UNIVARIATE TIME SERIES ANALYSIS OF CONSUMER PRICE INDEX ON FOOD AND NON-ALCOHOLIC BEVERAGES

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Abstract

The paper examined univariate time series forecast of consumer price index on the consumption of food and nonalcoholic beverages in Nigeria. It filled the knowledge gap by explicitly modeling and forecasting consumer price index in Nigeria using the univariate ARIMA model. The work was restricted to Nigerian Consumer Price Index. It was also restricted to food consumption (FC) data and food & nonalcoholic beverages consumption (FNBC) data from 1995-2021. This paper analyses were carried out using gretl 2019c, Minitab 16 and Micro software Excel (2010). The monthly and yearly means plots were done, so as to have a better understanding of the series behaviours. The series plots points to the fact that there is possibility that the time series are integrated of order 1 for food consumption series with no seasonality, while integrated of order 1 for food & nonalcoholic beverages consumption series with seasonality of order 12. Stationarity after second difference of the first differencing was obtained for both series. A suitable ARIMA Model was obtained for both series and was used for models forecast computation. Hence, the computed result suggested that ARIMA(0,1,1) and ARIMA(0,1,1)$(0,0,0)_{12}$ model were the best model for estimating and forecasting the two time series, using model selection criteria and accuracy measures. The plots of the forecasts generated for the FC and FNBC shows that the two variables are dependent and also shows that any gradual increase in the food consumption tends to pave way for increase in the food & nonalcoholic beverages consumption or a drastic drop in the food consumption will also drop the food & nonalcoholic beverages consumption in the same manner. It seems reasonable to conclude that there is significant relationship between the food consumption and food & nonalcoholic beverages consumption series. It was recommended that more detailed work should be carried out in the area of co-integration analysis of the two variables to enhance a better understanding and prediction distribution in Nigeria.

Keywords*:* *Consumer Price Index; Time Series Analysis; Non-alcoholic Beverages; Food Consumption; ARIMA Model*

1. Introduction

The environment of any organization is the aggregate of all conditions, events, and influences that surround and affect organizations (Levy, 1992). It is an aspiring sesame that economic environment in Nigeria is an embodiment of macro-economic variables associated with the factors of production of goods and services. The environment is dynamic and constantly changing. This then creates opportunities and threats for food and beverage sub sector of Nigeria. Prominent stakeholders and players in the Nigerian Food and Beverage subsector include Nestle Food Nigeria Plc, Cadbury Nigeria Plc, Nigeria Bottling company Plc, and Dangote flourmill Plc to mention but a few. An analysis of inflation in a country needs to be carried out to prevent and control inflation, to determine the causes of inflation, and as a basis for economic policy making to prevent an economic crisis, to maintain economic stability and to ensure the welfare of society (Siswanah, 2021).

Decision makers must make forecasts to help in decision making. To conduct these forecasts, most central banks take a number of variables into account. However, it is not an easy task, especially in developing countries, where economic processes are highly unstable and volatile. Moreover, the macroeconomic data on developing countries can be unreliable due to many reasons: measurement error, imperfect methods of measuring, etc. Nevertheless, there exist a number of empirical studies on inflation factors in developing countries. These studies show that inflation is a country-specific phenomenon, and its determinants differ across countries (Adams, 2014). Therefore, an effective monetary policy depends largely on the ability of economists to develop a reliable model that could help understand the on-going economic processes and predict future developments. In this regard, this study is important since it is aimed at forecasting Consumer Price Index (CPI), which is a component of inflation in the Nigeria economy. Consumer price index (CPI) is a measure that examines the weighted average of price of a basket of consumer goods and services, such as transportation, food and medical care; it is one of the most frequently used statistics for identifying period of inflation or deflation (Adams, 2014). Consumption of non-alcoholic beverages (NABs) such as Juice and carbonated drinks has been a basic form of refreshment among Nigerians of all ages, tribes and socioeconomic backgrounds (Phillip, 2013).

Forecasting is a global important part of econometric analysis, the most important probably for most people. How do we forecast economic variables, such as Food and non-alcoholic beverages, GDP, inflation, exchange rates, stock process, unemployment rates and myriad? Other economic variables problems involved in forecasting prices of financial assets, such as food and food & non-alcoholic beverages are of great concern. These asset prices are characterized by the phenomenon known as vitality clustery (Adams, 2014). It is no longer news that the global economic crisis has brought about shortage of financial resources and to a general down tone in consumption price index of food and food & non-alcoholic beverages across the globe. Therefore, forecasting consumption prices (food and food & non-alcoholic beverages) will help provide a way to expect and maybe avoid the risk of large change in prices.

Time series analysis is a statistical technique that deals with time series data, or trend analysis. Time series data means that data is in a series of particular time periods or intervals. The data is considered in three types: (1) Time series data: A set of observations on the values that a variable takes at different times. (2) Cross-sectional data: Data of one or more variables, collected at the same point in time. (3) Pooled data: A combination of time series data and cross-sectional data. (4) Exponential smoothing in time series analysis: This method predicts the one next period value based on the past and current value. It involves averaging of data such that the nonsystematic components of each individual case or observation cancel out each other. The exponential smoothing method is used to predict the short term predication. Alpha, Gamma, Phi, and Delta are the parameters that estimate the effect of the time series data. Alpha is used when seasonality is not present in data. Gamma is used when a series has a trend in data (Durbin, 2012). Delta is used when seasonality cycles are present in data. A model is applied according to the pattern of the data.

ARIMA modeling has been successfully used in various food and nonalcoholic beverage-market activities. With the growing economy, people need more funds to meet up the rapid expansion. At the same time people source for money in order to meet up with the numerous challenges. As such the food and nonalcoholic beverage market serves as an important tool in mobilizing and allocating savings among users who are critical to the growth and efficiency of the growing economy. This has led researchers to explore efficient ways of predicting food and nonalcoholic beverage market activities to enhance the benefit derived from them. No method has been discovered to accurately predict price movement of food and nonalcoholic beverage after numerous attempts by researchers.

As a result of insecurity and risk in some parts of Nigeria, manufacturing sector including food and beverage manufacturing sub-sector have lost substantial portion of their sales as it becomes problematic to penetrate some parts of the country. In view of these environmental challenges of Boko Haram vis-a-vis insurgency in the country, most Food and Beverage companies have got to relocate. This leads to decrease in Gross Domestic Product (GDP), promotes imports and demotes export among others. As earlier stated above, poor infrastructure associated with unstable power supply affect Food and Beverage subsector. This results in high cost of operation and low profit margin in the sub-sector.

In addition, consumer price index on the consumption of food and nonalcoholic beverage has become one of the well-known investments in the recent past due to its higher returns. It has become a great part of the global economy in the food and nonalcoholic beverage market influences both personal and corporate lives and economic life of a country. The Nigerian consumer price index forecasting is known more by its failure than success since its prices reveals the judgment and what investors expects base on the available information.

Base on this, the accuracy in forecasting the consumer price index on the consumption of food and nonalcoholic beverage or predicting the trend accurately is of importance of for anyone who wishes to invest in the dynamic global economy. Over the years, economists and financial analysts have constantly maintained that a market price that is not regulated is the best and stick to prove the true scarcity of a commodity or its worth. It is easy for one to evaluate the consumption of food and nonalcoholic beverage (CF and NABC) performance by the use of consumer price index or returns. You can predict consumer price returns from a variety of financial and macroeconomic variables which has been any attraction for equity investors. Of recent, attention has increased on the shift to the consumer price index as a way of measuring a sector of the consumption of food and nonalcoholic beverage market. The investing public has to a large extent an important indicator used by a bench mark by which investor or fund management compares the returns of his portfolio (Senol, 2012).

Thus, the need to predict the consumer price in order to meet the basic objectives of operators and investors of the consumer market for gaining more benefits cannot be overemphasized. This issue has brought to focus the attentions of statisticians and researchers all over the world. Consumer market is affected by numerous factors and this has created high controversy in the field. Many methods and approaches for models are present in the interaction. This study exclusively deals with the time series forecasting model and in particular the Autoregressive Integrated Moving Average (ARIMA) models which were described by Box-Jenkins.

This paper fills the lacuna by explicitly modeling and forecasting the consumer price index of food and nonalcoholic beverage consumption using the ARIMA model. It contributes to knowledge of forecasting consumer returns and expands forecasting literature in Nigeria. It may also spur further studies aimed at either sustaining or debunking its forecast model. The particular objectives are:

1. Describe the series plots, yearly mean plots, and monthly mean plots and obtain the stationarity of the series.
2. Determine the year with the highest Food Consumption (FC) and Food & Non-alcoholic Beverage Consumption (FNBC) rate.
3. Obtain a suitable model to fit the consumer price index of food and nonalcoholic beverage (Food Consumption and Non-alcoholic Beverage consumption) using the model selection criteria (AIC and BIC).
4. Estimate the forecasts from the obtained models considered (ARIMA model) covering the period of 2022-2023 using accuracy measures of forecast values.

1.2 Statement of the Problem

It is obvious that the global economic crises have led to a general down turn in consumer prices and shortage of financial resources across the globe. Most of the world’s reports on consumer markets highlight substantial decline in consumer price.

As a result of insecurity and risk in some parts of Nigeria, manufacturing sector including food and non-alcoholic beverage manufacturing sub-sector have lost substantial portion of their sales as it becomes problematic to penetrate some parts of the country. In view of these environmental challenges of BokoHaram vis-a-vis insurgency in the country, most Food and non-alcohol beverage companies have got to relocate. This leads to decrease in Gross Domestic Product (GDP), promotes imports and demotes export among others. As earlier stated above, poor infrastructure associated with unstable power supply affect Food and Beverage subsector. This results in high cost of operation and low profit margin in the sub-sector.

