ON THE MULTIVARIATE ANALYSIS OF STUDENTS' ACADEMIC PERFORMANCE IN WASSCE IN PUBLIC SENIOR SECONDARY SCHOOLS IN RIVERS STATE (2018-2020)

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Abstract

The paper examined students' performance in six subjects from WAEC examination from 2018 to 2020 using multivariate analysis through Hotelling T² distribution and paired t-test statistics. Four objectives where formulated and used for this study. Based on the factors in the objectives, relevant related literatures were reviewed. A secondary data extracted from the WAEC results from the public senior secondary schools under study were used for this study and the analyses of the data were done using Hotelling T² distribution, Quadratic form, and paired t-test statistics. All computations were done via Microsoft Excel 2010, SPSS (version 23.0) and MINITAB (version 16.0). The Hotelling T^2 statistics results between the students' academic performance for (2018 & 2019), (2019 & 2020) and (2018 & 2020) were all significant. Paired t-test statistics results showed a decrease in the Students' average performance for four subjects (Mathematics, English Language, Marketing and Biology), while an increase in the Students' average performance for Economics and Civic Education subjects. It was discovered that students' average performances in Economics and Civic Education subjects were better than other subjects. This research recommend the effective implementation of the Nigeria education policies that emphasizes on teachers qualification, years of teaching experience and the UNESCO policy on Teacher-Students ratio (this policy stipulates that the maximum number of students that should be in a secondary class is 25), since there is significant difference between students' average performance for four subjects.

Keywords: Multivariate Analysis; Academic Performance; WASSCE; Hotelling T^2 Distribution; Paired t-test Statistics

1. Introduction

Education, as a tool for change, lies at the heart of every country's desire for developing human capital for effective societal functioning. In Nigeria, education is a priceless tool for advancing the country's political, social, economic, scientific, and technological growth. Secondary education, which is the foundation of the entire educational system in Nigeria, is rapidly losing importance, owing, among other things, to students' unsatisfactory and bad performance in public examinations. In Nigeria, education is a "par excellence" tool for achieving national development. It has seen active participation from non-governmental organizations, communities, and individuals as well as government interventions. As a result, it is desirable for the country to state clearly and unequivocally the philosophy and goals that underpin its investment in education.

Several issues have recently been recognized by researchers and stakeholders in the education industry as the causes of students' low performance in public examinations. Poor school location, constant changes in government policies, school closures based on teachers' strike action, home-school distance, high student-teacher ratio, lack of supervision, monitoring, and evaluation machinery, lack of good textbooks, poor content and context of instruction are some of the factors identified, poor and nonconductive environment among others (Adeboyej et al., 2003; Adepoju, 2003).

In order to ensure that their children score better in the SSCE and, as a result, secure admission to universities of their choosing, some parents and guardians have made a specific choice of secondary school for their children, disregarding the school's location or financial implications. However, the distribution of secondary schools in both urban and rural areas (the urban-rural dichotomy) has a significant impact on students' private costs and academic achievement. For instance, secondary schools should be planned such that students living in all parts of a state can have cheap means of transport and easy access to them. In order to lower private costs, school size must be proportional to the prospective population of pupils within various towns or zones. The establishment of adjacent schools will surely aid in increasing enrolment rates and therefore bridging educational inequities within the state.

The importance of English Language and Mathematics as prerequisite subjects for admission to higher education in Nigeria and some West African countries such as Ghana, The Gambia, Sierra Leone, and Liberia (These countries share a colonial history and jointly established the WAEC) has made the two subjects compulsory or mandatory to pass at credit level by secondary schools students in public examinations. In the Nigerian setting, a credit level in either of the subjects has been utilized as one of the criteria for measuring and establishing a candidate's brilliancy. Of course, low performance in SSCE English Language and Mathematics by secondary school students has made it difficult for the majority of pupils to gain entrance to higher education institutions in recent years. According to Adepoju (2002), approximately 93 percent of secondary school leavers fail to qualify for university education in any given year. He also stated that 7.7% of students received credit in English Language in 1988, 9.0% in 1989, and 6.3 percent in 1990. The fall in students' academic achievement was more pronounced in Mathematics.

Academic achievement of pupils, according to Nwokocha and Amadike (2005), is the criterion for assessing a nation's educational excellence. As a result, it is necessary to maintain a high level of performance in internal and, for the most part, external examinations. Student's performance has been a subject of discussions and debate among scholars; because it is the most vital educational policy and indicator that stakeholders are interested in (Alaka, 2011). Xinyi (2006) informed that student's performance has been a subject of national interest and comparative studies among countries since the beginning of educational theory.

While stressing the importance of academic performance in the educational system, Aremu et al. (2001) believed that academic performance is a fundamental criterion by which all teaching-learning activities are measured, using some standards of excellence and the acquisition of specific grades in examinations to measure candidate's ability, mastery of the content, and skills in the classroom. Scholars agree, according to Arief (2019), that a student's academic attainment or performance is a 'net outcome' of their cognitive and non-cognitive traits, as well as the sociocultural framework in which the learning process occurs. Students' academic success is an important aspect of schooling (Anthony, 2018). It is regarded as the hub around which the entire educational system revolves. According to Abaidoo (2018), the success or failure of any academic institution is determined by the academic achievement of students. Similarly, some experts believe that a student's academic success serves as the foundation for acquiring knowledge and developing future talents. Additionally, some emphasized that the topmost priority of all educators is the academic performance of students.

Abdullah (2016) defined academic performance as the knowledge gained which is assessed by marks by a teacher and/or educational goals set by students and teachers to be achieved over a specific period of time. He went on to say that these objectives are assessed by ongoing evaluation or examination results. WASSCE is a standardized test that is administered in West African countries. Students who pass the exam receive a diploma indicating that they have completed secondary school. The West African Examination Council (WAEC) administers it, and it is only available to candidates who live in Anglophone West African countries.

Many Nigerian state governments have tried several times to make it a policy in public secondary schools not to have more than 30 students per class to improving the performance of students in public examinations. The major goal of this research was to determine the trend in students' SSCE performance in English Language and Mathematics in typical urban and rural secondary schools in Oyo State, Nigeria. It also sought to find out the percentage of those students who obtained grades from A1 – E8 as well as A1 – C6 with a view to providing useful data on the strengths and weaknesses of students' performance in the two subject areas in typical urban and rural schools for educational planners, educational policy makers, and curriculum planners

Multivariate statistical analysis is the study and solution of multi-index theories and methodologies using mathematical statistics methods. The past 20 years, with the computer application technology and the urgent need for research and production, multivariate

statistical analysis techniques are widely used in geology, meteorology, hydrology, medicine, industry, agriculture, economics, and a variety of other fields have evolved to effectively solve practical problems. Simplified system architecture to explore the system kernel, can use principal component analysis, factor analysis, correspondence analysis and other methods, a number of factors in each variable to find the best subset of information from a subset of the description found in multivariable system results, as well as the impact of various factors on the system.

Controlling for the model's prediction has two categories in multivariate analysis. The prediction model, which frequently uses multiple linear regression, stepwise regression analysis, discriminant analysis, or stepwise regression analysis in double screening modeling, is one example. The other is a descriptive model, which is a type of cluster analysis modeling technique that is widely utilized. Many prior studies have shown that a multivariate analysis system requires a similar nature of things or events grouped together in order to find the relationships between them and the underlying regularity are mostly qualitative treatment by a single factor, so the results do not ref the general characteristics of the system. For instance, a numerical classification model built using cluster analysis and discriminant analysis techniques.

Multivariate analysis (MVA) is based on multivariate statistics principles. MVA is typically utilized in cases where several measurements are taken on each experimental unit and the relationships between these measurements and their structures are critical. MVA is classified in a modern, overlapping manner as follows:

- 1. Multivariate normal and universal models, as well as distribution theory
- 2. The study and measurement of relationships
- 3. Probability computations of multidimensional regions
- 4. Data structures and patterns are investigated.

The desire to add physics-based analysis to compute the impacts of variables for a hierarchical "system-of-systems" can complicate multivariate analysis. Studies that want to apply multivariate analysis are frequently stymied by the problem's dimensionality. Surrogate models, which are very precise representations of the physics-based code, are frequently used to alleviate these difficulties. Surrogate models can be evaluated fast since they take the form of an equation. This becomes a key enabler for large-scale MVA studies: a Monte Carlo simulation spanning the design space, which is challenging with physics-based codes, becomes straightforward with this method when evaluating surrogate models, which often take the form of response-surface equations. In consumer and market research, quality control and quality assurance, process optimization and process control, and research and development, multivariate approaches are used to investigate datasets. Because social scientists are unable to conduct randomized laboratory experiments like those used in medical and natural sciences, these procedures are especially crucial in social science research. Multivariate approaches can be used to statistically estimate relationships between several

variables, as well as correlate how essential each one is to the final outcome and where dependencies exist.

The T-squared distribution of Hotelling is significant because it arises as a distribution of a series of statistics that are natural generalizations of the statistics underlying the T distribution of students. In particular, the distribution arises in multivariate statistics in undertaking tests of the differences between the (multivariate) means of different populations, where tests for univariate problem would make use of a t-test. It is proportional to the F distribution. Harold Hotelling created the distribution as a generalization of the student t-distribution.

If the notation T² p.m. is used to denote a random variable having Hotelling's T-squared distribution with parameters p and m then, if a random variable x has Hotelling's T-squared distribution $X \sim T^2$ p.m.

Then,

$$\frac{m-p+1}{pm}X \sim F_{p,m-p+1}$$
 (1.1)

where $F_{p,m-p+1}$ is the F= distribution with parameters PandM - P + 1

where Fp.m - P + 1 is the F = distribution with parameters P and M - P + 1

Hotelling's T-squared statistics is a generalization of student's t statistics that is use in multivariate hypothesis testing and is defined as follows. Let Np (N, Σ) denote a p- variate normal distribution with location μ and covariance Σ Let $X_1, \ldots, X_n \sim \text{Np}$ (N, Σ) be an independent random variables, It can be represented as a P x 1 real number column vector. It can be shown that $n(\bar{X} - N)^T \Sigma^{-1} (\bar{X} - N) \sim \varphi^2$ Where $\varphi^2 p$ is the chi-squared distribution with p degrees of freedom.

Multivariate techniques are complex and involve high level mathematics that requires a statistical program to analyze the data. These statistical programs are generally expensive. The results of multivariate analysis are not always easy to interpret and tend to be based on assumptions that may be difficult to assess. Multivariate approaches require a large sample of data to get meaningful conclusions; otherwise, the results are useless due to excessive standard errors. Standard errors define how confident you can be in the results, and the results from a large sample are more reliable than those from a small sample. Running statistical programs is very simple, but deciphering the results requires the assistance of a statistician.

