationships of lines, angles, curves, shapes, etc. The word *geometry* comes from two ancient Greek words, one meaning *earth* and the other meaning *measure*. Keith (2017) defined geometry as the branch of mathematics that exploits visual intuition (the most dominant of our senses) to remember theorems, understand proof, inspire conjecture, perceive reality and give global insight. He further asserts that these skills are transferable and are needed in all other branches of mathematics. In another vein, Sumzuma (2017) refers to geometry as that which helps us to acquire abilities such as making new discoveries, analyzing problems and making connections between mathematics and real life situations.

Throughout history, geometry has had a great importance in peoples' lives, originating with the need of human beings to specify quantities, to measure figures, land and earth and make maps. In other to represent and solve problems in survey and geo-informatics, sound geometry knowledge is necessary. Geometry is also used in other disciplines such as science of light (optics), geography (map design), music (note pattern design), art (model design), construction, and architecture, gardening and traffic signs. Artists, builders, draftsmen, masons, machinist, structural engineers and writers all make use of geometry daily.

The elements of geometry form an integral part of mathematics and further mathematics curriculum. The teaching and learning of geometry starts from the primary school, where it is an integral part of the primary school mathematics curriculum. The first element of geometry is introduced in Primary 5 under the heading *"plane shapes"*. The objectives at this early stage is to teach pupils how to;

i. Identify various plane figures.

ii. Calculate the perimeter and area of plane figures.

In Primary 6, the pupils do further work on the objectives at this level to teach the pupils how to calculate the area of some compound two-dimensional shapes.

At the secondary school, the elements of geometry are integrated into the mathematics and further mathematics curriculum. The geometric concepts taught at the primary school are usually revised at the Junior Secondary School level. The following topics are studied under geometry content as contained in the Senior Secondary School Mathematics Curriculum (2007);

a. Plane Geometry

b. Geometric construction

c. Mensuration

d. Trigonometry

e. Coordinate Geometry

By learning geometry students may be able to identify shapes and space around them. Geometry can also help them represent their surroundings. The comprehension of geometry models and their properties may give students a new perspective in their ability to analyze and communicate geometry-related things in their daily lives. The National Council of Teachers of Mathematics (NCTM, 2014), have emphasized the importance of geometry in school mathematics by stating that; geometry and spatial sense are fundamental components of mathematics learning. They offer ways to interpret and reflect on our physical environment. Geometry allows students to develop insight to understand other mathematical concepts and connect ideas across different areas of mathematics (Sunzuma, 2017). Furthermore, many ideas like symmetry or generalizations can help students increase insights into the nature and beauty of mathematics (NCTM, 2014). The importance of geometry is best stated by an inscription above the door of Plato's school; *"Let no one destitute of geometry enter my doors"* (Sunzuma, 2017).

Regardless of the importance of geometry in real life situations and the subsequent action of making it a content in mathematics curriculum, students have continued to dislike it and hence, poor performance has always been the outcome. Analysis of school certificate mathematics examinations results in Bayelsa State, reveals that students' earn consistently low scores, as less than 42% of registered candidates obtain credit pass. West African Senior School Certificate Examination Council, Chief Examiner's Report on students' areas of deficiency in school certificate examinations for eight (8) years (2010-2018), revealed that geometry content has consistently been termed as an area of weakness in terms of their achievement. Abakpa and Iji (2011) reported that students often avoid geometry questions or haphazardly attempt them. Mammana and Villiani (2018) also discovered that students' geometry achievement was always lower than the other areas of mathematics. They further noted that students shy away from the study of geometry. Zeynep (2010) reported that students have difficulties in learning geometry and in thinking geometrically. He further asserted that many geometrical problems require certain visualization in problem solving and students generally find it difficult to construct three-dimensional spaces.

Research reports have revealed that many reasons account for students' poor achievement in geometry. Among these are poor teaching approach (Olunoye, 2010), lack of interest and confidence in the subject (Arbain, 2015), poor learning environment (Olunoye, 2010). Similarly, Somalia (2019), Bilesanmi and Afuwape (2017), Ado (2018) reported that gender differences and students' choice of discipline are latent factors that affects students achievement in geometry.