The purpose of this study is to forecast and model the Consumer Price Index (CPI) on the consumption rate of food and consumption rate of food & non-alcoholic beverage using ARIMA time series. Forecasts of CPI of consumption food and non-alcoholic beverages are important because they affect many economic decisions. Without knowing future CPI rates, it would be difficult for lenders to price loans, which would limit credit and investments in turn have a negative impact on the economy. Investors need good CPI forecasts, since the returns to stocks and bonds depend on what happens to CPI. Businesses need CPI forecasts to price their goods and plan production. Homeowners' decisions about refinancing food and non-alcoholic beverage also depend on what they think will happen to CPI. Modeling CPI on the consumption rate of food and non-alcoholic beverage is important from the point of view of poverty alleviation and social justice.

It is also evident that consumer price index of food and nonalcoholic beverage market has been one of the most popular investments in Nigeria and the globe at large due to its high returns. Also, fluctuation in this market influences personal and cooperate financial hires and the economic health of a country. Accurately forecasting the consumer price index of food and nonalcoholic beverage consumption is of crucial importance for any future investor. Thus, there is need to predict the consumer price index to meet the fundamental objectives of investors and operators of the food and nonalcoholic beverage market. This has attracted the attention of Researchers so as to fill in this gap.

1.3 Significance of the Research

The study is significant to policy makers as it guides them in formulating macroeconomic policies by providing them with a long term perspective of food consumption and non-alcoholic beverage. Optimal policy will depend on optimal food and non-alcoholic beverage consumption forecasts. CPI uses include;

- As the main estimator of the rate of inflation. The percentage change of the CPI over a one-year period is what is usually referred to as the rate of inflation.
- A macroeconomic indicator. The CPI is used for general economic/social analysis and policy formulation particularly since it conveys important information about indirect tax revenue.
- As a tool in wage negotiation and indexation. CPI is used to adjust taxes and to determine, among other things, wage levels in the event of trade disputes, social security benefits, public service remuneration and pensions.
- As a deflator of expenditure. The prevailing CPI can be used to establish the real/constant value by deflating nominal values (previous cost) of goods and services.

Forecasting inflation in the food and non-alcoholic beverage industries generally improves financial planning in both the corporate and private sectors. Inflation affects actual cost of expenses and stock valuations on the corporate level. Forecasting changes can therefore help investors understand risks and hedge investments.

1.4 Scope of the Research

This paper focused on univariate time series forecast of consumer price index of food and non-alcoholic beverage consumption rate. It uses the components of food consumption rate of the consumer price index (CPI) and the food & non-alcoholic beverage consumption rate which will comprise the monthly value of the both. It fills this lacuna by explicitly modeling and forecasting consumer price index returns in Nigeria using the bivariate ARIMA model. The box Jenkins approach of model of identification, parameter estimation and diagnostic checking will be adopted in the analyses.

2. Literature Review

Many authors among who are: Lirby (2007), Malkeil (2013), and Durbin (2012) have compared, estimated, and forecasted for the future consumptions rate and commodities dealing with auto-correlation.

Abdullahi and Yakubu (2013) conducted a study on ‘Determinants of Non Alcoholic Beverages (NAB) Consumption in North-Western Nigeria: A study of Sokoto Metropolis’. This study seeks to determine the role played by socioeconomic characteristics in driving consumption of some selected NABs particularly juice and carbonated soda in Sokoto metropolis of Nigeria. To adequately capture the variables of interest, specifically, location and ethnicity variables, the metropolis was grouped into four clusters, namely Peri-urban, Sokoto main, G R As, and Resident community. In each of the clusters one hundred households were randomly selected to arrive at a sample size of four hundred households. Descriptive, OLSMR and probit analytical tools were used to analyse the data collected. The results shows that about 59% and 71% of the respondents consumed juice and soda respectively while the per capita consumption of juice was slightly higher than that of soda which were 7.57 and 7.32 litres respectively.

Partick et al. (2016) studied economic environment and performance of food and beverage sub-sector of a developing economy: Nigeria. This study examines the implications of economic environment on the performance of food and beverage sub-sector of Nigeria. The economic environment is an embodiment of dynamic variables characterized by significant challenges impacting on the food and beverage sub-sector. Performance in this sector is measured in terms of profitability, exchange rate, interest rate, current asset, turnover, market share and return on investment among others. This study therefore serves as report of investigation into the implications of these variables on the performance of food and beverage sub sector. The ordinary least square technique is adopted in the methodology and the result reveals a significant relationship between economic environmental variables and the food and beverage sub-sector. The study advocates a strong public private partnership between governments and the sector as well as encouragement of stable exchange rate so as to foster economic growth.

Phillip et al. (2013) studied the demand for non-alcoholic beverages among urban households in South-West, Nigeria. This study examined the roles of income, prices and household demography in household demand for non-alcoholic beverages (NABs) in two cities – Abeokuta and Ibadan in Southwest Nigeria. The study was based on primary data obtained from a cross-section of 407 households (211 from Abeokuta and 198 from Ibadan) drawn by multistage sampling technique across six Local Government Areas (LGAs) and 60 National Population Commission (NPC) enumeration areas (EAs). A structured questionnaire was used to collect data on households NABs expenditure, income, prices and other relevant socio-economic variables. The data were analysed within the framework of a linear approximation of an Almost Ideal Demand System. The study found that an average household, consisting of five (5) members, expended an average N5, 235.89 per month on NABs (approximately US$34.21 at N153.06/US$1 exchange rate in 2010). The bulk (67%) of the NABs expenditures were devoted to purchase of dairy products (36%) and cocoa-based products used in preparing chocolate drinks (31%). The estimated income elasticity of demand for the six categories of NABs studied were positive while all the own price elasticity of demand were negative. Demand for dairy products and cocoa based drinks were found to be price elastic, while those of carbonated drinks, malt drinks, fruit juice and other NABs were price inelastic. Increase in education of the household heads was found to be associated with significant increase in the budget shares of dairy products ($p<0.01$) and fruit juice ($p<0.10$), but a significant reduction in budget shares of cocoa-based products ($p<0.05$), carbonated drinks ($p<0.01$) and malt drinks ($p<0.05$). The study concludes that policies aimed at promoting increased demand and healthy choices of NABS must pay some attention to raising real income and increasing level of education among the citizenry. Profitability of business enterprises involved in dairies and cocoa-based products would also be better enhanced if the firms adopt cost saving strategies as against price hikes in a bid to enhance performance.

Nkpordee and Nduka (2018) conducted a research on application of time series analysis on the forecasting of the outbreak of malaria epidemic in Nigeria. The secondary data used for the study was obtained from National Bureau of Statistics (nbs), Social Statistics in Nigeria. The research work employed Box-Jenkins (1976) methodology to construct a suitable mathematical model by putting the ACF and PACF correlogram into consideration. The model; ARIMA(1,0,1) was used to forecast the monthly reported cases of malaria with a 16 months lead, which shows a gradual increase and decrease in the series. The research also evaluated the monthly mean of the reported cases of malaria. It was recommended that Government should ensure the provision of treated bed mosquito nets, insecticides, anti-malaria drugs etc. in the rural areas in Nigeria.

Al-Shiab (2006) examined the univariate ARIMA forecasting model on the Amman Stock Exchange (ASE) general daily index between 4/1/2004 and 10/8/2004; with out-of-sample testing undertaken on the following seven days. Four diagnostic tests were performed to select the best model describing the data, namely: R-square, adjusted R-square, Akaike's Information Criterion (AIC), and Schwarz Information Criterion (SIC). On the basis of these four diagnostic tests, ARIMA (*4, 1, 5*) model was chosen as the best model that explains the data and is suitable for accurate forecasting. The selected model predicted that the ASE would continue to grow by 0.195% for seven days starting on 11/8/2004. This forecast, however, was not consistent with actual performance during the period of the prediction since the ASE declined by 0.003%. He concluded that the forecast error implies the ASE tends towards weak form efficiency.