The aim of the research is to investigate students' performance in six subjects' areas in WAEC examination results from 2018 to 2020 using multivariate analysis through Hotelling T^2 distribution and paired t-test statistics. Specifically, this paper seeks to achieve the following objectives:

i. To estimate the mean vector, covariance matrix and correlation matrix for each of the years of the six subjects over the years (2018-2020).

ii. Estimate a quadratic form for each of the years using their covariance matrix, which will be used to show the homogenous function that consists of all possible second order terms.

iii. To test for significant difference between the students' academic performance for (2018 & 2019), (2019 & 2020) and (2018 & 2020) using Hotelling T^2 statistics.

iv.Using Paired t-test statistics to determine the difference between subjects (Students' performance in Mathematics, English Language, Marketing, Economics, Civic Education and Biology in WASSCE for; 2018, 2019 and 2020).

1.2 Statement of the Problem

Educators, parents, and the government have been concerned about the poor performance of students who sat for the West African Examination Council (WAEC). The flaws that are contributing to the downward trend in teaching and learning are numerous. Students find it increasingly difficult to pay attention to the teaching of subjects such as Mathematics in their schools, claiming that the topic is too difficult to master. The number of students who offer to learn subjects in the real sense and make career of it is diminishing progressively, as they have begun to lose interest in it.

Poor management of public schools on students' performance in Senior Secondary Schools in West African Examination council (WASSCE) is becoming worrisome to educational development in Nigeria and particularly in Rivers State. According to observations and complaints from examination boards, a substantial percentage of public secondary school students continue to do poorly in Senior School Certificate Examinations due to inefficient resource management. Over the years, the majority of students that sat for the May/June West African Examinations Council (WASSCE) have been recording mass failure, not only in the area of overall performance of the students but also in the core subjects like English, Mathematics, and other compulsory subjects like Civic Education, Marketing, Economics and Biology.

Armed robbery, rapping, cultism, abduction, and other social vices would rise in a state or country where a greater number of teenagers drop out of school. Some graduates of today's secondary education system are unable to function in society or progress to further education without the assistance of their parents or forgery. They are incapable of thinking for themselves or of respecting others' opinions and feelings. Except for items that would make them money quickly, they do not regard the dignity of labour. Every year, the number of pupils and teachers in the classroom decreases. It is based on these factors that this study is designed to address the situation of poor performance of students in WAEC in some selected secondary schools (Public Schools) in Rivers State; Akuku Toru LGA as a case study. The study also looked out for the factors responsible for the poor performance and the way out. These, therefore, have been a source of concern to the researcher in taking a decision to examine students' academic performance in WASSCE over the recent years (from 2018 to 2020).

1.3 Significance of the Research

The findings of this study will aid in the efficient implementation of Nigerian education policies that place a premium on teacher qualifications, years of experience, and the UNESCO teacher-student ratio policy (this policy stipulates that the maximum number of students that should be in a secondary class is 25). It will also encourage curriculum revision and improvement to ensure that the subject's content and scope are adequately covered.

It will also assist students to recognize that the bulk of key subjects learning are vested on their innate urge and willingness to perform and do well. The essential premise that learning or teaching is child-centered underpins all of their other activities and businesses.

Given the importance of secondary education in the Nigerian educational system and the rise in WASSCE failure in public senior secondary schools, it is necessary to discuss some of the issues basic truth to staff of public Secondary Schools in Rivers State which will in turn enlighten them to know the level of performance of students within the Zones, whether they are performing very well or below expectation.

It would also be of significance to the respective school authorities in the state to take corrective measures within their authority through adequate planning of resources to meet the demands and guide their action and future of public schools" staffs and students for successful teaching and learning process in schools. It would also be of significance to parents to supervise their child's/children's work at home in order to improve their performances. Finally, the study would also be of significance to future researchers who may be interested in carrying out further research in this current area.

1.4 Scope of the Research

This study examines the analysis of students' academic performance in West African Secondary Schools Certificate Examination (WASSCE) in Public Senior Secondary Schools which implied that this work is limited to a Public Senior Secondary Schools in Akuku Toru LGA of Rivers State, Nigeria (CCS Abonima) between the years 2018-2020. The scope of the study is restricted to one examination center among the public schools under the Education Zone that have presented students for WASSCE examination for at least three years. Due to the large number of Public Senior Secondary Schools in this Zone, the researchers will not cover all public schools in the educational Zone of the State.

2. Literature Review

According to Nwaozuzu (2012), poor teacher quality is responsible for pupils' poor performance in WAEC exams. She emphasized that previous research had revealed a concerning rate of pupil failure in the English language. She pointed out that some English Language teachers in secondary schools, particularly in private schools, are primarily secondary school graduates with little prior teaching experience. However, things have changed as many private and public schools have qualified teachers as per the certificate they possess but these teachers are still not able to deliver as expected. Their poor teaching inability to handle the teaching and learning of English is still very significant.

According to Adebayo (2008), most pupils who attend public schools have low standardized test results. He went on to say that children at private schools perform better in English than students in public schools because certain private schools indulge in examination misconduct.

Abdullahi (2009) pointed out that mathematics like an Octopus has its numerous tentacles in all branches of knowledge. Previous researches conducted show that there have been mass failures in Mathematics. Some studies also show that students' negative attitude towards Mathematics has led to poor performance in the Subject. With regards sex factor as an influence in attitude and performance of students in Mathematics, it was discovered that when males and females performance were compared in an analysis, there existed a sex factor.

Adeniran (2009) investigated the many reasons that contribute to pupils' low performance in mathematics, as well as potential solutions. Students' negative attitudes toward mathematics, as well as their performance in the subject, are examples of such factors. Another issue contributing to students' poor performance in mathematics is certain teachers' inability to effectively teach the subject and convey its abilities. Out of all the subjects in the school curriculum, mathematics has been the hurdle or hindrance to many students' advancement. In publicly administered examinations, mathematics records the most valuable and heartbreakingly outcomes. All stakeholders in the educational system, including the government, educators, proprietors, principals, teachers, and guardians, have been concerned, worried, and anxious about kids' dismal poor performance in mathematics year after year.

Umoru (2010) opines that the development of any nation depends on advancement in Science and technology. He stressed that people of the world are living in a changing world where Science and technology have been part of the world's tradition and any country that fails to recognize this fact at the risk of being technologically backward. As a result, the National Policy on Education emphasizes the need of students being well-trained in order to fulfill the demands of the current age of science and technology. Students should have a strong grounding in science subjects in order to reach this goal.

Reginald (2009) evaluated students' performance in WAEC Science Subjects in a few selected schools in the southeast and discovered that more students scored better in Biology and Chemistry than Physics. In his opinion, the low performance of pupils in Physics is primarily due to schools' failure to employ excellent and competent Physics teachers as well as inadequate laboratory equipment for physics practical.

Lamenting on students poor performance in Physics, Chemistry and Biology DanAzumi (2008), reiterated that one of the most repeatedly mentioned problem causing poor performance in these Subjects since the introduction of SSCE is lack of equipment and materials to conduct practical. Lawal (2006) found no significant link between laboratory equipment adequacy and student academic performance in Science (Physics, Chemistry, and Biology) in SSCE in a multivariate research his study on availability and impact of material resources on performance in Physics, Chemistry and Biology in selected secondary schools in Zaria metropolis.

Leonard (2012) who conducted a study of students' performance in WAEC in Art Subjects in a few selected schools in the south cast, and it was discovered that 70% of the students that sat for the WAEC performed tremendously very well while 30% of the population of the students failed. Also in 2012, it was published in a media that a girl had nine (9) Al in WASSCE examination in Art Subjects from St. Louis Secondary School Umuahia. From the analysis of some of the WAEC results on students' performance in Art Subjects it can be deduced that students performed fairly in Art Subjects than Science Subjects.

Nwobia (2007) conducted a study of students' performance in WAEC in Art Subjects from 2007 to 2010 in a few selected secondary schools in Southeast and it was discovered that 75% of students obtained Al - A3 in Art Subjects while the remaining 25% all in the category of pass and fail. Also an analysis of WAEC result by WAEC office as at December 2012 shows that 62.03% students performed poorly in art Subjects especially English language.

Ojo (2009) opines that teacher quality matters. In fact, it is the single most important schoolrelated element impacting pupils' art achievement. Teacher competency in teaching Art Subjects can contribute to positive achievement on students' performance. According to David (2007), the Arts Subject contributes to children's growth, and it is critical that students are well-taught in order to attain success.

Omekara and Kelechi (2012) evaluated Multivariate Analysis of the performance of students using Hotelling T^2 Statistic. The goal of this study is to see if there is any evidence of a substantial difference in academic achievement between two groups of pupils. Its goal is to see how effective the Hotelling's T-square test statistic is at determining such a difference, establishing this distinction will aid in identifying the high-performing group for a more indepth investigation into the causes for the disparities. This will help education researchers who are working to improve student performance. The study used data from Michael Okpara University of Agriculture (MOUA), Umudike's College of Natural and Applied Sciences (CNAS). Two departments (Chemistry and Statistics) were chosen at random and their first year students of 2009/2010 academic were considered of which their results were analyzed. Hotelling's T-square test statistic was used to analyze the performance of 135 and 120 firstyear students from the Departments of Chemistry and Statistics, respectively. The results reveal a huge disparity in the students' performances in the two Departments. The much improved performance of statistics students suggests that this technique could be used to examine comparative performance of students in order to better understand the better performers and uncover variables that contribute to their superior performance.

Atanda (2011) conducted A Survey of Secondary Students Achievement in English Language and Mathematics in Nigeria: Lessons for Secondary School Administrators in Nigeria. Secondary school education prepares students for institutions of higher learning. Notably, the transition to the institutions of higher learning depends on their level of performance in at least five subjects, Mathematics and English language inclusive in most cases. Thus, the study investigated the senior secondary school students' performance in the two core subjects in Senior secondary school certificate examination (SSCE) conducted by the West African Examination Council (WAEC) with the view to draw lessons for effective secondary school administration. The descriptive research design was adopted while secondary data on statistics of student academic performance in SSCE were used. The data were analysed with simple percentages. The study revealed poor academic performance in the two subjects in the six geopolitical zones in Nigeria. Only 21% of the candidates passed both Mathematics and English in South-West, 29% passed in South-South; 21% passed in South- East, 7% passed in North-Central; 7.3% passed in North-East; while 11.7% passed in North-West. The performance was poorer in the three geopolitical zones in the northern part of Nigeria. The study recommended among others moderate average student-teacher ratio, good guidance counselling service, provision of regular feedback to the students, provision of adequate instructional materials and encouragement of participatory method of teaching.