Ado (2018) defined interest as a subjective feeling of concentration or persisting tendency to pay attention and enjoy some activity or content. Imoko and Agwagah (2016) defined interest as the

feeling of one whose attention or curiosity is particularly engaged by something. Interest is an important variable in learning because when one becomes interested in an activity, one is likely to be more deeply involved in that activity. Relating it to this study, interest is the motivation of student to like circle theorem concepts through the use of mobile application instructional technique in teaching and learning process. One is likely to do well in a discipline of interest. Abdulcarismo (2009) submitted that some children may be intellectually and physically capable they may never learn until their interest is stimulated. Once the students' interest is stimulated, they will continue to learn as long as the teacher is capable of sustaining their interest in the subject matter. McPhan (2018) opined that interest is the mother of attention; once there is direct interest, attention is guaranteed and learning is assured. Ebele and Sam (2015) reported that low interest in mathematics emanates from anxiety and phobia. Phobia has been observed by Abdulcarismo (2009) to be an academic sickness whose virus has not yet been fully diagnosed for an effective treatment in the class and the symptoms of this phobia are usually expressed on the faces of mathematics students in their classes.

Gender and mean interest scores are factors that have not produced conclusive results in sciences and mathematics. Okigbo and Okeke (2013) carried out a study on the influence of gender on mean interest scores of students taught mathematics using games and analogies as advanced organizers and reported a no significant difference on the mean interest scores of male and female students in mathematics. The study also established that the interaction effect of gender and treatment on interest scores was not significant. On the contrary Ajaegba and Ekwueme (2019), Mman and Tukunkaya (2019) reported a significant difference in the mean interest rating and a significant interaction effect on interest in favour of males. Thus, there is need for more studies on the instructional strategies that can enhance equal achievement and interest among male and female students in geometry which necessitates this study.

In Nigeria, there are two categories of public secondary schools, when schools are been classified based on ownership; those owned by the Federal Government and those owned by the state government (Akpe, 2016). The secondary schools owned by the Federal Government are also called Federal Unity Schools and there are one hundred and four (104) across the nation. Learners' choice of discipline has to do with science/engineering and arts/commercial classes that students belong to depending on their future career (Bilesanmi & Afuwape, 2017). Science students refers to students offering science subjects such as Physics, Chemistry, Biology as their core subject, while arts students are those offering Literature-in-English, Government, Economics, Commerce, Christian Religious Study (CRS) as their core subjects. In Federal Government Colleges, students are usually divided into four streams, two sciences and two arts classes The choice of discipline on students' achievement in mathematics is a major issue that has not produced conclusive results. So, many studies on science and arts students' performance in mathematics exist, but there is scarce research evidence on issues relating to differences in the interest of Arts and Science students in geometry content only. Somalia (2019) researched on the influence of choice of discipline on the performance of secondary school students in mathematics and reported that

science students performed better than their arts counterparts. Bilesanmi and Afuwape (2017) reported that when the right teaching strategy is adopted, learning outcome of both science and arts students will improve. It is worthwhile to check how the achievement and interest of science and arts students will be in geometry, when mobile app instructional technique is utilized.

Different teaching and learning methods have been experimented to motivate and increase learners' interest in geometry. Examples of such learning method are, Mastery Learning Approach (Abakpa & Iji, 2011), the Use of GeoGebra (Arbain, 2015), the Use of Geometry Learning Media based on Augmented Reality (Rohendi, 2018), Geometers Sketchpad (Dimakos, 2010), Digital Geometric Software (Hasan, 2012), Cabri Geometry Plus II (Derya, 2012), Digital Daily Life Photograph (Zeycep, 2010), Daily-Life Story Using Dynamic Geometry Software (Abdelfatah, 2010), Personal Math Concept Chart (Rhonda, 2017).