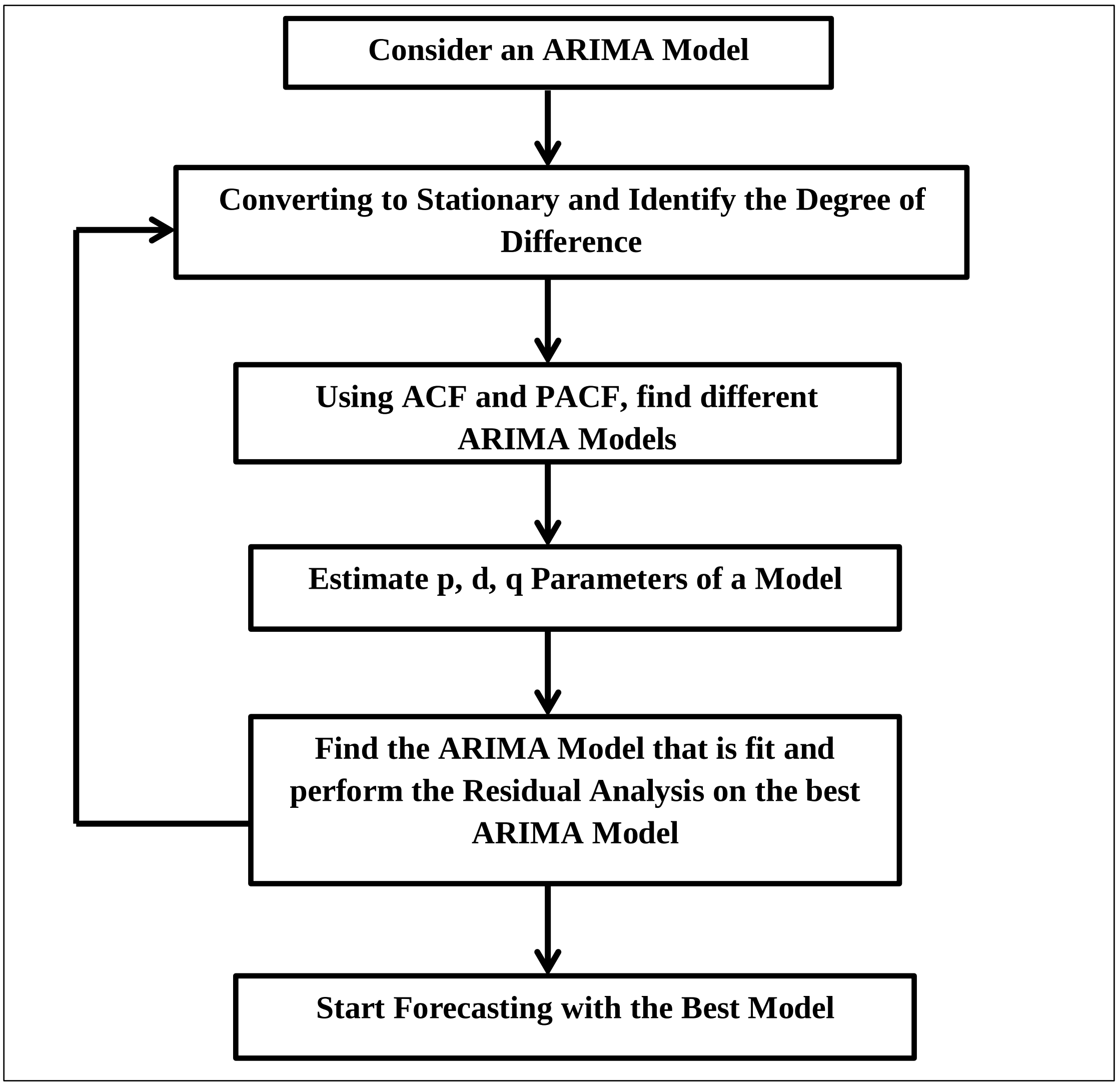
Stanley, Biu and Enegesele (2020) conducted a study on comparison of univariate and bivariate time series forecasts of Nigerian stock exchange variables. The study examined univariate and bivariate time series forecast of Nigerian stock exchange variables: All Share Index (ASI) and the External Reserves (ER) which comprise of monthly value from 1985 to 2018 of them both. It filled the lacuna by explicitly modeling and forecasting stock returns in Nigeria using the univariate ARIMA and bivariate VAR models. The monthly and yearly means plots were done, to have a better understanding of the series behaviours. The order of the regular autoregressive and moving avenge model that is necessary to adequately represent the time series model was determined. The series plots showed that ASI series is integrated of order 1 without seasonality while ER series is integrated of order 1 with the seasonality of order 12. A suitable ARIMA and VAR Model were obtained for both series using model selection criteria (MSC) and the models were used to generate forecasts. The univariate and bivariate model forecasts were compared and the result shows that the bivariate model is better to predict the two series than the univariate model from the result of forecast accuracy measures (i.e. MAPE and MSE).

3. Materials and Methods

3.1 Research Design: The study seeks to examine univariate time series forecast of consumer price index of food and non-alcoholic beverage consumption rate, using the bivariate ARIMA model. The Box-Jenkins approach of model identification, parameter estimation, and diagnostic checking will be adopted in the analyses. This study is restricted to the consumer price index of Nigeria. It is also restricted to food consumption (FC) and non-alcoholic beverage consumption (FNBC) rate data. The secondary data used for the study were collected from the National Bureau of Statistics (NBS) Statistical Bulletin. It is the monthly data of food consumption (FC) and non-alcoholic beverage consumption (FNBC) on the consumer price index ranging from 1995 to 2021.

3.2 Nature and Source of Study Data: The accumulated and used data for this study is a secondary statistics extracted from survey information performed in Nigeria on the consumer price index (specifically on the components of food consumption rate and the food & non-alcoholic beverage consumption rate) from the National Bureau of Statistics from January 1995 to December 2021. Implicit composite consumer price index table of the rebased GDP figures (Base Period: November 2009 = 100) evaluating several interest sectors of the Nigeria economy data was used. The data are presented in Appendix A and B. One impartial bivariate model has been proposed to predict the trend in both food consumption rate and food & non-alcoholic beverage. The following programmes are used to acquire the parameters which constitute the model; a number of which include Gretl 2019c and MINITAB 16. To facilitate records evaluation, the researcher made use of Microsoft Excel 2010, Gretl 2019c and MINITAB 16. Gretl 2019c and MINITAB 16 had been utilized in estimating the parameters for the ARIMA model.

3.4 Flowchart of Method



3.5 Model Specification

Autoregressive Model AR (p)

An autoregressive process is denoted by AR (P) and this can be denoted by

$$X_t = \sum_{k=1}^{p} \phi k(B^{k})X_t + \varepsilon_t \tag{3.1}$$

where

$$\phi(B) = 1 - \sum_{k=1}^{p} \phi\varepsilon_t B^k = 1 - (\phi_1 B + \phi_2 B^2 + ... + \phi_k B^k)$$

but since $\phi(B)X_t = \varepsilon_t$ for an AR(p) process we have $(1 - \phi_1 B - \phi_2 B^2 - ... - \phi_k B^k)X_t = \varepsilon_t$

$$\Rightarrow X_t - \phi_1 X_{t-1} - \phi_2 X_{t-2} - ... - \phi_k X_{t-k} = \varepsilon_t$$

$$X_t = \phi_1 X_{t-1} + \phi_2 X_{t-2} + ... + \phi_k X_{t-k} + \varepsilon_t \tag{3.2}$$

$\phi_1, \phi_2, ..., \phi_k$ are constants and ε_t is a sequence of independent (uncorrelated) random variables with mean 0 and variance σ^2 such sequence of random variables is called the White noise $\varepsilon_t \approx N(0, \sigma^2$

Moving Average Model MA(q)

The moving average of order q is denoted by

$$X_t = \sum_{k=1}^{p} \theta_k (B^k)\varepsilon_t \tag{3.3}$$

$$\theta(B) = 1 + \sum_{k=1}^{p} \theta_k B^k = 1 + \theta_1 B + \theta_2 B^2 + ... + \theta_k B^k$$

But since $X_t = \theta(B)\varepsilon_t$ for an MA process, then we have

$$X_t = (1 + \theta_1 B + \theta_2 B^2 + ... + \theta_q B^q)\varepsilon_t$$

$$= \varepsilon_t + \theta_1 \varepsilon_{t-1} + \theta_2 \varepsilon_{t-2} + ... + \theta_q \varepsilon_{t-q} \tag{3.4}$$

Where $\theta_1, \theta_2, ..., \theta_n$ are the fixed constants with $\theta_0 = 1$ and ε_t is a sequence of independent (uncorrelated) random variables with mean 0 and variance σ^2 such sequence of random variables is called the White noise $\varepsilon_t \approx N(0, \sigma^2)$.

Autoregressive Moving Average (ARMA) Model

Box and Jenkins (1976) noted that the mixed autoregressive moving average model is the combination of *AR (p) and MA (q).* Let's say that X_t is the deviation from the mean µ, and then ARMA (p, q) model can be written as

$$y = \phi_1 x_{t-1} + \phi_p x_{t-p} + \theta_1 \varepsilon_{t-1} + ... + \theta_q \varepsilon_{t-q} . \tag{3.5}$$

Using the backward shift operator

$\phi(L)x_t = \theta(L)\varepsilon_t$, where $\phi(L)$ and $\theta(L)$ are polynomial of order p and q respectively. Thus,

$$\phi(L) = 1 - \phi_1 L - ... - \phi_p L^p.$$

If the root of the equation, $\phi(L) = 1 - \phi_1 L - ... - \phi_p L^p$, lies outside the unit circle, then the ARMA(p,q) model is stationary.

Mixed Autoregressive Integrated Moving Average (ARIMA) Model

The seasonal ARIMA model incorporated non-seasonal and seasonal factors in the multiplicative model. One shorthand notation for the model is ARIMA (p,d,q)*(P,D,Q)$_s$. Where P = non-seasonal AR order, d = non-seasonal differencing, q = non-seasonal MA order, s = time of the repeating seasonal pattern.