Christian (2015) investigated panel data analysis on students' academic performance in West Africa senior school certificate examination (WASSCE). This work focused on panel data analysis on students' academic performance. This study is significant because it is necessary to understand some of the variables that have contributed to the drop in student performance in West Africa Senior High School Certificate Examination (WASSCE) over the years. The effects of student-teacher ratio and teacher years of experience on academic achievement of chosen secondary school pupils in Lagos, Nigeria were explored in this study. Ten Senior Secondary Schools in Lagos' Ajeromi Ifelodun Local Government Area were chosen using a simple random sampling method. The study was guided by three research questions and hypotheses. They were analyzed using the fixed and random effect models at 0.05 level of significance. The findings revealed that there is a link between student-teacher ratio and academic achievement, as well as the years of experience of the teachers. The results show that the student-teacher ratio has a significant impact on student performance in these selected schools, and that as class sizes grow students' performance declines. Similarly, the results show that teachers' years of experience have a considerable impact on their students' performance. This suggests that as the years of experience increases, the students are may likely perform very well. In addition, when class sizes grew larger, students' performance declined across the board. On this note, it can be deduced that when the class size keep increase, learning process becomes difficult which in turn affects the performance of the students. The findings of comparing the fixed and random effects models indicated that the fixed effects model best fit the data. On the basis of these findings, proposals for the government and educational stakeholders on how to address this dreadful situation were developed.

Adepoju and Oluchukwu (2011) conducted a researcher on a study of secondary school students' academic performance at the senior school certificate examinations and implications for educational planning and policy in Nigeria. Between 2005 and 2007, this study assessed and investigated secondary school students' academic performance in two main subjects (English Language and Mathematics) at the Senior School Certificate Examinations (SSCE) in ten secondary schools in five randomised Local Government Areas of Oyo State, Nigeria. A descriptive survey research design was used in this study. The study's data was gathered using a tool called the Students' Academic Performance in English Language and Mathematics Questionnaire (SAPEMQ). The ten secondary schools that

participated were chosen using a basic random sampling technique and statistical techniques employed were such as Percentages, mean scores, and multiple regression which were used to analyze the data (backward procedure). The study was guided by four research questions and one null hypothesis. The results revealed, among other things, that students in urban and rural schools performed significantly differently on the SSCE, with impressive mean scores obtained in urban schools (Urban = 69.8, 54.4, and 60.2 in 2005, 2006, and 2007 respectively; Rural = 36.4, 24.9, and 23.8 in 2005, 2006, and 2007). The findings were reviewed in terms of their significance for educational planning and policy in Nigeria.

Ali and Bisandu (2018) examined the application of Hotelling's t-squared statistic and twoway ANOVA model. This study was on the application of Hotelling's T-Squared Statistic and Two-way analysis of variance without replication on the comparison between boarding and day school student performance using the selected subjects; Mathematics, English Language, Biology and Economics, from 2014 to 2017. The paper's main goal is to see if there is a substantial difference in average performance between boarding and day school pupils taking a school test operating both systems. The data collected were presented in tabular form. The data were further analyzed using the above mentioned statistical tools. According to Hotelling's T-Squared test, there is a considerable difference in average performance between boarding and day students. The two-way analysis of variance, on the other hand, shows a significant variation in student performance across subjects but provides little evidence to support the conclusion that there is significant difference between the years under study.

From the literature reviewed on analysis of students' performance in WAEC, it is of great importance that schools, both Public and Private should monitor their teachers to make sure that students are well-taught, and teachers should use appropriate instructional resources when instructing them. Also, the public schools should try to discourage students and teachers from engaging in examination malpractice because it has been observed that students generally depend on cheating in examinations and as a result develop nonchalant attitude towards studying and reading their books which usually lead to poor performance of students in WAEC examination.

Related literatures from several scholarship study such as Christian (2015), Adepoju and Oluchukwu (2011), Oluwatoyin (2015), Ali and Bisandu (2018), Atanda (2011), Omekara and Kelechi (2012) were reviewed to support this present paper. From the researcher's observation, none of these researchers conducted their research study on the same population, study area, uses same tools, same statistical models, same period and the same title as this present study. The theory that was used in this present study was based on multivariate analysis specifically; Hotelling's T^2 Distribution propended by Hotelling (1931).

3. Materials and Methods

The researchers used secondary data extracted from the WAEC results from the schools under study for this work. The researchers personally went to the schools to collect the WAEC results from the head of the schools (Principal's office). To determine an adequate Hotelling T^2 distribution to checkmate the students' performance in each of the subject, a yearly statistics data of a periodic range of 3 years 2018 - 2020 was used. The data used for this study can be provided on demand.

The following programmes are used to obtain the parameters which constitute the models; some of which include MINITAB (version 16.0), Microsoft Excel (2010) and SPSS (version 23.0). To facilitate data analysis, the researcher made use of Microsoft Excel (2013), MINITAB (version 16.0) and SPSS (version 23.0). Microsoft Excel 2010 and MINITAB (version 16.0) were used in estimating the parameters for covariance matrix, correlation matrix, Hotelling T^2 statistics and F-value. SPSS (version 23.0) was used to determine the parameters for paired t-test and goodness of fit parameters.

3.1 Method of Data Analysis and Model Specification

3.1.1 Mean Vectors

If $x_1, x_2, ..., x_n \neg Np(\mu, \Sigma)$, with the samples independently drawn from two or more multivariate normal distribution with same mean, where

$$\boldsymbol{X}_{i} = \begin{bmatrix} \boldsymbol{x}_{i1} \\ \boldsymbol{x}_{i2} \\ \vdots \\ \boldsymbol{x}_{ip} \end{bmatrix}$$
(3.1)

where

- x_1 = Students' score in Mathematics
- x_2 = Students' score in English Language
- x_3 = Students' score in Marketing
- x_4 = Students' score in Economics
- x_5 = Students' score in Civic Education and
- x_6 = Students' score in Biology

The sample mean vector \overline{x} can be found either as the average of the *n* observation vectors or by calculating the average of each of the ρ values separately:

$$\overline{X}_{1} = \frac{1}{n_{1}} \sum_{i=1}^{n_{1}} X_{i1} = \begin{bmatrix} \overline{x}_{1} \\ \overline{x}_{2} \\ \vdots \\ \overline{x}_{p} \end{bmatrix}$$
(3.2)

Where, for example,

$$\overline{X}_{2} = \frac{1}{n_{2}} \sum_{i=1}^{n_{2}} X_{i2} = \begin{bmatrix} \overline{x}_{1} \\ \overline{x}_{2} \\ \vdots \\ \overline{x}_{p} \end{bmatrix} \text{ and so on}$$
(3.3)

Again, the mean of x over all possible values in the population is called *population mean vector* or the expected value of x. It is defined as a vector of expected values of each variable,

$$\mathbf{E}(x) = \mathbf{E} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_p \end{bmatrix} = \begin{bmatrix} \mathbf{E}(x_1) \\ \mathbf{E}(x_2) \\ \vdots \\ \mathbf{E}(x_p) \end{bmatrix} = \begin{bmatrix} \mu_1 \\ \mu_2 \\ \vdots \\ \mu_p \end{bmatrix} = \mu$$
(3.4)

Where μ_j is the population mean of the jth variable. Therefore, we say that \bar{x} is an unbiased estimator of μ .

3.1.2 Covariance Matrix

The sample covariance matrix $S = (S_{jk})$ is the matrix of sample variance and covariance of the p variables:

$$S = (S_{jk}) = \begin{bmatrix} s_{11} & s_{12} & \cdots & s_{1p} \\ s_{21} & s_{22} & \cdots & s_{2p} \\ \vdots & \vdots & \vdots & \vdots \\ s_{p1} & s_{p2} & \cdots & s_{pp} \end{bmatrix}$$
(3.5)

To obtain S, we simply calculate the individual elements in S_{jk} .

$$S_{jj} = S_j^2 = \frac{1}{n-1} \sum_{i=1}^n \left(x_{ij} - \bar{x}_j \right)^2$$
(3.6)

$$=\frac{1}{n-1}\left(\sum_{i=1}^{n}x_{ij}^{2}-n\bar{x}_{j}^{2}\right)$$
(3.7)

The sample covariance matrix *S* can also be expressed in terms of the observation vectors:

$$S = \frac{1}{n-1} \sum_{i=1}^{n} (x_i - \bar{x}) (x_i - \bar{x})'$$
(3.8)

$$=\frac{1}{n-1}\left(\sum_{i=1}^{n}x_{i}x_{i}'-n\overline{x}\overline{x}'\right)$$
(3.9)

If x is a random vector taking on any possible value in a multivariate population the population covariance matrix is defined as

$$\Sigma = Cov(x) = \begin{bmatrix} \sigma_{11} & \sigma_{12} & \cdots & \sigma_{1p} \\ \sigma_{21} & \sigma_{22} & \cdots & \sigma_{2p} \\ \vdots & \vdots & \vdots & \vdots \\ \sigma_{p1} & \sigma_{p2} & \cdots & \sigma_{pp} \end{bmatrix}$$
(3.10)

The diagonal elements $\sigma_{jj} = \sigma_j^2$ are the population variance of the *x*'s and the off-diagonal elements σ_{jk} are the population covariances of all possible pairs of *x*'s. The population covariance matrix in (3.10) can also be found as

$$\Sigma = \mathbf{E}\left[\left(x_i - \mu\right)\left(x_i - \mu\right)'\right]$$
(3.11)

Since $E(S_{jk}) = \sigma_{jk}$ for all *j*, *k*, the sample covariance matrix *S* is an unbiased estimator for Σ :

$$\mathcal{E}(S) = \Sigma \tag{3.12}$$

3.1.3 Correlation Matrix

The sample correlation between the j^{th} and k^{th} variables is calculated as follows:

$$r_{x_1x_2} = \frac{S_{x_1x_2}}{S_{x_1}S_{x_2}} = \frac{\sum_{i=1}^n (x_{i1} - \bar{x}_1)(x_{i2} - \bar{x}_2)}{\sqrt{\sum_{i=1}^n (x_{i1} - \bar{x}_1)^2 \sum_{i=1}^n (x_{i2} - \bar{x}_2)^2}}$$
(3.13)

Which can be further defined as

$$r_{jk} = \frac{S_{jk}}{\sqrt{S_{jj}S_{kk}}} = \frac{S_{jk}}{S_{j}S_{k}}$$
(3.14)

The sample correlation matrix is similar to the covariance matrix, but instead of covariances, it has correlations:

$$R = (r_{jk}) = \begin{bmatrix} 1 & r_{12} & \cdots & r_{1p} \\ r_{21} & 1 & \cdots & r_{2p} \\ \vdots & \vdots & \vdots & \vdots \\ r_{p1} & r_{p2} & \cdots & 1 \end{bmatrix}$$
(3.15)

Again, the population correlation of two random variables x_1 and x_2 is

$$r_{x_1x_2} = Corr(x_1, x_2) = \frac{\sigma_{x_1x_2}}{\sigma_{x_1}\sigma_{x_2}} = \frac{E[(x_1 - \mu_{x_1})(x_2 - \mu_{x_2})]}{\sqrt{E(x_1 - \mu_{x_1})^2}\sqrt{E(x_2 - \mu_{x_2})^2}}$$
(3.16)

3.1.4 Quadratic Form (Q.F)

A quadratic form in p variables $X_1, X_2, ..., X_p$ is a homogenous function that consists of all possible second order terms.

$$Q(x) = a_{11}X_1^2 + a_{22}X_2^2 + \dots + a_{pp}X_p^2 + a_{12}X_1X_2 + \dots + a_{p-1}X_{p-1}X_p = \sum_i \sum_j a_{ij}X_iX_j = X^T A X \quad (3.17)$$

Note: A quadratic form is called positive definite if;

$$Q(x) = X^T A X \rangle 0 \forall x \neq 0.$$

It is called positive semi-definite if

$$Q(x) = X^T A X \ge 0$$

where

$$A = (n-1)S_{as} S = \frac{A}{n-1}$$
 (3.18)

3.1.5 Multivariate Test Statistics (Hotelling T² Distribution)

The Hotelling T^2 Distribution is the multivariate extension of the student distribution.