Teachers have the potential of exerting strong influence on students' learning. Uloko and Usman (2018) reported that there is a positive correlation between good teaching approach and students' learning in geometry. Also, Abakpa and Iji (2011) stated that good strategy improves both low and high ability students in geometry at upper basic education class. That is good teaching approach produces high achievement among learners, while poor teaching approach will lead to poor learning and low achievement. Roosing (2012) submitted that changes in technology will continue to alter possibilities for learning and create new challenges for pedagogy. Also Ectuban (2018) opined that increased development in technology coupled with a range of needs and expectations from a range of stakeholders have made it imperative for educational organizations to constantly upgrade their strategies and policies in teaching and learning as a way to remain effective and competitive. Ectuban (2018) noted that students all over the world are very much comfortable with electronic gadgets and equipment and the need to use these gadgets and equipment in teaching is highly evident. Also mobile computing is explored primarily because many of the learners spend more time with their mobile devices than any other toy or learning materials. Some of the learners have even become addictive to the use of their Mobile devices for other activities such as games.

Mobile application instructional techniques also known as *mobile application instructional techniques* are a series of applications designed to assist learners in performing single or various related tasks with the purpose of creating learning. The use of mobile application instructional techniques represents a technology that is ubiquitous in nature, wireless, highly portable and endowed with multimedia capabilities bringing a new dimension to curriculum delivery, (Fuchs, 2014). The last ten years have witnessed an impressive increase in the use of mobile application instructional technique in schools (Ectuban, 2018). Although, created for non-educational environment, tools like tablets and smartphones have made their way into the classroom. These devices with learning applications have attracted interest from the educational communities mainly due to their gaming capabilities. The use of mobile application instructional technique allows students to engage in problem-solving based learning activities, permits students to work on tasks that are goal oriented and open-ended with a strong gaming component. They empower students to develop their own understanding through active involvement and sense-making. Furthermore,

learning experiences like digital simulations or manipulations have the capacity to bring interactivity, thus enhancing cognitive and affective processes (Barros & Marcos, 2010).

Ectuban (2018) observed that the use of mobile devices such as cell phones and tablets are among the six new rising advances that have significantly affected instruction delivery. Particularly, in the subject matter of mathematics, Ruthven and Hennessy (2012) made a comparison between the learning outcomes of mobile-based teaching and mathematics thematic teaching; results revealed that mobile-based learning significantly improved the advancement of scientific aptitude and the development of a more profound perceptual ability for the students. The Nigerian government recognizes the importance of ICT as a tool for development of the country. It has stressed that ICT has a role to play in education both directly as a subject and indirectly as a tool to assist in instruction delivery and management (NMC, 2015). As a way to match actions with words, the School Based Knowledge Centre Project, launched in 2014 by the Federal Government, through the Nigerian Communication Commission (NCC), supplied smartphones to all Federal Unity schools and some state owned secondary schools (NMC, 2015). This marked the advent of the use of mobile devices as an instructional aide in teaching and learning in Government owned secondary schools across the country.

Mobile app designed specifically for geometry used in this study is Apollonius which is an interactive, measurement, and simulation app. It allows you to create geometric constructions, such as those that can be made using a ruler and a compass and subsequently move parts of your constructions to explore the relationship between the geometric objects on the screen. Its interface is especially designed for touch screen, providing one of the smoothest experiences of any existing interactive geometry app (Handal & Herrington, 2013). It is inspired by software such as GeoGebra, the Geometer's Sketchpad and Cabri. The choice of Apollonius was predicated on the fact that it can be installed on every kind of smartphone (both Android OS and IOS devices) and can be used in the teaching and learning of mathematics without internet connections and it is designed specifically for geometry.

Purpose of the Study

The aim of this study is to determine the effects of teaching geometry using mobile application instructional techniques on secondary school students' interest in Bayelsa State. Specifically, the study sought to:

i. Determine the mean interest scores of students taught geometry using mobile application instructional technique and those taught using lecture method.

ii. Determine the mean interest scores of male and female students taught geometry using mobile application instructional techniques.

iii. Determine the mean interest scores of science and arts students taught geometry using mobile application instructional techniques.