Without differencing operations the model could be written more formally as:

$$\phi_1(B)^p \phi_2(B^s)^P \nabla^d \nabla^D X_t = \theta_1(B)^q \theta_2(B^s)^Q \varepsilon_t \quad (3.6)$$

The non-seasonal components are:

AR: $\phi(B) = 1 - \phi_1 B - ... - \phi_p B^p$

MA: $\theta(B) = 1 + \theta_1 B + ... + \theta_q B^q$

The seasonal components are:

Seasonal AR: $\phi(B^s) = 1 - \phi_1 B^s - ... - \phi_p B^{Ps}$

Seasonal MA: $\theta(B^s) = 1 + \theta_1 B^s + ... + \theta_q B^{Qs}$

Box and Jenkins (1976) noted that the mixed autoregressive moving average model is the combination of *AR (p) and MA (q).* Let's say that X_t is the deviation from the mean μ, and then ARMA (p, q) model can be written as

$$X_t - \phi_1 x_{t-1} - \phi_2 x_{t-2} - ... - \phi_p x_{t-p} = \varepsilon_t - \theta_1 \varepsilon_{t-1} - \theta_2 \varepsilon_{t-2} - ... - \theta_q \varepsilon_{t-q} \quad (3.7)$$

Thus,

$$\phi(B)x_t = \theta(B)\varepsilon_t \quad (3.8)$$

The equation (3.7) can be written as

$$x_t = \phi^{-1}(B)\theta(B)\varepsilon_t$$

$$= \frac{\theta(B)}{\phi(B)}\varepsilon_t = \frac{1 - \theta_1 B - ... - \theta_q B^q}{1 - \phi_1 B - ... - \phi_p B^p}\varepsilon_t \quad (3.9)$$

The ARIMA model is based on prior values in the autoregressive terms and the error made by the previous prediction. The order of ARIMA model is given by *p, d, q* where, *p* represents the autoregressive component, d stands for the differencing to achieve stationarity and q is the order of the moving average.

Seasonal Autoregressive Integrated Moving Average (SARIMA) model applies to time series with seasonal and non-seasonal behavior. SARIMA model has a multiplicative and additive part. The multiplicative is so applied because of the assumption that there exists a significant parameter resulting from the multiplication between nonseasonal parameters. By the use of ∇ and B notation, ARIMA (p, d, q) model can be written as

$$\phi(B)w_t = \theta(B)\varepsilon_t \tag{3.10}$$

where the polynomial in B is given as

$$\phi(B) = 1 - \phi_1(B) - \ldots - \phi_p B^p and \theta(B) = 1 - \theta_1(B) - \ldots - \theta_q B^q$$

This study focused on the multiplication model because of the assumption that there is a major parameter between the non-seasonal and seasonal models. This is denoted by $ARIMA\ (p,d,q) \times (P,D,Q)_s$ written as

$$\phi_p(B)\phi_P(B^s)\nabla^d \nabla_s{}^D \chi_t = \theta_q(B)\theta_Q(B^s) \tag{3.11}$$

Where

$$\phi(B) = 1 - \phi_1 B - \phi_2 B^2 - \ldots - \phi_p B^p; \varphi(B^s) = 1 - \Phi_{1,} sB^s - \Phi_{2,} sB^{ss} - \ldots - \varphi_{p,} sB^{ps}$$

$$\nabla^d = 1 - B - B^2 - \ldots - B^d; \nabla_s{}^D = 1 - B^s - B^{2s} \ldots - B^{2D}$$

$$\theta(B) = 1 - \theta_1 B - \theta_2 B^2 - \ldots - \theta_q B^q; \Theta(B^s) = 1 - \Theta_{1,} sB^s - \Theta_2 sB^{2s} - \ldots - \Theta_{q,} sB^{Qs}$$

where χ_t is the time series at period t, ε_t stands for the white noise, B represents the backshift operator, is the duration of the seasonal model which could be weekly, quarterly, or yearly, p is the autoregressive parameter, P is the seasonal autoregressive parameter, d is the order of the monthly difference (quarterly difference), D is the order of seasonal difference, q is the moving average parameter and Q is the seasonal moving average parameter.

3.2 Model Selection Criteria (MSC)

The AR and MA order p and q have to be determined by examining the regular and seasonal autocorrelation and partial autocorrelation function; ACF, PACF, SACF, and SPACF for Yt. before an ARMA (p, q) is estimated. The idea is to fit all ARMA (p,q) models with order $p \leq p_{max}$ and $q \leq q_{max}$ and choose the value of p and q which minimizes some model selection criteria. For ARMA (p, q), the model selection criteria are given by

$$MSC(p,q) = Ln(\sigma^2(p,q)) + C_T.\varphi(p,q) \tag{3.12}$$

where $\sigma^2(p,q)$ is the MLE of $\text{var}(\varepsilon_t)C_T$ is a sequence indexed by the sample size T, and $\varphi(p,q)$ is a penalty function that penalizes large ARMA(p, q) model.

The three most common information criteria for selection models are the Akaike Information Criteria (AIC), Schwarz-Bayesian Information Criteria (BIC), and Hannan-Quinn Information Criteria.

Akaike Information Criteria

The AIC is a measure of the relative goodness of fit of a statistical model. The AIC value is given by

$$AIC = T \ln\left[\frac{RSS}{T}\right] + 2P \tag{3.13}$$

where T is the number of data points (observations); ln is the natural logarithm; RSS is the residual sum of square (σ^2) or the error variance of the model which is an unbiased estimator of the true variance and p is the number of parameters in the model (Akaike, 1983).

Schwartz-Bayesian Information Criteria (SBIC or BIC)

The BIC is a model selection criterion that involves selections among a finite set of models. The BIC is given by

$$BIC = T \ln\left[\frac{RSS}{T}\right] + P \ln(T) \tag{3.14}$$

where the parameters are defined as previous Equation (2.10)

Hunnan and Quinn (HQ) criterion

Hunnan and Quinn (1979) developed a procedure which is known as HQ criteria. Statistic of the procedure can be represented as

$$\text{HQ}=\left(\frac{ESS}{T}\right)(InT)\left(\frac{2K}{T}\right) \tag{3.15}$$

The value of HQ will decrease provided there are at least 16 observation (Ramanathan. 1995)

3.3 Model Accuracy Measures

To gauge the accuracy of our estimates, the estimated errors will be used to compare the two models forecasts. This is done by subtracting the estimated forecast values (EFV) from the original values or [actual values (AV)] to obtain the estimate errors. The estimated error is denoted by

$$e_i = AV_i - EFV_i, i = 1,2,\ldots,V \tag{3.16}$$

where v is the number of forecast values

Then accuracy measures considered in this paper are: Mean Error (ME), Mean Absolute Error (MAE), and Mean Square Error (MSE).

Mean Error (ME)

The first descriptive Statistics of Error used is called the Mean Error. It indicates the deviation between the actual values and estimates, Mean Error is given as

$$ME = \left[\frac{1}{V}\sum_{i=1}^{v} e_i\right] \tag{3.17}$$

Mean Square Error (MSE)

MSE also indicates the fluctuations of the deviations and it can be calculated as

$$MSE = \left[\frac{1}{V}\sum_{i=1}^{v} e_i^{\,2}\right] \tag{3.18}$$

Mean Absolute Percentage Error (MAPE)

This accounts for the percentage of deviation between the actual values and estimates. This can be obtained as

$$MAPE = 100 \times \left[\frac{1}{V}\sum_{i=1}^{V}\left|\frac{e_i}{AV_i}\right|\right] \qquad (AV_i \neq 0) \tag{3.19}$$

Mean Absolute Error (MAE)

$$MAE = \frac{1}{n}\sum_{t=1}^{n}\left|\hat{e}_t\right| \tag{3.20}$$

4. Results

This research work collected two data sets from the National Bureau of Statistics on Consumer Price Index on Food Consumption Report (1995-2021) and Consumer Price Index on Consumption of Food & Non Alcoholic Beverage (1995-2021). The data sets are Consumer Price Index on Food Consumption: 1995-2021 and Consumer Price Index on Consumption of Food & Non Alcoholic Beverage: 1995-2021; which were used to study the univariate time series analysis of series forecasts in Nigeria.

Monthly Plot, Yearly Plot and Series of the Data Sets

In this section, the monthly mean, yearly mean and the series plots (Consumer Price Index on Consumption of Food and Non Alcoholic Beverage) were done to examine the relationships, trend component and seasonality effect, if present in the data sets.

4.1 Monthly Plot, Yearly Plot and Series plot of Food Consumption

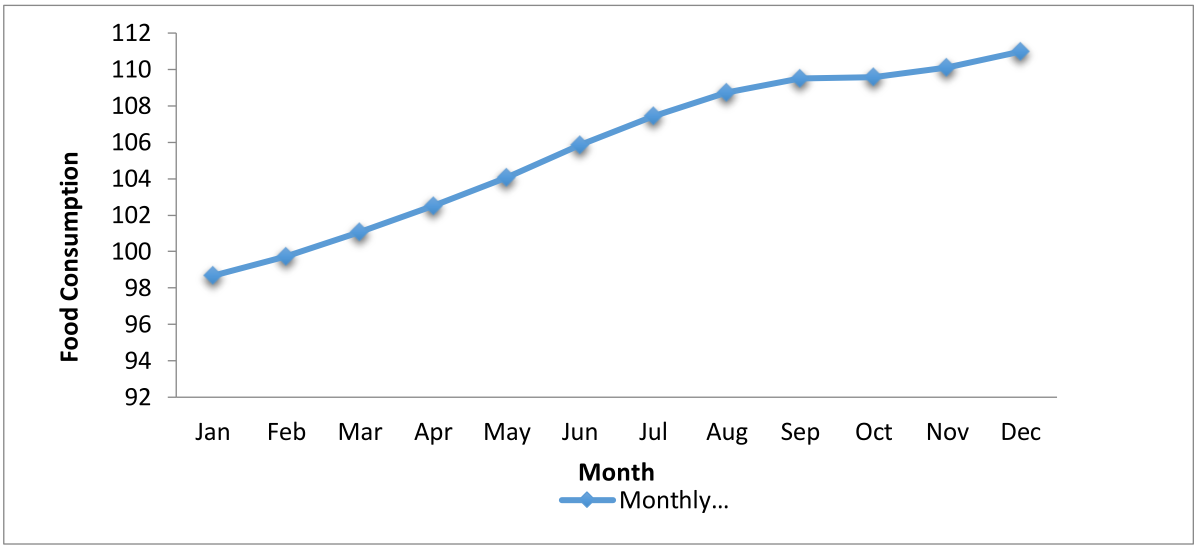


Figure 1: Monthly Means Plot of Food Consumption

Figure 1 shows the monthly mean behaviour of the food consumption index, where the peak is in December and least food consumption rate is in January. However, the monthly mean series show an increase from the beginning (or swing upward); January to December.