1. One Sample Test

<u>Hypothesis:</u> $H_0: \bar{x} = \mu_0$

VS

$$H_1: \overline{x} \neq \mu_0$$

Test Statistics:

 $T^{2} = n(\bar{x} - \mu_{0})^{T} S^{-1}(\bar{x} - \mu_{0})$ (3.19)

where :

 \overline{x} is the sample mean vector

 μ_0 is the known population mean vector

n is the total sample size

Decision Rule:

Reject
$$H_0: \bar{x} = \mu_0$$
 if $T^2 \ge \left\lfloor \frac{p(n-1)}{n-p} \right\rfloor F_{p,(n-p)}^{\alpha}$, otherwise accept H_0 .

where;

P is the number of variables

n is the sample size and

n-*p* is the degree of freedom.

2. Two Sample Multivariate Test

Hypothesis:
$$H_0: \bar{x}_1 = \bar{x}_2$$

VS

$$H_1: \bar{x}_1 \neq \bar{x}_2$$

Test Statistics:

$$T^{2} = \frac{n_{1}n_{2}}{n_{1} + n_{2} - 2} (\bar{x}_{1} - \bar{x}_{2})^{T} \Sigma^{-1} (\bar{x}_{1} - \bar{x}_{2})$$
(3.20)

where

$$\Sigma = \frac{(n_1 - 1)\Sigma_1 + (n_2 - 1)\Sigma_2}{n_1 + n_2 - 2}$$
(3.21)

Decision Rule:

Reject
$$H_0$$
 if $T^2 \ge \left\lfloor \frac{p(n_1 + n_2 - p - 1)}{n_1 + n_2 - 2} \right\rfloor F_{p,(n_1 + n_2 - 2)}^{\alpha}$, otherwise accept H_0 .

where;

P is the number of variables

 n_1 is the sample size of the first variable

 n_2 is the sample size of the second variable and

 $n_1 + n_2 - 2$ is the degree of freedom.

3.1.6 Paired t-test

1. Hypotheses

The null hypothesis for a paired t-test is: H_0 : $\Box_d = \Box_0$ where:

- \Box \Box_d = the population mean of the differences
- \Box \Box_0 = the hypothesized mean of the differences

You can choose from three different hypotheses:

$H_1: \square_d > \square_0$	One-tailed test
H ₁ : $\Box_d < \Box_0$	One-tailed test

H₁: \square_d \square_0 Two-tailed test

2. Test Statistic

$$t = \frac{X - \mu_0}{S_d / \left(\sqrt{n}\right)} \tag{3.22}$$

where:

 μ_0 = the hypothesized population mean of the differences

 \overline{X} = the average of the differences between paired samples

 S_d = is the sample standard deviation of the paired sample differences

n = the sample size.

3. Confidence Interval

$$\overline{X} - t_{\alpha/2} \left(S_d / \sqrt{n} \right)$$
 to $\overline{X} + t_{\alpha/2} \left(S_d / \sqrt{n} \right)$

where:

$$S_d = \sqrt{\frac{\Sigma(x - \overline{x})^2}{n - 1}}$$
(3.23)

 $\overline{X} = \Sigma X / n$, where $X = x_1 - x_2$ and x_1 and x_2 are paired observations from populations 1 and 2, respectively

 $t_{\alpha/2}$ = the inverse cumulative probability of a t distribution with n-1 degrees of freedom at 1-

 $\alpha/2$; $\alpha = 1$ - confidence level /100

- S_d = the standard deviation of the differences
- n = number of pairs of values

3.2 Percentage and Data samples from WAEC results Collected

The secondary data extracted from the WAEC results from the various schools under study percentage and data sample determined in this work. The data sets are presented in tables below were used to determine the sample used for each years. The Chi-square test for each year's result can be seen in Appendix V.

YEA R	TOTAL NO OF STUDENTS	TOTAL NO. THAT WROTE MARKETING	% OF STUDENTS THAT WROTE MARKETING
2018	115	110	110/115 X 100 = 95.65%
2019	119	86	86/119 X 100% = 72.27%
2020	199	195	195/199 X 100% = 97.99%

YEA R	TOTAL NO OF STUDENTS	TOTAL NO. THAT WROTE ECONOMIC	% OF STUDENTS THAT WROTE ECONOMIC
2018	115	110	110/115 X 100 = 95.65%
2019	119	86	86/119 X 100% = 72.27%
2020	199	195	195/199 X 100% = 97.99%

YEA R	TOTAL NO OF STUDENTS	TOTAL NO. THAT WROTE BIOLOGY	% OF STUDENTS THAT WROTE BIOLOGY
2018	115	110	110/115 X 100 = 95.65%
2019	119	110	110 X 100 = 92.44%
2020	199	195	195/199 X 100% = 97.99%

YEA R	TOTAL NO OF STUDENTS	TOTAL NO. THAT WROTE ENGLISH LANGUAGE	% OF STUDENTS THAT WROTE ENGLISH LANGUAGE
2018	115	110	110/115 X 100 = 95.65%
2019	119	110	110 X 100 = 92.44%
2020	199	195	195/199 X 100% = 97.99%

YEA R	TOTAL NO OF STUDENTS	TOTAL NO. THAT WROTE MATHEMATICS	% OF STUDENTS THAT WROTE MATHEMATICS
2018	115	113	113/115 X 100% = 98.26%
2019	119	86	86/119 X 100% = 72.27%
2020	199	195	195/199 X 100% = 97.99%

YEA R	TOTAL NO OF STUDENTS	TOTALNO.THATWROTECIVICEDUCATION	% OF STUDENTS THAT WROTE CIVIC EDUCATION
2018	115	110	110/115 X 100 = 95.65%
2019	119	86	86/119 X 100% = 72.27%
2020	199	195	195/199 X 100% = 97.99%

Note: The smallest number of students that wrote each subjects was used as the sample sizes, that is $n_1 = 110$, $n_2 = 86$ and $n_3 = 195$.

4. Results

4.1 Presentation of Data

This section deals with the results for description of the variable; mean vector, covariance matrix and correlation matrix for each of the years of the six subjects, quadratic form for each of the years using their covariance matrix, Hotelling T^2 statistics, Paired t-test statistics and discussion of findings. However, the descriptive statistics of the variables and test for significant difference between the students' academic performance for each of the subjects for (2018-2019), (2019-2020) and (2018-2020) were also done.

4.2 Data Analysis

4.2.1 Mean Vectors for 2018, 2019 and 2020

$$\overline{X}_{2018} = \begin{bmatrix} 61.13 \\ 55.082 \\ 63.762 \\ 60.712 \\ 76.10 \\ 53.654 \end{bmatrix}, \quad \overline{X}_{2019} = \begin{bmatrix} 67.535 \\ 54.209 \\ 64.73 \\ 70.52 \\ 73.52 \\ 55.708 \end{bmatrix}, \quad \overline{X}_{2020} = \begin{bmatrix} 65.954 \\ 61.102 \\ 68.194 \\ 56.370 \\ 68.026 \\ 67.780 \end{bmatrix}$$

4.2.2 Covariance Matrix for 2018, 2019 and 2020

	136.63	30.934	21.591	- 0.435	3.374	21.898
	30.934	60.515	11.544	5.9344	- 3.454	11.905
G	21.591	11.544	58.606	- 5.955	- 8.38918	14.929
$S_{2018} =$	- 0.435	5.9344	- 5.955	37.062	- 5.734	0.9272
	3.374	-3.454 -	8.38918	- 5.734	117.04	-1.775
	21.898	11.905	14.929	0.9272	-1.775	48.411
	18.063	3.640	5.450	5.797	-10.760	-1.583
	3.640	29.626	1.0395	6.361	5.191	0.087
с —	5.450	1.0395	131.60	0.241	8.078	1.108
$S_{2019} =$	5.797	6.361	0.241	82.90	16.82	4.166
	-10.760	5.191	8.078	16.82	189.51	-16.81
	-1.583	0.087	1.108	4.166	-16.81	23.741

	66.618	11.087	-4.534	2.7426	30.043	3.2461
	11.087	29.959	-2.471	- 3.9949	14.6725	- 0.359
с _	- 4.534	- 2.471	61.928	1.4997	- 4.986	- 4.741
$S_{2020} =$	2.7426	- 3.9949	1.4997	45.349	7.2702	-1.0428
	30.043	14.6725	- 4.986	7.2702	134.025	5.4347
	3.2461	-0.359	- 4.741	-1.0428	5.4347	20.042