Research Questions

The following research questions were formulated to guide the study;

1. What are the mean interest scores of students taught geometry using mobile application instructional technique and those taught using lecture method?

2. What are mean interest scores of male and female students taught geometry using mobile application instructional techniques?

3. What are the mean interest scores of science and arts students taught geometry using mobile application instructional techniques?

Research Hypotheses

The following null hypotheses were formulated and tested at 0.05 level of significance

Ho_1: There is no significant difference between the mean interest scores of students taught geometry using mobile application instructional techniques and those taught using lecture method.

Ho_2: There is no significant difference between the mean interest scores of male and female students taught geometry using mobile application instructional techniques.

Ho_3: There is no significant difference between the mean interest scores of science and arts students taught geometry using mobile application instructional techniques.

METHODOLOGY

A pre-test, post-test, non-equivalent control group quasi–experimental design was adopted for the study. The choice of this design, allows investigation of intact groups in real-life classroom setting, since it was not possible to randomly assemble students for any intervention during school hours

so as to avoid artificial conditions. The population for this study was all the mathematics students in Bayelsa State. A sample of one hundred and seventy nine (179) second year senior secondary school (SS2) students were used for the study. The two co-educational Federal Government Colleges (FGC, Odi and FSTC, Tungbo) were purposively selected, because they are the only schools with the Knowledge-Based Centers, where the mobile devices are available. Also, SS2 class was purposively selected because there was no impending external examination that could distract students from full participation in the study and the content used was meant for them as contained in the senior secondary mathematics curriculum. In each school, there are 4 classes (2 science classes and 2 arts classes) making a total of 8 classes in both schools selected for the study. Simple random sampling by flipping of the coin was used to select one out of the two science classes and one out of the two arts classes from each school, thereby making it a total of four classes to participate in the study. Allocation of the classes into experimental and control group was done by random sampling through balloting, which resulted to two (2) classes assigned to experimental group and the other two (2) to the control group. All the one hundred and seventy nine (179) SS2 students in the four streams of the selected schools formed the sample of the study. The instruments for data collection was Geometry Interest Inventory (GII) developed by the researcher from vocational interest (Creswell, 2018) and other interest inventories. The GII was used to help students express their feelings towards geometry. It consists of two sections. Section A sought general information about respondents, while Section B bothered on their interest in geometry. The GII is a 30-item inventory with a five-point response type of *"Highly Interested",* if you like it very much to engage in the activity; *"Interested",* if you like to engage in it; *"undecided"*, if you like to neither like nor dislike it; *"Not interested",* if you dislike engaging in the activity and "*Highly Not Interested"*, if you very much dislike engaging in it. Like very much, like, neither like nor dislike, dislike and dislike very much all had values of 5, 4, 3, 2 and 1 respectively. The instruments were validated by two (2) mathematics educators in Science Education Department and one (1) measurement and evaluation expert. The reliability index of the GII was established using Cronbach Alpha reliability estimate. Cronbach alpha was used because the GII items were polytomously scored. The reliability index was found to be 0.80. Statistical Package for Social Science (SPSS) software version 23 was used to analyze the data. The research questions were answered using mean ($\bar{x}$) and standard deviation (SD). The research hypotheses were tested at 0.05 levels of significance using Analysis of Covariance (ANCOVA). ANCOVA is an extension of Analysis of Variance that allows for the exploration of the differences between groups, while statistically controlling for an additional variable called the Covariate (Pallant, 2018). The pre-GII scores served as the covariates. The pre-GII scores were also subjected to independent t-test analysis to determine the equivalence of the groups.

Analysis and Results

Research Question 1: What are the mean interest scores of students taught geometry using mobile app instructional technique and those taught using lecture method?