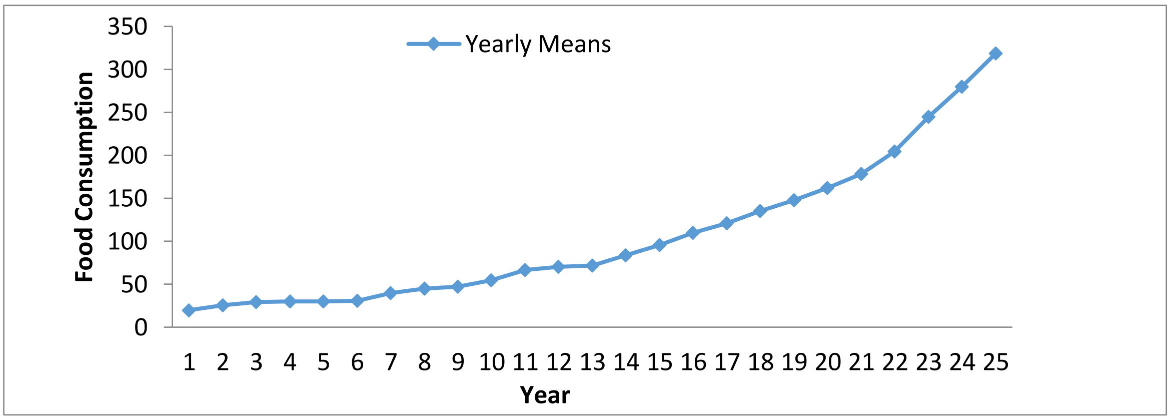


Figure 2: Yearly Means Plot of Food Consumption

Figure 2 shows an upward trend and then upward movements in a random manner. There seem to be evidence of peak in the year 2021 represented by 25 and increases almost all through the periods.

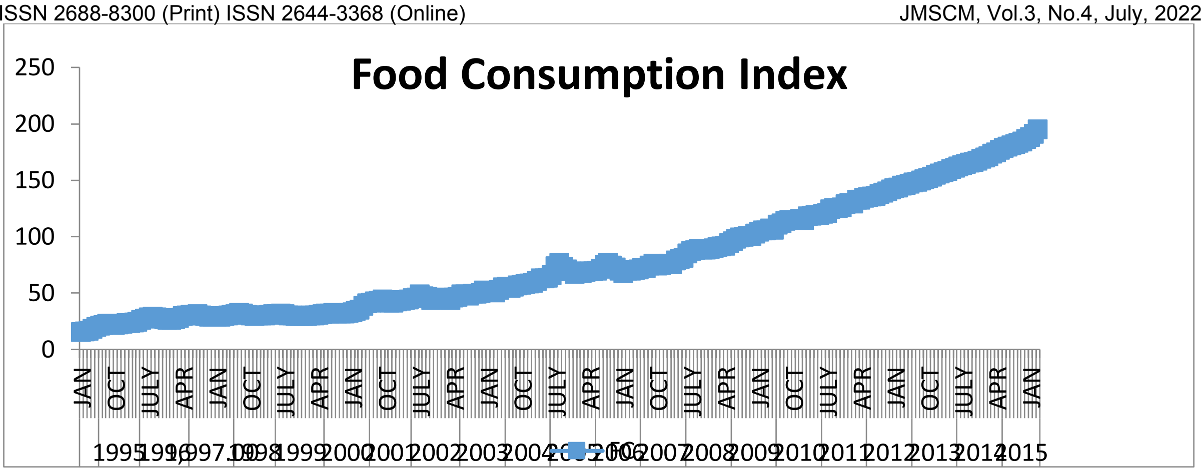


Figure 3: Series Plot of total Food Consumption

Figure 3 shows an upward trend from the year 1995 to 2021. There seem to be an evidence of peak in the year 2021 and show continuous swing upward movements almost all through the latter periods.

4.2 Monthly Plot, Yearly Plot and Series Plot of Food & Non-Alcoholic Beverage Consumption

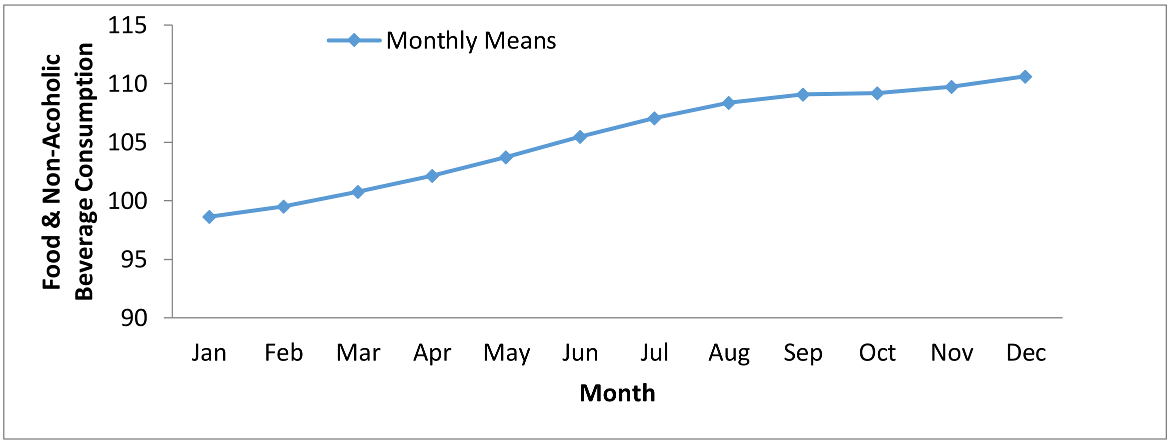


Figure 4: Monthly Means Plot of Food & Non-Alcoholic Beverage Consumption

Figure 4 shows the monthly mean behavior of the food & non-alcoholic beverage consumption, where the peak is in December and least food & non-alcoholic beverage consumption rate is in January. However, the monthly mean series shows an increase from the beginning (or swing upward) of January and a decrease in October. Then, show an upward movement in a random manner from November to December.

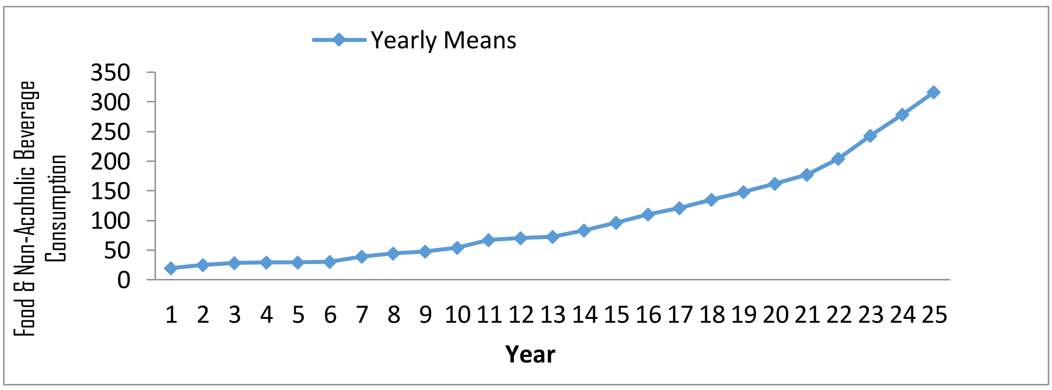


Figure 5: Yearly Means Plot of Food & Non-Alcoholic Beverage Consumption

Figure 5 is similar to Figure 2, which shows an upward trend and then upward movements in a random manner at the last years. There seem to be an evidence of peak in the year 2007 and depressions almost all through the latter periods.