4.2.3 Correlation Matrix for 2018, 2019 and 2020

	1	0.34017	0.24128	-0.00611	0.02667	0.26922
	0.34017	1	0.19386	0.12531	-0.04104	0.21995
P _	0.24128	0.19386	1	-0.12778	-0.10129	0.28029
$\Lambda_{2018} -$	-0.00611	0.12531	-0.12778	1	-0.08705	0.02189
	0.02667	-0.04104	-0.10129	-0.08705	1	-0.02358
	0.26922	0.21995	0.28029	0.02189	-0.02358	1
	1	0.157	0.112	0.15	-0.184	-0.076
	0.157	1	0.017	0.128	0.069	0.003
D	0.112	0.017	1	0.002	0.051	0.02
Λ ₂₀₁₉ -	0.15	0.128	0.002	1	0.134	0.094
	- 0.184	0.069	0.051	0.134	1	-0.25
	- 0.076	5 0.003	0.02	0.094	-0.25	1

$$R_{2020} = \begin{bmatrix} 1 & 0.248 & -0.07 & 0.05 & 0.318 & 0.089 \\ 0.248 & 1 & -0.06 & -0.11 & 0.232 & -0.01 \\ -0.07 & -0.06 & 1 & 0.028 & -0.05 & -0.13 \\ 0.05 & -0.11 & 0.028 & 1 & 0.093 & -0.03 \\ 0.318 & 0.232 & -0.05 & 0.093 & 1 & 0.105 \\ 0.089 & -0.01 & -0.13 & -0.03 & 0.105 & 1 \end{bmatrix}$$

4.2.4 Quadratic Form for 2018, 2019 and 2020 to show the Homogenous Function

1. Quadratic Form for 2018

$$X_{2018} = \begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \\ X_5 \\ X_6 \end{bmatrix}, n=110$$

$$X_{2018} = \begin{bmatrix} X_1, X_2, X_3, X_4, X_5, X_6 \end{bmatrix}$$

$$S_{2018} = \begin{bmatrix} 136.63 & 30.934 & 21.591 & -0.435 & 3.374 & 21.898 \\ 30.934 & 60.515 & 11.544 & 5.9344 & -3.454 & 11.905 \\ 21.591 & 11.544 & 58.606 & -5.955 & -8.38918 & 14.929 \\ -0.435 & 5.9344 & -5.955 & 37.062 & -5.734 & 0.9272 \\ 3.374 & -3.454 & -8.38918 & -5.734 & 117.04 & -1.775 \\ 21.898 & 11.905 & 14.929 & 0.9272 & -1.775 & 48.411 \end{bmatrix}$$

$$Q(x) = a_{11}X_1^2 + a_{22}X_2^2 + \dots + a_{pp}X_p^2 + a_{12}X_1X_2 + \dots + a_{p-1}X_{p-1}X_p = \sum_i \sum_j a_{ij}X_iX_j = X^T A X_i A X_j$$

and p = 6; then,

$$X^{T}AX = 136.6X_{1}^{2} + 60.5X_{2}^{2} + ... + 48.4X_{6}^{2} + 61.9X_{1}X_{2} + 43.2X_{1}X_{3} - 0.87X_{1}X_{4} + ... - 3.55X_{5}X_{6}$$

2. Quadratic Form for 2019

$$X_{2019} = \begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \\ X_5 \\ X_6 \end{bmatrix}, n=86$$

$$X_{2019} = \begin{bmatrix} 18.063 & 3.640 & 5.450 & 5.797 & -10.760 & -1.583 \\ 3.640 & 29.626 & 1.0395 & 6.361 & 5.191 & 0.087 \\ 5.450 & 1.0395 & 131.60 & 0.241 & 8.078 & 1.108 \\ 5.797 & 6.361 & 0.241 & 82.90 & 16.82 & 4.166 \\ -10.760 & 5.191 & 8.078 & 16.82 & 189.51 & -16.81 \\ -1.583 & 0.087 & 1.108 & 4.166 & -16.81 & 23.741 \end{bmatrix}$$

Similarly, the Quadratic Form for 2019

$$X^{T}AX = 18.1X_{1}^{2} + 29.6X_{2}^{2} + ... + 23.7X_{6}^{2} + 7.3X_{1}X_{2} + 10.9X_{1}X_{3} + 11.8X_{1}X_{4} + ... - 33.6X_{5}X_{6}$$

3. Quadratic Form for 2020

$$X_{2020} = \begin{bmatrix} 12927\\ 11976\\ 1934\\ 9898\\ 11891\\ 8337 \end{bmatrix}, n=196$$

$$S_{2020} = \begin{bmatrix} 66.618 & 11.087 & -4.534 & 2.7426 & 30.043 & 3.2461\\ 11.087 & 29.959 & -2.471 & -3.9949 & 14.6725 & -0.359\\ -4.534 & -2.471 & 61.928 & 1.4997 & -4.986 & -4.741\\ 2.7426 & -3.9949 & 1.4997 & 45.349 & 7.2702 & -1.0428\\ 30.043 & 14.6725 & -4.986 & 7.2702 & 134.025 & 5.4347\\ 3.2461 & -0.359 & -4.741 & -1.0428 & 5.4347 & 20.042 \end{bmatrix}$$

Likewise, the Quadratic Form for 2020

$$X^{T}AX = 66.6X_{1}^{2} + 29.96X_{2}^{2} + \dots + 20.04X_{6}^{2} + 22.1X_{1}X_{2} - 9.1X_{1}X_{3} + 5.5X_{1}X_{4} + \dots + 10.86X_{5}X_{6}$$

Note: the quadratic form obtained for each years can be used to determine if $Q(x) = X^T A X > 0$, $\forall x \neq 0$ that is the covariance matrix is positive definite or positive semi-definite or not.

4.2.5 Multivariate Test Statistics (Hotelling T² Distribution)

1. Hotelling (T²) Statistics Summary of the Significant Difference Between the Students' Academic Performance for 2018 and 2019

<u>Hypothesis 1:</u> $H_0: \bar{x}_{2018} = \bar{x}_{2019}$

VS

$$H_1: \bar{x}_{2018} \neq \bar{x}_{2019}$$

Test Statistics:

$$T^{2} = \frac{n_{1}n_{2}}{n_{1} + n_{2} - 2} (\bar{x}_{2018} - \bar{x}_{2019})^{T} \Sigma^{-1} (\bar{x}_{2018} - \bar{x}_{2019})$$

$$n_1 = 110, n_2 = 86,$$

$$\overline{X}_{2018} = \begin{bmatrix} 61.13 \\ 55.082 \\ 63.762 \\ 60.712 \\ 76.10 \\ 53.654 \end{bmatrix}, \quad \overline{X}_{2019} = \begin{bmatrix} 67.535 \\ 54.209 \\ 64.73 \\ 70.52 \\ 73.52 \\ 55.708 \end{bmatrix},$$

$$(\bar{x}_{2018} - \bar{x}_{2019}) = \begin{bmatrix} 01.13 \\ 55.082 \\ 63.762 \\ 60.712 \\ 76.1 \\ 53.654 \end{bmatrix} - \begin{bmatrix} 07.533 \\ 54.209 \\ 64.73 \\ 70.52 \\ 73.52 \\ 55.708 \end{bmatrix} = \begin{bmatrix} -0.403 \\ 0.873 \\ -0.968 \\ -9.808 \\ 2.58 \\ -2.054 \end{bmatrix}$$

 $(\bar{x}_{2018} - \bar{x}_{2019})^T = -6.405$ 0.873 -0.968 -9.808 2.58 -2.054

$$\Sigma = \frac{(n_1 - 1)\Sigma_1 + (n_2 - 1)\Sigma_2}{n_1 + n_2 - 2}$$

	2353.419	1258.296	6388.054	-649.095	-914.421	1627.26	1	463.25	88.3575	11	186 20.	485	686.63	94.1
	- 47.415	646.8496	-649.095	4039.758	-625.006	101.064	8	492.745	540.685	20	.485 70	46.5	1429.7	354.
	367.766	-376.486	-914.42062	-625.006	12757.36	-193.47	5	-914.6	441.235	68	6.63 14	29.71	5853.35	-1428.85
$\Sigma = $	2386.882	1297.645	1627.261	101.0648	-193.475	5276.79	9	-134.55	5 7.395	9	4.18 354.	11 - 142	28.85	2017.985
<u> </u>						194								
	16428.0)3 3	3681.206	2816.669	44	5.33 - 54	46.8	334	2252.327					
	3681.20)6 9	9114.454	1346.654	118	37.535	54.7	'49	1305.04					
	2816.66	59 1	346.654	17574.05	- 62	.8.61 -	227	.791	1721.441					
	445.33	11	187.535	-628.61	11()86.26 8	304.	.69	455.1748					
	- 546.83	34 (54.749 -	227.791804	4.694	28610.7	1 -	1622.	33					
Σ –	2252.32	27 1	305.04	1721.44145	5.1748	-1622.32	25 7	7294.7	84					
2 -				194										
	84.6805	4 1	8.97529	14.51891	2.29	95515	-2	2.8187	11.	609				
	18.9752	9 4	6.98172	6.941513	6.12	21312	0.	.334	6.7	27				
Σ –	14.5189	1 6	.941513	90.58791	-3.2	4026	-]	1.174	8.8	73				
2 -	2.29551	5 6	.121312	-3.24026	57.1	4566	4	.1479	2.3	846				
	- 2.8187	3 0	.333758	-1.17418	4.14	17907	14	47.478	-8.	363				
	11.6099	3 6	.72701	8.873407	2.34	16262	-8	8.3625	37.	601				
	0.013	3558	-0.00483	-0.00	151	1.42E	- 06	5	9.101E-()5	-0.0029	94		
	- 0.00)483	0.024031	-0.00	091	-0.002	231		-0.00023		-0.0025	5		
_ 1	- 0.00)151	-0.00091	0.011	596	0.000	911		-8.392E-	05	-0.0021	8		
Σ^{-1}	= 1.42E	E-06	-0.00231	0.000	911	0.017	881		-0.00055		-0.0010)4		
	9.1E	- 05	-0.00023	-8.4E	- 05	-0.000)55		0.00689		0.00159	99		
	- 0.00)294	-0.0025	-0.002	218	-0.001	104		0.00160		0.02888	86		
	L											_		
		-	0.08332	-0.08332	_	0.08332		-0.0	8332	-0.0)833	-0	.08332	
		().080022	0.080022	0	.080022		0.08	0022	0.08	8002	0.	080022	
Σ^{-1}	$(\overline{x}, \overline{x})$) _ -	0.007	- 0.007		- 0.007		- (0.007	- 0.	00699		- 0.00	7
L	$(x_{2018} - x_2)$	$ _{2019} = _{-100}$	0.17757	-0.17757	-(0.17757		-0.1	7757	-0.1	7757	-0	.17757	
		().01917	0.01917	0	.01917		0.01	917	0.0	1917	0.	019172	
		-	0.02622	-0.02622	-(0.02622		-0.0	2622	-0.0	26219	-0	.02622	
		L												

3371.806

6596.244

-47.415

646.8496

2353.419

1258.296

367.766

-376.486

2386.882 |

1297.645

1535.355

309.4

14892.67

3371.806

463.25

88.3575

309.4

2518.21

-914.6

441.235

492.745

540.685

-134.55

7.395

94.18 354.1

$$\frac{n_1 n_2}{n_1 + n_2 - 2} = \frac{110 * 86}{110 + 86 - 2} = \frac{9460}{194} = 48.7629$$

$$T_{cal.}^2 = 119.7237$$

$$T_{Crit.}^2 = \left\lfloor \frac{p(n_1 + n_2 - p - 1)}{n_1 + n_2 - 2} \right\rfloor F_{p,(n_1 + n_2 - 2)}^{\alpha} = \left\lfloor \frac{6(110 + 86 - 6 - 1)}{110 + 86 - 2} \right\rfloor * F_{6,194}^{0.05} = 5.845361 * 2.14$$

$$T_{Crit.}^2 = 12.50907$$

Decision Rule:

Since the calculated T^2 of 119.7237 is greater than the critical T^2 of 12.50907, we reject the null hypothesis at 0.05 level of significance. This implies that there is significant difference between the students' academic performance in WAEC for 2018 and 2019.