Table 1: Mean Interest Scores, Standard Deviations and Mean Gain of Students Taught Using MAIT and Lecture Method

Groups	N	Pre-GII		Post-GII		Mean Gain
		$\bar{x}$	SD	$\bar{x}$	SD	
Experimental	86	1.77	0.66	3.26	1.04	1.49
Control	93	1.72	1.55	2.31	0.76	0.59

The result in Table 1 shows that the students in the experimental group had a mean interest score of 1.77 and standard deviation of 0.66 in the Pre-GII, while students in control group had a mean interest score and standard deviation of 1.72 and 1.55 respectively. Similarly, in Post-GII, mean interest score of 3.26 and standard deviation of 1.04 students in experimental group respectively, while mean achievement score and standard deviation of students in control group are 2.31 and 0.76 respectively. The standard deviations of students taught geometry using MAIT increased from pre-GII to post-GII indicating that the scattering of the scores increases as the mean increased, while the students taught with lecture method decreased from pre-GII to post-GII indicating that the scattering of the scores decreases as the mean increased. The scattering of the scores was higher for those taught geometry using MAIT when compared to those taught geometry using lecture method. The mean interest gain between Pre-GII and Post-GII for students taught in experimental and control group are 1.49 and 0.59 respectively. This implies that the mean interest gain of students taught geometry using MAIT was higher than those taught using Lecture method.

Research Question 2: What are the mean interest scores of male and female students taught geometry using mobile application instructional techniques?

Table 2: Mean Achievement Scores and Standard Deviation of Male and Female Students Taught Geometry Using Mobile application instructional techniques

Sex	N	Pre GII		Post-GII		Mean Gain
		$\bar{x}$	SD	$\bar{x}$	SD	
Male	43	1.89	1.55	2.27	1.31	0.38
Female	43	1.73	0.61	2.31	1.26	0.58

Table 2 shows that the male students in the experimental group had a mean interest score of 1.73 and standard deviation of 1.55 in the Pre-GII and a mean interest score of 2.27 and standard deviation of 1.31 in the Post-GII. It was also revealed that the female students in the experimental group obtained a mean interest score of 1.73 and standard deviation of 0.61 in the Pre-GII and a mean interest score of 2.31 and standard deviation of 1.26 in the Post-GII. The standard deviations of female students taught geometry using MAIT increased from pre-GII to post-GII indicating that

the scattering of the scores increases as the mean increased, while the male students taught with MAIT decreased from pre-GII to post-GII indicating that the scattering of the scores decreases as the mean increased. The scattering of the scores was higher for females taught geometry using MAIT when compared to their male counterparts. The mean interest gain between Pre-GII and Post-GII of male and female students taught geometry using mobile application-based instructional techniques is 0.38 and 0.58 respectively. This implies that the mean interest gain for female was higher than their male counterparts.

Research Question 3: What are the mean interest scores of science and arts students taught geometry using mobile application instructional techniques?

Table 3: Mean interest Scores and Standard Deviation of Arts and Science Students Taught Using Mobile Application

Choice of discipline	N	Pre GII		Post-GII		Mean Gain
		$\bar{x}$	SD	$\bar{x}$	SD	
Science students	52	1.73	0.63	2.61	1.28	0.88
Arts students	34	1.73	1.57	2.30	1.23	0.57

The Table 3 above shows that the science students in the experimental group had a mean interest score of 1.73 and standard deviation of 0.63 in the Pre-GII and a mean interest score of 2.61 and standard deviation of 1.28 in the Post-GII. It was also revealed that the arts students in the experimental group obtained a mean interest score of 1.73 and standard deviation of 1.57 in the Pre-GII and a mean interest score of 2.30 and standard deviation of 1.23 in the Post-GII. The standard deviations of science students taught geometry using MAIT increased from pre-GII to post-GII indicating that the scattering of the scores increases as the mean increased, while the arts students taught with MAIT decreased from pre-GII to post-GII indicating that the scattering of the scores decreases as the mean increased. The scattering of the scores was higher for those science students taught geometry using MAIT when compared to their arts counterparts. The mean interest gain between Pre-GII and Post-GII of science and arts students taught geometry using mobile application-based instructional techniques is 0.88 and 0.57 respectively. This implies that the mean interest gain of science students taught using MAIT was higher than the mean interest gain of arts students taught with MAIT.