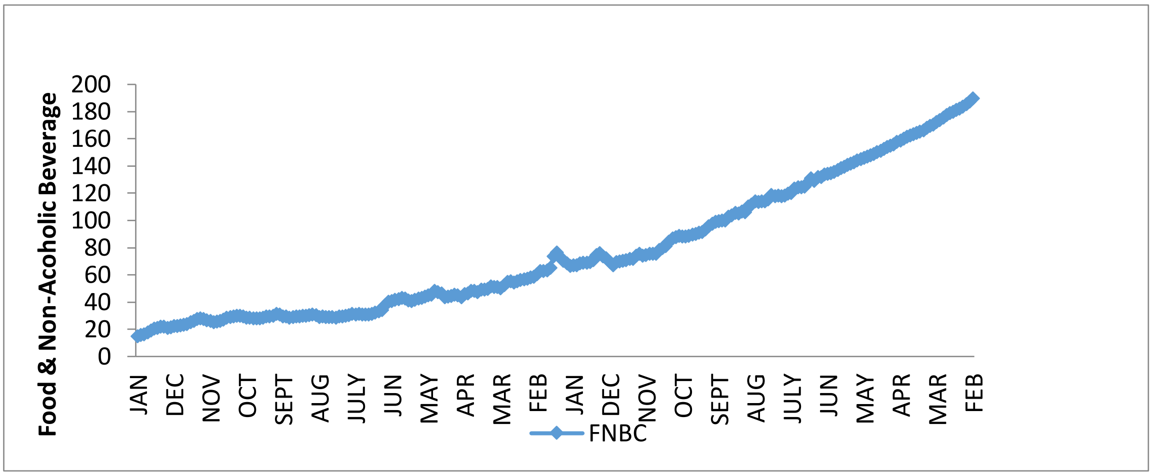


Figure 6: Series Plot of Food & Non-Alcoholic Beverage Consumption

Also Figure 6 is similar to Figure 3, which shows an upward trend from the year 1995 to 2009, then show upward movements in a random manner from 2009 to 2021. There seem to be an evidence of peak in the year 2021 and show continuous swing upward and downward movements almost all through the latter periods. In addition, continuous swing upward and downward movements in a random manner in the early years seem to indicate that these series have seasonal variation. Next, we compare the two plots in Figure 7;

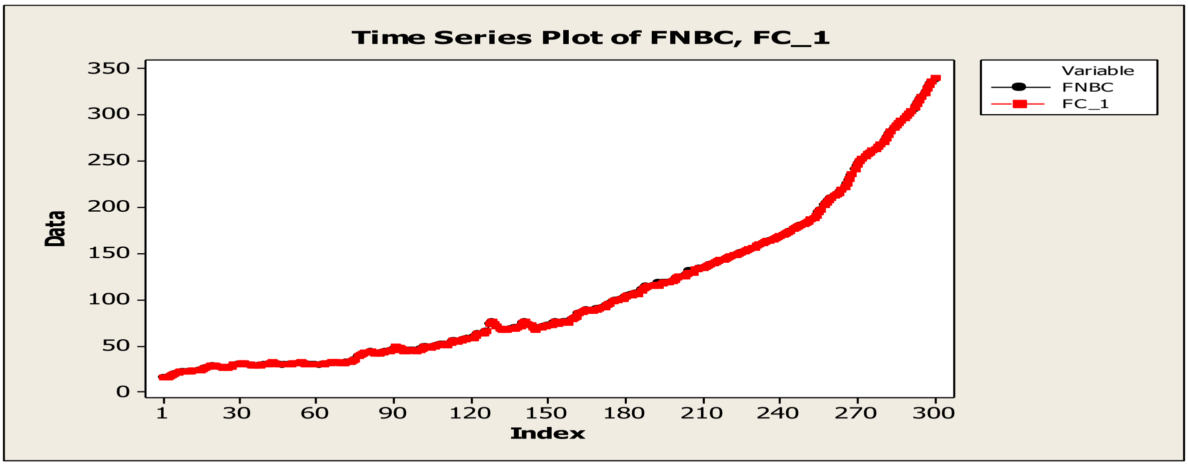


Figure 7: The Food Consumption and Food & Non-Alcoholic Beverage comparison

In Figure 7, comparing the two series, it is noticed that there is a similar behaviour between the two series which show an upward trend component. It also indicated that the two series are not stationary.

4.3: Stationarity of the Data Sets

Next, since these series are not stationary, then first difference was applied to obtained stationarity below (Figure 8 to 9).

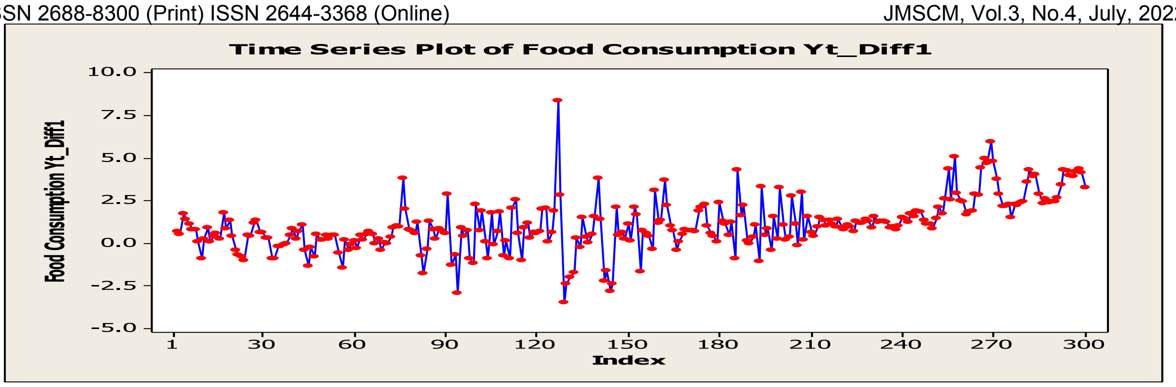


Figure 8: Food Consumption first difference series

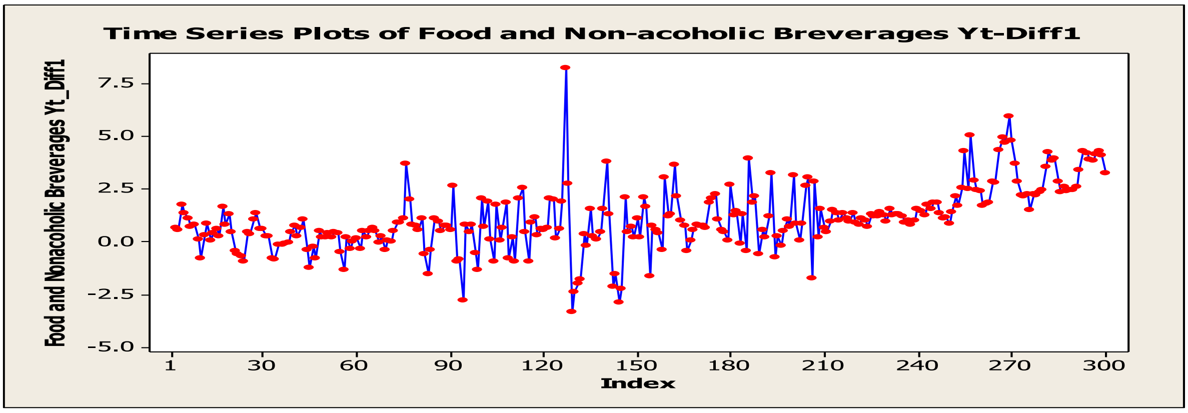


Figure 9: Food & Non-Alcoholic Beverage first difference series

Figure 8 and 9 are the first difference series which shows the series are sine wave pattern in nature with mean greater than zero and non-constants variance. However, both the series are not stationary after first difference (or do not behave better).

Next, since these series are not yet stationary, then, difference of the first difference was applied to obtained stationarity below (Figure 10 to 11).

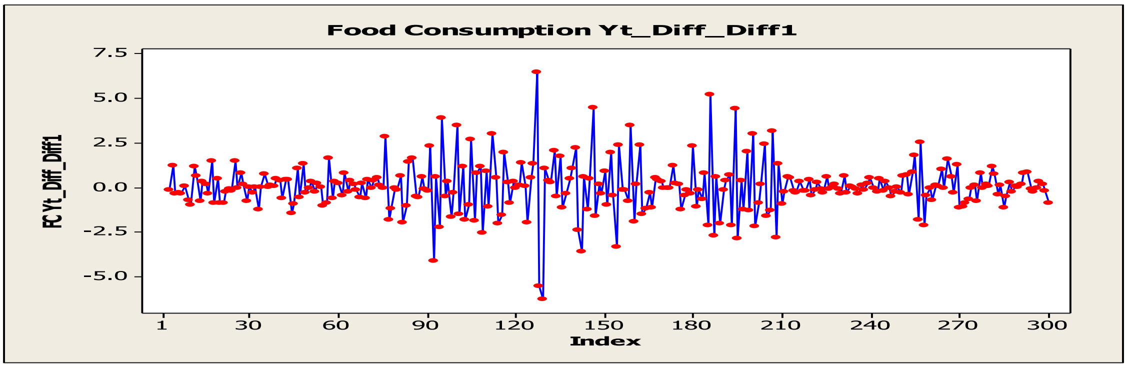


Figure 10: Food Consumption difference of the first difference series

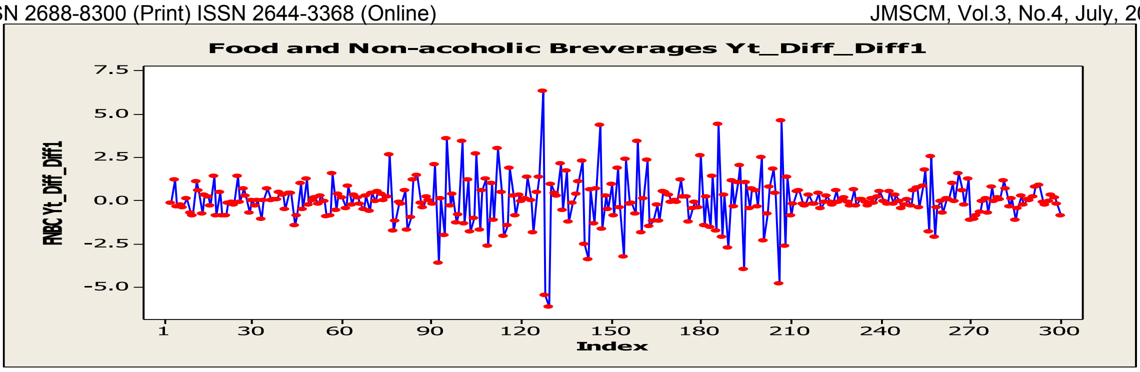


Figure 11: Food & Non-Alcoholic Beverage difference of the first difference series

Figure 10 and 11 are the difference series which shows the series are sine wave pattern in nature with mean zero and constant variance. However, both the series are now stationary after first difference (or behaves much better).