1. Hotelling (T²) Statistics Summary of the Significant Difference Between the Students' Academic Performance for 2019 and 2020

Hypothesis 2: $H_0: \bar{x}_{2019} = \bar{x}_{2020}$

VS

 $H_1: \overline{x}_{2019} \neq \overline{x}_{2020}$

Test Statistics:

$$T^{2} = \frac{n_{1}n_{2}}{n_{1} + n_{2} - 2} (\bar{x}_{2019} - \bar{x}_{2020})^{T} \Sigma^{-1} (\bar{x}_{2019} - \bar{x}_{2020})$$

 n_1 =86, n_2 =196,

$$\overline{X}_{2019} = \begin{bmatrix} 67.535 \\ 54.209 \\ 64.73 \\ 70.52 \\ 73.52 \\ 55.708 \end{bmatrix}, \quad \overline{X}_{2020} = \begin{bmatrix} 65.954 \\ 61.102 \\ 68.194 \\ 56.370 \\ 68.026 \\ 67.780 \end{bmatrix},$$

$$(\bar{x}_{2019} - \bar{x}_{2020}) = \begin{vmatrix} 67.535 \\ 54.209 \\ 64.73 \\ 70.52 \\ 73.52 \\ 55.708 \end{vmatrix} - \begin{vmatrix} 65.954 \\ 61.102 \\ 68.194 \\ 56.370 \\ 68.026 \\ 67.780 \end{vmatrix} = \begin{vmatrix} 1.581 \\ -6.893 \\ -3.464 \\ 14.15 \\ 5.494 \\ -12.072 \end{vmatrix}$$

 $(\bar{x}_{2019} - \bar{x}_{2020})^T = 1.581 - 6.893 - 3.464 \ 14.15 \ 5.494 \ -12.072$

$$\Sigma = \frac{(n_1 - 1)\Sigma_1 + (n_2 - 1)\Sigma_2}{n_1 + n_2 - 2}$$

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	- 914.6	441.235	686.63	1429.71	5853.35	-1428.85		5858.385	2861.138	-972.2	27	1417.689	26134.88
Σ –	-134.555	7.395	94.18	354.11	-1428.85	2017.985		632.9895	-70.005	-924.4	195	-203.346	1059.767
2 -								280					
	14525.	87	2471.365	-42	0.88	1027.55	52	4943.2	785	498.4	345		
	2471.3	65	8360.215	-39	3.4875	-238.32	21	3302.3	373	-62.61			
	- 420.8	8	-393.488	232	61.96	312.926	55	-285.6	54	-830.3	315		
	1027.5	52	-238.321	312	.9265	15889.5	56	2847.	389	150.76	54		
	4943.7	85	3302.373	-28	5.64	2847.38	39	41988	3.23	-369.0)84		
Γ	498.43	45	-62.61	- 830).315	150.764	ŀ	-369.0)84	5926.	175		
2 =					28	80							
	51.8'	78	8.826	-1.5	03	3.670		17.656		1.780			
	8.820	6	29.858	-1.4	05	- 0.851		11.794	-	0.224			
Σ.	1.50	03	-1.405	83.0	78	1.118		-1.020	-	2.965			
<u> </u>	- 3.670	0	- 0.851	1.11	8	56.748		10.169		0.538			
	17.65	56	11.794	-1.0	20	10.169		149.958	-	1.318			

[1	1.780	- 0.224	-2.965	0.538	-1.318	21.165		
	0.0210	-	0.0055	0.0002	1 -0.0	0012	-0.0020	-0.0019
	- 0.0055	0.	0361	0.0004	9.0	0129	-0.0023	0.00074
Σ^{-1} _	0.00021	0.	00049	0.0121	1 -0.0	0003	5.2E - 05	0.0017
2 =	- 0.0011	0.	0013	- 0.0002	.0.0	179	- 0.0012	-0.0005
	- 0.0020	-0	0.0023	5.21E -	05 -0.0	0012	0.00717	0.00063
	- 0.0019	0.	00074	0.00169	9 -0.(005	0.00063	0.04770

	0.066963	0.066963	0.06696336	0.066963	0.066963	0.066963
	- 0.26215	-0.26215	-0.26215488	-0.26215	-0.26215	-0.26215
$\Sigma^{-1}(\overline{x}, \overline{x}) =$	- 0.06902	-0.06902	-0.0690206	-0.06902	-0.06902	-0.06902
$\sum (x_{2019} - x_{2020}) =$	0.243195	0.243195	0.24319537 4	0.243195	0.243195	0.243195
	0.027197	0.027197	0.02719734 2	0.027197	0.027197	0.027197
	- 0.59294	-0.59294	-0.59294338	-0.59294	-0.59294	-0.59294

463.25

88.3575

11186

20.485

492.745

540.685

20.485

7046.5

-914.6

441.235

686.63

1429.7

-134.555

7.395

94.18

354.11

| 12990.51

2161.965

- 884.13

534.807

2161.965

5842.005

-481.845

-779.006

-884.13

-481.845

12075.96

292.4415

534.807

-779.006

292.4415

8843.055

309.4

2518.21

88.3575

540.685

1535.355

309.4

463.25

492.745

5858.385

2861.138

-972.27

1417.689

632.9895

-70.005

-924.495

-203.346

1059.767

3908.19

$$\frac{n_1 n_2}{n_1 + n_2 - 2} = \frac{86^{*}196}{86 + 196 - 2} = \frac{16856}{280} = 60.20$$

$$T_{cal.}^2 = 776.618$$

$$T_{Crit.}^2 = \left\lfloor \frac{p(n_1 + n_2 - p - 1)}{n_1 + n_2 - 2} \right\rfloor F_{p,(n_1 + n_2 - 2)}^{\alpha} = \left\lfloor \frac{6(86 + 196 - 6 - 1)}{86 + 196 - 2} \right\rfloor * F_{6,280}^{0.05} = 5.89286 * 2.13$$

$$T_{Crit}^{2} = 12.5518$$

Decision Rule:

Since the calculated T^2 of 776.618 is greater than the critical T^2 of 12.5518, we reject the null hypothesis at 0.05 level of significance. This implies that there is significant difference between the students' academic performance in WAEC for 2019 and 2020.

4.2.5.3 Hotelling (T²) Statistics Summary of the Significant Difference Between the Students' Academic Performance for 2018 and 2020

<u>Hypothesis 3:</u> $H_0: \bar{x}_{2018} = \bar{x}_{2020}$

VS

 $H_1: \overline{x}_{2018} \neq \overline{x}_{2020}$

$$T^{2} = \frac{n_{1}n_{2}}{n_{1} + n_{2} - 2} (\bar{x}_{2018} - \bar{x}_{2020})^{T} \Sigma^{-1} (\bar{x}_{2018} - \bar{x}_{2020})$$

$$n_1 = 110, n_2 = 196,$$

$$\overline{X}_{2018} = \begin{bmatrix} 61.13 \\ 55.082 \\ 63.762 \\ 60.712 \\ 76.10 \\ 53.654 \end{bmatrix}, \quad \overline{X}_{2020} = \begin{bmatrix} 65.954 \\ 61.102 \\ 68.194 \\ 56.370 \\ 68.026 \\ 67.780 \end{bmatrix},$$

$$(\bar{x}_{2018} - \bar{x}_{2020}) = \begin{vmatrix} 61.13 \\ 55.082 \\ 63.762 \\ 60.712 \\ 76.10 \\ 53.654 \end{vmatrix} - \begin{vmatrix} 65.954 \\ 61.102 \\ 68.194 \\ 56.370 \\ 68.026 \\ 67.780 \end{vmatrix} = \begin{vmatrix} -4.824 \\ -6.02 \\ -4.432 \\ 4.342 \\ 8.074 \\ -14.126 \end{vmatrix}$$

 $(\bar{x}_{2018} - \bar{x}_{2020})^T = -4.824$ - 6.02 -4.432 4.342 8.074 -14.126

$$\Sigma = \frac{(n_1 - 1)\Sigma_1 + (n_2 - 1)\Sigma_2}{n_1 + n_2 - 2}$$

1	4892.67	3371.806	2353.419	-47.415	367.766	2386.882	12990.51	2161.965	-884.13	534.807	5858.385	632.9895
3	371.806	6596.244	1258.296	646.8496	-376.486	1297.645	2161.965	5842.005	-481.845	-779.006	2861.138	-70.005
2	353.419	1258.296	6388.054	-649.095	-914.421	1627.261	- 884.13	-481.845	12075.96	292.4415	-972.27	-924.495
-	47.415	646.8496	-649.095	4039.758	-625.006	101.0648	534.807	-779.006	292.4415	8843.055	1417.689	-203.346
3	67.766	-376.486	-914.42062	-625.006	12757.36	-193.475	5858.385	2861.138	-972.27	1417.689	26134.88	1059.767
$\Sigma = \frac{\lfloor 2 \rfloor}{2}$	386.882	1297.645	1627.261	101.0648	-193.475	5276.799	632.9895	-70.005	-924.495	-203.346	1059.767	3908.19
						50	4					
	2788	3.18	5533.77	1 1	469.289	487.	392	6226.15	1	3019.872		
	5533	.771	12438.2	5 7	76.451	-132	2.156	2484.65	52	1227.64		
	1469.	.289	776.451	1	8464.014	-356	5.654	-1886.6	9	702.766		
	487.3	392	-132.150	5 -	356.6535	1288	82.81	792.683	i	-102.281		
	6226	.151	2484.65	2 -	1886.691	792.	683	38892.2	.4	866.2915		
$\Sigma =$	3019	.872	1227.64	7	702.766	-102	2.281	866.291	5	9184.989		
-					3	04						
	91.72	099	18.20319	4.3	8331875	1.6032	63	20.48076	9.9	933788		
	18.20	319	40.91529	2.:	554115132	-0.434	72	8.173196	4.(038289		
Σ –	4.833	188	2.554115	60	0.73688816	-1.173	2	-6.20622	2.3	31173		
$\Sigma =$	1.6032	263	-0.43472	-1	.1732023	42.377	67	2.60751	-0	.33645		
	20.48	076	8.173196	-6	.20621914	2.6075	1	127.935	2.8	849643		
	9.933	788	4.038289	2.	311730263	-0.336	45	2.849643	30	.21378		