Ho_1: There is no significant difference between the mean interest scores of students taught geometry using mobile app instructional techniques and those taught using lecture method.

Table 4: Analysis of Co-variance (ANCOVA) for Students' Mean Interest Scores by Mobile Application Instructional Technique and lecture method

Source	Type III Sum of Squares	Df	Mean Square	F	Sig.	Partial Eta Squared
Corrected Model	149.020[a]	2	74.510	91.168	.000	.509
Intercept	310.434	1	310.434	379.839	.000	.683
PREGII	.087	1	.087	.106	.745	.001
APPROACH	148.501	1	148.501	181.702	.000	.508
Error	143.841	176	.817			
Total	1245.760	179				
Corrected Total	292.861	178				

Summary of data analysis presented in Table 4 shows that the Pre-GII scores for both groups (Control and Experimental) has f-calculated value of 0.106 and a p-value of 0.745 which is greater than the critical p-value of 0.05 which implies that they are equivalent and that both groups have the same baseline knowledge before the treatment was implemented. It also shows that the main effect, teaching approach has f-calculated value of 178.749 and a p-value of 0.00 which is less than the critical p-value of 0.05. This is based on 1 degree of freedom for numerator and 178-degree of freedom for denominator. This implies that the null hypothesis is rejected. That is the difference between the mean interest scores of students taught geometry, using mobile app instructional techniques and those taught using lecture method is statistically significant.

Ho_2: There is no significant difference between the mean interest scores of male and female students taught geometry using mobile application instructional technique.

Table 5: ANCOVA Analysis of Mean Interest Scores of Male and Female Students in Experimental Group

Source	Type III Sum of Squares	Df	Mean Square	F	Sig.	Partial Eta Squared
Corrected Model	149.048[a]	4	37.262	45.084	.000	.509
Intercept	305.030	1	305.030	369.059	.000	.680
PREGII	.103	1	.103	.124	.725	.001
METHOD	147.738	1	147.738	178.749	.000	.507

GENDER	.022	1	.022	.026	.871	.000
METHOD * GENDER	.006	1	.006	.008	.931	.000
Error	143.812	174	.827			
Total	1245.760	179				
Corrected Total	292.861	178				

The results presented in Table 5 above shows that the Pre-GII scores for both groups (Male and Female) has f-calculated value of 0.124 and a p-value of 0.725 which is greater than the critical p-value of 0.05 which implies that they are equivalent and that both groups have the same baseline knowledge before the treatment was implemented. It also shows that the main effect, gender, has f-calculated value of .026 and a p-value of .871 which is greater than the critical p-value of 0.05. This means that the null hypothesis is not rejected. That is, the difference between the mean interest scores of male and female students taught geometry using mobile app instructional technique is not statistically significant.

Ho_3: There is no significant difference in the mean interest scores of science and arts students taught geometry using mobile application instructional techniques.

Table 6: ANCOVA Analysis of Mean Interest Scores of Science and Arts Students in Experimental Group

Source	**Type III Sum of Squares**	**Df**	**Mean Square**	**F**	**Sig.**	**Partial Eta Squared**
Corrected Model	152.384[a]	4	38.096	47.187	.000	.520
Intercept	312.097	1	312.097	386.576	.000	.690
PREGII	.164	1	.164	.203	.653	.001
METHOD	134.632	1	134.632	166.761	.000	.489
CHOICE	1.625	1	1.625	2.013	.158	.011
METHOD * CHOICE	1.621	1	1.621	2.008	.158	.011
Error	140.477	174	.807			
Total	1245.760	179				
Corrected Total	292.861	178				