4.4 Discussion of Results

Parameters Estimates and ARIMA Model Identification

The ACF and PACF Plots for the actual series and the difference series for both food consumption and food & nonalcoholic beverages series were obtained in Figure 12, 13, 14 and 15 for food consumption and Figure 16, 17, 18 and 19 for food & nonalcoholic beverages.

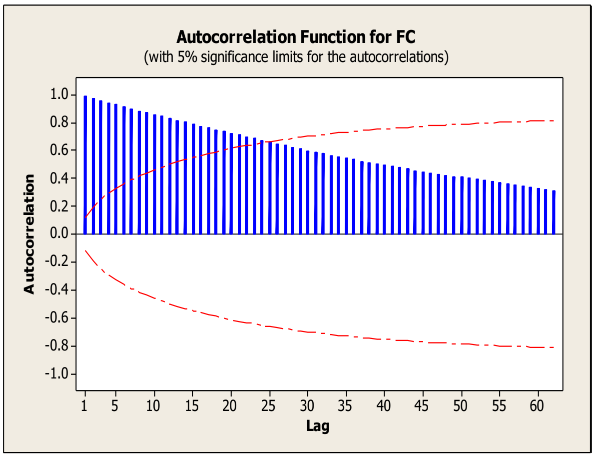


Figure 12: ACF for FC

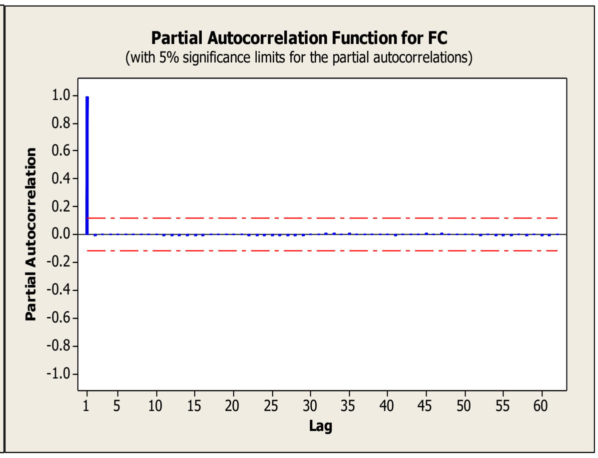


Figure 13: PACF for FC

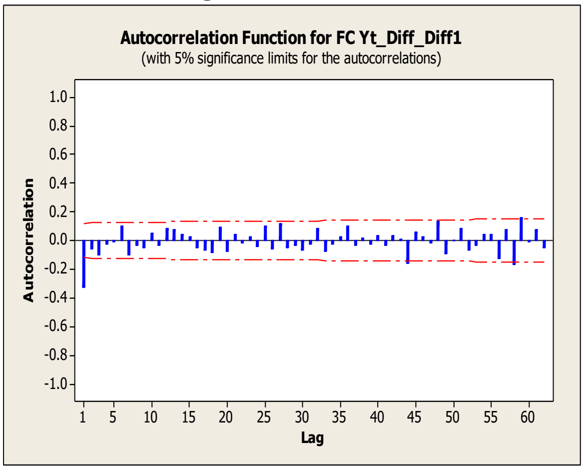


Figure 14: ACF for FC difference of the first difference Series

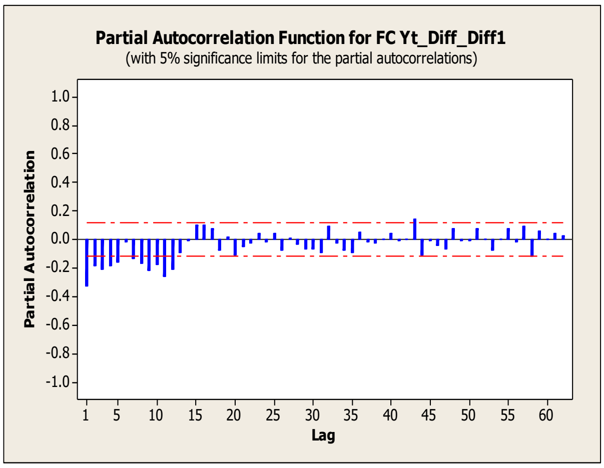


Figure 15: PACF for FC difference of the first difference Series

The ACF plot in Figure 12 spikes dies down extremely slowly indicating AR(p) process (where p=1 or p=2), which is indicated in the difference of the first difference series in Figure 4.14. In addition, PACF plots spikes are close to white noise except lags 1 or 2 cut on, which is also an indication of AR(P); where p=1 or p=2) and MA(q), where q=1.

However, Various ARMA(p,q) models were fitted to the food consumption series with respective residuals as white noise and is summarized in Table 1. The model selection criteria used to select the best model amongst models is AIC and BIC is also detailed out in Table 1.

Table 1:

ARIMA Models	AR(p) Estimates ϕ_1	ϕ_2	ϕ_3	MA(q) Estimates θ_1	θ_2	θ_3	Modified Box-Pierce (Ljung-Box) Chi-Square statistic k=12	k=24	k=36	k=48
ARIMA(1,1,0)	-0.6002 (0.000***)						65.0 (10)	74.4 (22)	93.7 (34)	114.2 (46)
ARIMA(2,1,0)	-0.6292 (0.000***)	-0.3823 (0.086*)					66.8 (9)	81.9 (21)	105.0 (33)	118.8 (45)
ARIMA(3,1,0)	-0.9537 (0.000***)	-0.6528 (0.000***)	-0.3263 (0.000***)				58.9 (8)	78.6 (20)	97.1 (32)	119.8 (44)
ARIMA(0,1,1)				**1.0000 (0.000***)**			**44.0 (10)**	**57.1 (22)**	**77.1 (34)**	**95.5 (46)**
ARIMA(2,1,1)	-1.5849 (0.000***)	-0.6026 (0.000***))		-0.9809 (0.000***)			65.7 (8)	74.4 (20)	93.0 (32)	112.8 (44)

ARIMA Models	**RSS (σ^2)**	**AIC**	**Rank**	**BIC**	**Rank**	**Average Rank**
ARIMA(1,1,0)	968.940	353.37	4	357.07	4	4
ARIMA(2,1,0)	827.777	308.45	3	315.85	3	3
ARIMA(3,1,0)	740.036	277.06	2	288.15	2	2
ARIMA(0,1,1)	**576.684**	**198.74**	**1**	**202.44**	**1**	**1**
ARIMA(2,1,1)	961.829	355.18	5	366.27	5	5

FOOTNOTE: ***-sig. at 1%, **-sig. at 5%,*-sig. at 10%.