$$\Sigma^{-1} = \begin{bmatrix} 0.012663 & -0.00493 & -0.00085531 & -0.00048 & -0.00167 & -0.00329 \\ -0.00493 & 0.027056 & -0.00076276 & 0.000486 & -0.00095 & -0.00184 \\ -0.00086 & -0.00076 & 0.016713387 & 0.000417 & 0.00101 & -0.00099 \\ -0.00048 & 0.000486 & 0.000417266 & 0.023661 & -0.00042 & 0.000363 \\ -0.00167 & -0.00095 & 0.001009895 & -0.00042 & 0.008206 & -0.00018 \\ -0.00329 & -0.00184 & -0.00098623 & 0.003239 & 0.003239 & 0.003239 \\ -0.11521 & -0.11521 & -0.11521415 & -0.11521 & -0.11521 & -0.11521 \\ -0.04146 & -0.04146 & -0.04145883 & -0.04146 & -0.04146 & -0.04146 \\ 0.091695 & 0.091695 & 0.091694821 & 0.091695 & 0.091695 & 0.091695 \\ -0.076234 & 0.076234 & 0.076233516 & 0.076234 & 0.076234 & 0.076234 \\ -0.4562 & -0.4562 & -0.45619758 & -0.4562 & -0.4562 & -0.4562 \end{bmatrix}$$

$$\frac{n_1 n_2}{n_1 + n_2 - 2} = \frac{110*196}{110+196-2} = \frac{21560}{304} = 70.9211$$

$$T_{cal.}^2 = 590.035$$

$$T_{Crit.}^{2} = \left\lfloor \frac{p(n_{1} + n_{2} - p - 1)}{n_{1} + n_{2} - 2} \right\rfloor F_{p,(n_{1} + n_{2} - 2)}^{\alpha} = \left\lfloor \frac{6(110 + 196 - 6 - 1)}{110 + 196 - 2} \right\rfloor * F_{6,304}^{0.05} = 5.90132 * 2.12$$

 $T_{Crit}^{\ 2} = 12.5108$

Decision Rule:

Since the calculated T^2 of 590.035 is greater than the critical T^2 of 12.5108, we reject the null hypothesis at 0.05 level of significance. This implies that there is significant difference between the students' academic performance in WAEC for 2018 and 2020.

Variable	2018-2019	2019-2020	2018-2020
$T_{cal.}^{2}$	119.724	776.618	590.035
$F^{\alpha}_{p,(n_1+n_2-2)}$	2.14	2.13	2.12
$T_{Crit.}^2$	12.5091	12.5518	12.5108

Table 1: Comparison of the Hotelling (T^2) Statistics Result for the Three Years

Table 4.1 shows the results summarized of the hotelling T^2 statistics between the students' academic performance for (2018 & 2019), (2019 & 2020) and (2018 & 2020) are significant. Next, to determine the significant difference between subjects, we applied Paired t-test statistics in the section below.

4.2.6 Paired t-test Analysis

Table 2: Paired t-test Analysis to Determine the Difference between Subjects in WASSCE for; 2018 and2019

		Mean	Std. Deviation	Std. Error Mean
Pair 1	MATHS 2018	60.8721	12.02526	1.29672
	MATHS 2019	67.5349	4.25011	.45830
Pair 2	ENGLISH 2018	54.4070	8.03870	.86684
	ENGLISH 2019	54.2093	5.44300	.58693
Pair 3	MARKETING 2018	63.6543	7.57654	.84184
	MARKETING 2019	64.7284	11.47172	1.27464
Pair 4	ECONS 2018	60.6812	6.07670	.73155
	ECONS 2019	70.5217	9.10496	1.09611
Pair 5	C. EDU 2018	76.5176	11.12442	1.20661
	C.EDU 2019	73.5176	13.76643	1.49318
Pair 6	BIOLOGY 2018	53.0154	7.05003	.87445
	BIOLOGY 2019	55.7077	4.87251	.60436

Paired Differences

			Std.	Std. Error	95% Interval Difference	Confide of	nce the		Sig.	(2-
		Mean	Deviation	Mean	Lower	Upper	t	df	tailed)	
Pair 1	MATHS 2018 - MATHS 2019 -	-6.66	12.595	1.358	-9.363	-3.962	-4.906	85	0.000**	
Pair 2	ENGLISH 2018 - ENGLISH 2019	0.198	10.197	1.099	-1.989	2.384	.180	85	0.858	
Pair 3	MARKET 2018- MARKET 2019	-1.074	14.088	1.565	-4.189	2.041	686	80	0.495	
Pair 4	ECONS 2018 - ECONS 2019	-9.841	11.636	1.401	-12.636	-7.045	-7.025	68	0.000**	
Pair 5	C.EDU 2018 - C.EDU 2019	3.000	18.444	2.001	978	6.978	1.500	84	0.137	
Pair 6	BIOLOGY 2018 - BIOLOGY 2019	-2.692	8.132	1.009	-4.707	-0.677	-2.669	64	0.010**	

Table 3: Summary of the Paired Samples Test between Subjects in WASSCE for 2018 and 2019

Footnote: p-value **= sig. at 5%.

Table 3 shows that three subjects (Mathematics, Economics and Biology) are significant. It implies a decrease in the Students' average performance in those subjects, since the mean difference between the subjects for the two years is negative.

Table 4: Paired t-test Analysis to Determine the Difference between Subjects in WASSCE for; 2019 and2020

	Mean	Std. Deviation	Std. Error Mean
MATHS 2019	67.5349	4.25011	.45830
MATHS 2020	67.0930	7.21050	.77753
ENGLISH 2019	54.2093	5.44300	.58693
ENGLISH 2020	61.1047	5.85919	.63181
MARKETING 2019	64.7284	11.47172	1.27464
MARKETING 2020	68.0741	7.19162	.79907
	MATHS 2019 MATHS 2020 ENGLISH 2019 ENGLISH 2020 MARKETING 2019 MARKETING 2020	Mean MATHS 2019 67.5349 MATHS 2020 67.0930 ENGLISH 2019 54.2093 ENGLISH 2020 61.1047 MARKETING 2019 64.7284 MARKETING 2020 68.0741	MeanStd. DeviationMATHS 201967.53494.25011MATHS 202067.09307.21050ENGLISH 201954.20935.44300ENGLISH 202061.10475.85919MARKETING 201964.728411.47172MARKETING 202068.07417.19162

Pair 4	ECONS 2019	70.5217	9.10496	1.09611
	ECONS 2020	57.7826	6.37290	.76721
Pair 5	C. EDU 2019	73.5176	13.76643	1.49318
	C.EDU 2020	71.2588	10.57305	1.14681
Pair 6	BIOLOGY 2019	55.7077	4.87251	.60436
	BIOLOGY 2020	67.7538	4.65373	.57722

Table 5: Summary of the Paired Samples Test between Subjects in WASSCE for 2019 and 2020

. . .

		Paired	Difference	es						
					95% Interval	Confiden of t	ce he			
		Mean	Std. Deviation	Std. Error Mean	Difference Lower	Upper	t	df	Sig. tailed)	(2-
Pair 1	MATHS 2019- MATHS 2020	-0.442	8.904	0.960	-1.467	2.351	0.460	85	0.647	
Pair 2	ENGLISH 2019 - ENGLISH 2020	6.895	8.089	0.872	-8.629	-5.161	-7.905	85	0.000**	
Pair 3	MARKET 2019- MARKET 2020	3.346	13.253	1.473	-6.276	-0.415	-2.272	80	0.026**	
Pair 4	ECONS 2019 - ECONS 2020	-12.739	10.567	1.272	10.201	15.278	10.014	68	0.000**	
Pair 5	C.EDU 2019 - C.EDU 2020	-2.259	17.829	1.934	-1.587	6.104	1.168	84	0.246	
Pair 6	BIOLOGY 2019 - BIOLOGY 2020	12.046	57.423	0.921	-13.886	-10.207	-13.083	64	0.000**	

Footnote: p-value **= sig. at 5%.

Table 5 shows that four subjects (English Language, Marketing, Economics and Biology) are significant. It implies a decrease in the Students' average performance for three subjects (English Language, Marketing and Biology), since the mean difference between the subjects for the two years is negative. While an increase in the Students' average performance for

Economics subject, since the mean difference between the subjects for the two years is positive.

		Mean	Std. Deviation	Std. Error Mean
Pair 1	MATHS 2018	61.1273	11.68871	1.11447
	MATHS 2020	66.4091	7.11027	.67794
Pair 2	ENGLISH 2018	55.0818	7.77921	.74172
	ENGLISH 2020	60.9091	5.61052	.53494
Pair 3	MARKETING 2018	63.7619	7.65547	.74710
	MARKETING 2020	68.6476	7.71609	.75301
Pair 4	ECONS 2018	60.7115	6.08782	.59696
	ECONS 2020	57.4904	6.55817	.64308
Pair 5	C. EDU 2018	76.1009	10.81832	1.03621
	C.EDU 2020	71.2202	10.37211	.99347
Pair 6	BIOLOGY 2018	53.6538	6.95781	.78782
	BIOLOGY 2020	67.5641	4.46222	.50525

Table 6: Paired t-test Analysis to Determine the Difference between Subjects in WASSCE for; 2018 and2020

Table 7: Summary of the Paired Samples Test between Subjects in WASSCE for 2018 and 2020

		Paired	Difference	es						
			95% C Interval Std. Difference Std. Error		Confide of	ence the	Sig.	(2-		
		Mean	Deviation	Mean	Lower	Upper	t	df	tailed)	(-
Pair 1	MATHS MATHS 20	20185.282 020	13.907	1.326	-7.910	-2.654	-3.983	109	0.000**	
Pair 2	ENGLISH ENGLISH	20185.827 2020	8.829	0.842	-7.495	-4.159	-6.923	109	0.000**	

Pair 3	MARKET 20184.886 11. MARKET 2020	1.060 1.079	-7.026	-2.745	-4.526 104	0.000**
Pair 4	ECONS 2018 -3.221 9.4 ECONS 2020	497 0.931	1.374	5.068	3.459 103	0.001**
Pair 5	C.EDU 2018 -4.881 13. C.EDU 2020	3.692 1.311	2.281	7.480	3.722 108	0.000**
Pair 6	BIOLOGY 2018	889 0.893	-15.688	-12.131	-15.572 77	0.000**

Footnote: p-value **= sig. at 5%.