The result presented in Table 6 above shows that the Pre-GII scores for both groups (Arts and Sciences) has f-calculated value of 0.203 and a p-value of 0.653 which is greater than the critical p-value of 0.05 which implies that they are equivalent and that both groups have the same baseline knowledge before the treatment was implemented. It also shows that the main effect, choice of discipline, has f-calculated value of 2.013 and a p-value of .158 which is greater than the critical p-value of 0.05. This means that the null hypothesis is not rejected. Therefore, the difference in the mean interest scores of science and arts students taught geometry using mobile app instructional techniques is not, statistically significant

Discussion of Findings

Method, and interest scores in geometry

The findings of this study show that the mobile app instructional technique increases the interest of students in geometry. Result from research question 4 shows that there was a higher mean interest score for the group taught using mobile app instructional technique over the group taught using the lecture method. This was further confirmed by the result from research hypothesis 4 which intends to find out if there exists a significant difference between the mean interest scores of the two groups. This could be attributed to the use of mobile application instructional technique which is a platform that can sustain the interest of learners. This finding aligns with Ectuban (2018) who reported that mobile devices used for teaching mathematics attracted the interest of students. It also supports the findings of Khristin, Keith and Barron (2018),Yeng and Chieng(2019) and Grant (2015) who reported that the use of mobile apps improved students' interest scores significantly.

Method, gender and interest scores in geometry

Analysis of result from research question 5 revealed that the mean interest scores of male and female students taught geometry using mobile application instructional technique. The result revealed that both male and female students in the experimental group gained from the method used. This was further confirmed by the result from hypothesis 5 which intends to find out if there exists a significant difference between the mean interest scores of male and female students in the experimental group. Analysis shows that gender was not significant to students' interest in geometry. This could be attributed to the use of mobile app instructional technique (MAIT) which help students to modify and build on their view points as well as create avenues for brainstorming exercise that allows the exchange of ideas between students. This indicated that the use of mobile application instructional technique bridged gender gap of students' interest in mathematics. This finding supports Khristin & Barron, (2015), Hilao & Wichadee(2017), Mman & Tukandaya(2019) who reported non-significant difference in the mean interest scores of male and female students when mobile application and devices was used as a medium of instruction. This finding disagrees with Chung and Rong-Chil (2017) who reported that mean interest scores does not interact with gender.

Method, choice of discipline and interest scores in geometry

Analysis of result from research question 6 revealed the mean interest scores of Science and Arts students taught geometry using mobile application instructional technique. The result revealed that both Science and Arts students in the experimental group gained from the method used. This was further confirmed by the result from hypothesis 6 which intends to find out if there exists a significant difference between the mean interest scores of Science and Arts students in the experimental group. Analysis confirmed that choice of discipline was not significant to students' interest in geometry. This could be attributed to the use of mobile application instructional technique (MAIT) which creates an avenue for students irrespective of their choice of discipline to learn geometry by practice during classroom activities. This indicated that the use of mobile app instructional technique bridged the gap that existed between science and arts students' interest in mathematics. This finding aligns with Al-Takhyneh (2018) who reported that the use of mobile applications enables the students in arts, science and commercial to interact directly with the educational content thereby increasing their interest in geometry. This finding also supports Bilesanmi and Afuwape (2017), who reported that the use of mobile devices enabled students to represent construction graphically, carry out suitable and over lapping geometric transformations, and control the properties of the angles and lines on the contrary. This finding contradicts Somalia (2019), Ado (2019) who reported a significant difference in the mean achievement in mathematics in favour of science students, when compared with their arts counterparts.

Conclusion

The study established that mobile application instructional technique improved the interest of students, bridged the gap resulting from the difference in interest of male and female students as well as difference in the interest of science and arts students in geometry.

Recommendations

Considering the above findings, of the following recommendations were made;

1. Mobile Application Instructional Technique should be implemented in the teaching and learning of geometry and mathematics in Nigerian secondary schools in order to enhance students' achievement and interest.

2. Government and other stakeholders should ensure that teachers are trained adequately to enable them utilize mobile devices for teaching and learning purpose.

3. Students must have regular access to technologies that support learning so as to advance their mathematical thinking, reasoning, problem-solving, and communication skills.

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