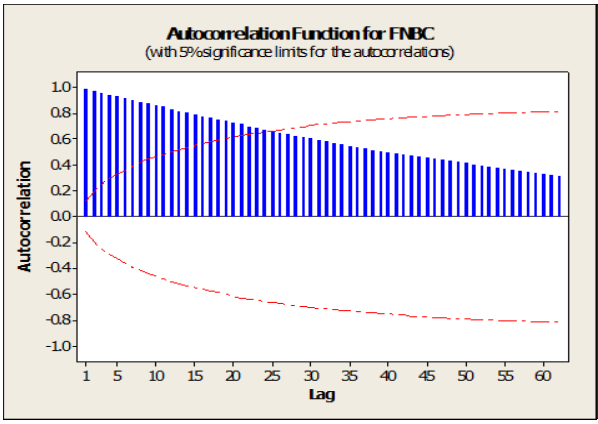


Figure 16: ACF for FNBC

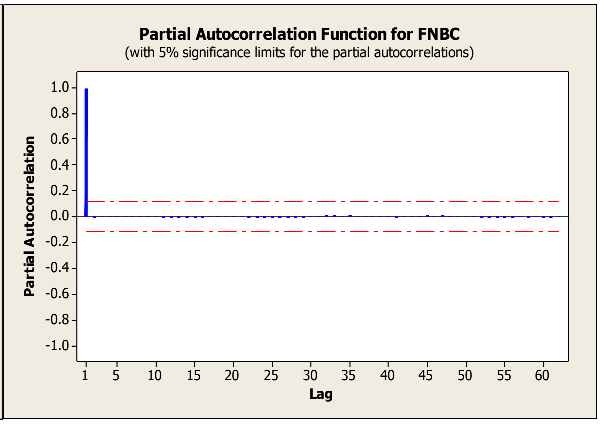


Figure 17: PACF for FNBC

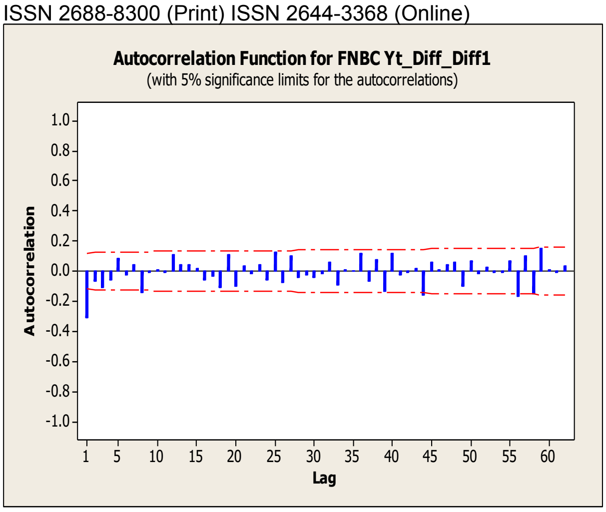


Figure 18: ACF for FNBC difference of the first difference Series

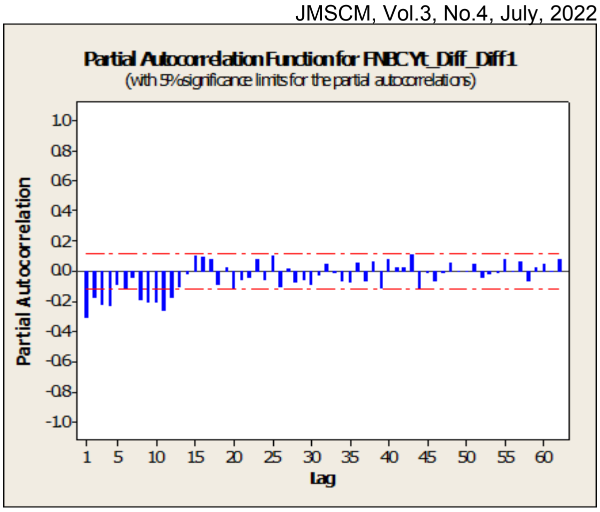


Figure 19: PACF for FNBC difference of the first difference Series

The ACF plot in Figure 16 spikes dies down extremely slowly indicating AR(p) process (where p=1 or p=2), which is also indicated in the difference of the first difference series in Figure 18. In addition, PACF plots spikes are close to white noise except lags 1 or 2 cut on, which is also an indicating of AR(P); where p=1 or p=2) and MA(q), where q=1. Also the early lags indicated that these series have seasonal variation of order 12. However, Various SARIMA(P, D, Q) models were fitted to the Food & Nonalcoholic beverage consumption series with respective residuals as white noise and it is summarized in table 4.2. The model selection criteria used to select the best model amongst models is AIC and BIC is also detailed out in Table 2.

Table 2:

SARIMA 0f Order 12	AR(p) Estimates ϕ_1	ϕ_2	ϕ_3	MA(q) Estimates θ_1	θ_2	θ_3	Modified Box-Pierce (Ljung-Box) Chi-Square statistic k=12	k=24	k=36	k=48
ARIMA(1,1,0)(1,0,0)12	0.0810 (0.165)					ARIMA(1,1,0)(1,0,0)12	61.9 (9)	17.7 (21)	86.1 (33)	104.2 (45)
ARIMA(0,1,0)(1,0,0)12	0.0713 (0.221)					ARIMA(0,1,0)(1,0,0)12	120.0 (10)	148.4 (22)	187.4 (34)	236.1 (46)
ARIMA(0,1,0)(2,0,0)12		-0.1232 (0.035***)				ARIMA(0,1,0)(2,0,0)12	116.7 (9)	144.7 (21)	171.5 (33)	228.9 (45)
ARIMA(1,1,0)(0,0,1)12	-0.5950 (0.000***)			-0.0915 (0.117)		ARIMA(1,1,0)(0,0,1)12	62.1 (9)	72.0 (21)	86.6 (33)	105.0 (45)
ARIMA(0,1,0)(0,0,1)12				-0.0979 (0.092*)		ARIMA(0,1,0)(0,0,1)12	118.8 (10)	147.0 (22)	187.6 (34)	237.0 (46)
ARIMA(0,1,0)(0,0,2)12				-0.1133 (0.051*)	-0.1270 (0.030***)	ARIMA(0,1,0)(0,0,2)12	114.6 (9)	142.0 (21)	168.6 (33)	227.1 (45)
ARIMA(1,1,0)(0,0,0)12	-0.5947 (0.000***)					ARIMA(1,1,0)(0,0,0)12	62.1 (10)	70.9 (22)	83.6 (34)	101.4 (46)
ARIMA(0,1,1)(0,0,0)12				**1.0000 (0.000***)**		**ARIMA(0,1,1)(0,0,0)12**	**46.3 (10)**	**62.1 (22)**	**82.8 (34)**	**109.6 (46)**

SARIMA 0f Order 12	**RSS (σ^2)**	**AIC**	**Rank**	**BIC**	**Rank**	**Average Rank**
ARIMA(1,1,0)(1,0,0)12	930.206	341.22	2	344.91	2	2
ARIMA(0,1,0)(1,0,0)12	1440.81	471.61	8	475.31	7	7.5
ARIMA(0,1,0)(2,0,0)12	1418.91	469.04	6	476.44	8	7
ARIMA(1,1,0)(0,0,1)12	929.447	342.97	3	350.37	4	3.5

ARIMA(0,1,0)(0,0,1)$_{12}$	1438.17	471.06	7	474.76	5	6
ARIMA(0,1,0)(0,0,2)$_{12}$	1412.09	467.61	5	475.0	6	5.5
ARIMA(1,1,0)(0,0,0)$_{12}$	936.353	343.18	4	346.88	3	3.5
ARIMA(0,1,1)(0,0,0)$_{12}$	**556.385**	**188.06**	**1**	**191.76**	**1**	**1**

FOOTNOTE: ***-sig. at 1%, **-sig. at 5%,*-sig. at 10%.

From table 1, it was reveals that the ARIMA(0,1,1) was the best and suitable model that should be used to forecast the behaviour of Food consumption in Nigeria which was determined by equation below;

$$Y_{1t} = 1.0000 X_{1t-1} + \varepsilon_{1t}$$

From table 2, it is reveals that the ARIMA(0,1,1)(0,0,0)$_{12}$ was the best and suitable model that should be used to forecast the behaviour of Food & Nonalcoholic Beverages consumption in Nigeria which was determined by equation below;

$$Y_{2t} = 1.0000 X_{2t-1} + \varepsilon_{2t}$$

4. 5: Forecast

In this stage, the ARIMA models identified can now be used to generate forecast. We have observed 298 data points, the start of the origin is 1 and the end is 298. Appendix A shows the fitted values and the forecasts for t = 299, 300, to 322. Twenty- four (24) months' forecast values with confidence intervals of the forecasts (i.e. January 2022 to December 2023).

The plots of the forecasts generated for the food consumption and food & nonalcoholic beverages consumption are shown in Appendices G and H respectively. The graph shows that the two variables are dependent and also shows that any gradual increase in the food consumption tends to pave way for increase in the food & nonalcoholic beverages consumption or a drastic drop in the food consumption will also drop the food & nonalcoholic beverages consumption in the same manner.

1. Models Forecasts (The ARIMA Model Identified Forecasts)

To gauge the accuracy of our estimates, the estimated errors were used to compare the two models forecasts in Section 3.3 and 3.5. This is done by subtracting the estimated forecast values (F_i) from the original values or [actual values (A_i)] to obtain the estimate errors. The ARIMA model identified forecasts in Section 4.5 is compared to determine the suitable model between the two models for forecasting food consumption and food & nonalcoholic beverages consumption, using accuracy measures

Accuracy Measures.

Variable		**ARIMA model**
Food Consumption (FC) ARIMA(0,1,1)	MAE	0.0456
	MSE	1.955
	MAPE	1.092%
Food & Nonalcoholic Beverages Consumption (FNBC) $ARIMA(0,1,1)(0,0,0)_{12}$	MAE	0.7805
	MSE	1.886
	MAPE	1.020%

The univariate is better to predict the two series, using accuracy measure of (i.e. MAPE, MAE and MSE).

5. Conclusion

It is reasonable to conclude that there is significant relationship between the food consumption and food & nonalcoholic beverage consumption series. In addition, univariate model such as (**ARIMA(0,1,1)** and **$ARIMA(0,1,1)(0,0,0)_{12}$**) seems better to predict the two series, using accuracy measure of univariate (i.e. MAPE, MAE and MSE).

6. Recommendation

It is recommended that more detailed work should be carried out in the area of co-integration analysis of the two variables to enhance a better understanding and prediction distribution in Nigeria. This will help in creating a strong and adequate model that can improve the consumption of food and food & nonalcoholic beverages in Nigeria in the nearest future.

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APPENDIX A

Forecasts from period 298

		95% Limits		
Period	Forecast	Lower	Upper	Actual
299	-0.19993	-2.89221	2.49235	
300	-0.20086	-2.89314	2.49142	
301	-0.20179	-2.89407	2.49049	
302	-0.20272	-2.89500	2.48956	
303	-0.20365	-2.89593	2.48863	
304	-0.20458	-2.89686	2.48770	
305	-0.20551	-2.89779	2.48677	
306	-0.20644	-2.89872	2.48584	
307	-0.20737	-2.89965	2.48491	
308	-0.20830	-2.90058	2.48398	

309	-0.20922	-2.90150	2.48305
310	-0.21015	-2.90243	2.48213
311	-0.21108	-2.90336	2.48120
312	-0.21201	-2.90429	2.48027
313	-0.21294	-2.90522	2.47934
314	-0.21387	-2.90615	2.47841
315	-0.21480	-2.90708	2.47748
316	-0.21573	-2.90801	2.47655
317	-0.21666	-2.90894	2.47562
318	-0.21759	-2.90987	2.47469
319	-0.21852	-2.91080	2.47376
320	-0.21945	-2.91173	2.47283
321	-0.22037	-2.91265	2.47191
322	-0.22130	-2.91358	2.47098