Table 7 shows all WASSCE subjects considered are significant. It implies a decrease in the Students' average performance for four subjects (Mathematics, English Language, Marketing and Biology), since the mean difference between the subjects for the two years is negative. While an increase in the Students' average performance for Economics and Civic Education subjects, since their mean difference positive.

5. Conclusion

A quadratic form was obtained for each year using their covariance matrix, which was used to show the homogenous function that consists of all possible second order terms. The Hotelling T^2 statistics results between the students' academic performance for (2018 & 2019), (2019 & 2020) and (2018 & 2020) are all significant. Paired t-test statistics results a decrease in the Students' average performance for four subjects (Mathematics, English Language, Marketing and Biology), while an increase in the Students' average performance for Economics and Civic Education subjects. It was discovered that students' average performance in Economics and Civic Education subjects better than other subjects.

6. Recommendation

This research recommend the effective implementation of the Nigeria education policies that emphasizes on teachers qualification, years of teaching experience and the UNESCO policy on Teacher-Students ratio (this policy stipulates that the maximum number of students that should be in a secondary class is 25), since there is significant difference between Students' average performance for four subjects.

7. Limitations of the Study

The timeframe for this study was very short, so the researchers focuses their attention on only one WAEC center since they couldn't go round several schools. This is because some of the School to be visited will require crossing of the sea to such schools, and with the activities of sea pirates, they (the researchers) could only go to one public secondary school in the educational zone that has WASSCE examination centre. Another limitation was transportation constraint; this was a serious bottleneck that tends to hinder the completion of this research paper. Thirdly, the behaviour of the school Principal in granting the researchers' permission to access the WAEC results was another serious problem encountered in the process of completing this study.

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APPENDIX I

CONSENT LETTER

Department of Mathematics/Statistics Ignatius Ajuru University of Education, Rumuolumeni, Port Harcourt March, 2022

Sir/Ma,

Request for Permission to Conduct Research Experiment and obtain Research Information/Gather Data Your School

We are Nkpordee Lekia and Ogolo Ibinabo Magnus, postgraduate students of the above institution conducting a research project on the multivariate analysis of students' academic performance in WASSCE in public senior secondary schools in Rivers State (2018-2020).

To achieve this goal, I plead for your assistance in permitting me to access necessary documents on students' WAEC results for 2018, 2019 and 2020 in order to extract relevant secondary data which will be used for this study only.

I will ensure that whatever information gotten will be treated anonymously.

Thanks for your anticipated co-operation.

Yours faithfully,

2018 RESULT

NKPORDEE Lekia and OGOLO Ibinabo Magnus

Researchers

-						
S/N	MATHS	ENGLISH	MARKETING	ECONS	C. EDU	BIOLOGY
1	44	49		54	98	
2	48	68	78	68	59	38
3	39	45	67	60	65	49
4	53	46	65	62	77	45
5	46	45	61	65	86	47
6	49	42	52	53	66	43
7						
8	54	47	66	57		
9					66	
10	51	53	59	66		46
11	54	39	49	54	79	44
12	56	47	65	64	69	45

APPENDIX II

13	45	55	50	57	74	40
14	65	51	53	50	80	42
15	52	54	58	63	70	41
16	50	52	55	61	68	
17	60	55	56	55	89	40
18	59	58	64	67	95	
19						
20	56	56	51	70	94	49
21	54	49	59	64	79	44
22	59	59	79	69	69	54
23	69	49	54	69	95	49
24	49	44	52	54	59	48
25	79	58	59	59	96	47
26	99	69	58	62	98	54
27	54	69	66	69	95	54
28	52	40	57	61	63	52
29	45	66	64	50	79	
30	59	64	69	64	79	54
31	69	54	69	69	79	59
32	69	54	69	69	69	54
33	69	52	59	59	78	54
34	69	54	54	59	79	49
35	46	48	60	66	70	50
36	58	41	55	60	80	53
37	55	49	58	65	76	
38	67	45	50	67	73	46
39	53	51	65	68	76	58
40	67	53	61	62	72	
41	54	46	67	64	65	61
42	52	54	65	67	90	
43	65	52	66	65	96	
44	69	64	70	56	74	54
45			66			49
46	77	60	65	59	91	
47	51	50	66	56	65	52
48	79	55	70	65	90	64
49	65	63	64		73	61
50	75	59	67	60	68	53
51	61	46	65	61	81	62
52	83	57	72	69	82	67
53	66	55	68	59	73	59
54	54	52	65	48	69	
55	59	54	69		80	55
56	55	50	60		66	56
57	44	49		54	69	54

58	69	54	65	52	89	50
59	69	59	69	58	66	
60	77	55	67	59	89	54
61	54	54	69	64	78	59
62	69	59	65	53	69	52
63	50	49	66	57	80	50
64	55	54	60	55	70	
65	44	45	58	51	65	
66	64	53	77	58	67	56
67	67	51	66	56	68	53
68	59	64	68	57	91	58
69	64	50	62	59	65	
70	61	59		58	94	59
71	53	54	69		96	54
72	49	50		64	68	67
73	69	59	64	54	89	59
74	64	54	64	55	70	
75	79	50	65	57	90	
76	67	97	69	55	72	
77	50	54	54		66	64
78	98	62	60	50	69	60
79	59	60	85	69	70	63
80	60	52	50	54	65	50
81	69	64		53	67	
82	65	53	67	52	87	54
83	54	54	64		82	49
84	70	61	91	51	93	59
85	68	59		54	84	
86	53	55	65	59	78	61
87	58	63	66		68	51
88	49	54	60	58	68	50
89	69	57	69	60	65	66
90	99	65	71	65	70	64
91	59	69	54	55	67	56
92	45	50	63	69	73	
93	59	59	64	66	69	54
94	54	53	68	58	90	59
95	50	64	60	67	66	50
96	65	56	65	68	75	49
97	52	51	60	64	68	
98	51	52	55	61	59	
99	68	58	66	69	72	
100	59	54	59	62	69	
101						
102	64	59	64	69	96	

103	69	50	62	65	70	53
104	95	55	67	65	81	58
105	54	64	65	56	64	
106	50	60	70		75	54
107	59	44	59	69	69	
108	64	54	55	64	73	
109	65	69	93	69	79	57
110	69	65	69	55	90	68
111	61	62	60	54	69	
112	62	59	66	59	77	64
113	67	56	73	57	75	
114	79	53	68	68	69	53
115	69	64	54	58	96	64

Source: Office of the Principal, CCS Abonima

2019 RESULT S/N MATHS C. EDU ENGLISH MARKETING ECONS BIOLOGY

APPENDIX III

27	69	38	54	68	78	54
28	68	64	68	69	97	53
29	66	53	54	67	69	64
30						
31	69	53	67	68	67	
32	67	59	64		79	54
33	69	53	59	97	67	
34						
35	69	54	69	68	67	
36	68	58	67	95	97	54
37	68	59	69	67	68	58
38	67	54	58		79	59
39	67	54	59	78	65	49
40	67	52	65	69	64	
41						
42						
43	69	54	67	73	68	53
44	68	53	68		69	54
45	67	59	77	68	69	60
46	67	58	53		65	57
47	67	53	68	69	98	47
48						
49	69	54	67	65	66	54
50						
51	67	59	64	65	69	54
52						
53						
54						
55	62	46	54	57	53	
56	69	58	48		67	55
57						
58						
59			65		95	
60	65	60	54		75	56
61	68	54		70		52
62						
63			69			
64	78	58	70	66	69	64
65	67	48		69	78	53
66						
67	67	54	95	78		
68	68	45	46	62	53	53
69	67	51	65	65	75	63
70	65	49	73	67	68	
71	69	55	68	65	70	54

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72	65	50	69	71	91	58
73						
74	79	54	66	65	79	51
75	66	50	65		62	63
76	69	49	70		65	60
77						
78	66	55	72	59	65	53
79	67	59		70	68	58
80		67			65	
81						
82	68	56	66	65	70	52
83						
84						
85	69	45	69	71	74	
86	64	68		73	88	57
87	60		77	69	63	59
88						
89	65	50	67	66	69	
90	53	49	56	67	92	
91						
92						
93						
94	78	59		68	66	53
95	66	57		65	6	56
96	63	54			67	
97	67	56	90	69	65	54
98	68	50	66		79	52
99	69	54	54		69	64
100	59	44	53	79	66	49
101						
102	69	47	64		69	54
103	66	64	89	69	65	64
104	65	48	67		77	51
105	68	52	93	67	95	
106	67	54	52		79	52
107						
108	69	54	69		69	54
109	69	52	69	69	77	54
110	65	55	61	66	90	50
111						
112	69	59	69	69	69	
113	66	64	67	66	88	69
114	78	61	68	78	71	64
115						
116	67	48	38	68	60	54

117	65	53	65	70	79	50
118	69	58	66		84	63
119	69	49	59	67	68	

Source: Office of the Principal, CCS Abonima

APPENDIX IV

2020	2020 RESULT						
S/N	MATHS	ENGLISH	MARKETING	ECONS	C. EDU	BIOLOGY	
1	68	60	65	54	91		
2	79	66	69	50	65		
3	70	69	66	64	68	69	
4	69	59	59		69		
5	65	64		57	66	68	
6	67	55	68	54	67		
7	44	54	67	49	49		
8	54	59	65	44	66	58	
9	59	50	70	50	75	64	
10	64	59	54	52	65	65	
11	78	67	79	65	89	79	
12	69	64	69	51	67	59	
13	57	60	67	69	65	64	
14	70	55	64	59	69	68	
15	63	69	68	55	66	66	
16	69	54	69	56	79		
17	79	64	55	58	70		
18	65	67		64	73	78	
19	62	50	60	52	67		
20	68	59	64	61	48		
21	57	55	67	49	69		
22	69	69	66		65	69	
23	67	61	78	63	74	61	
24	65	64	59	55	79		
25	79	69	68	59	69	67	
26	77	65	60		79	69	
27	69	49	67	48	65		
28	50	67	69	64	54	68	
29	59	64	79	69	68	66	
30	68	59	65	65	44		
31	65	69	70	67	94	79	
32	69	59	79		79	79	
33	69	54	64	59	69	69	
34	64	44	69		64	69	
35							
36	69	69	69	69	92	69	
37	69	54	69	59	79		

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38	65	60		55	63	66
30	67	57	74	61	66	00
40	99	63	74	68	90	69
40	69	64	/1	50	64	65
42	69	60	60	52	69	69
42	68	49	54	67	93	59
43	63	47 68	65	55	93 87	55
44	69	69	69	54	70	69
45	65	61	65	54	67	07
40	67	54	67	58	65	78
47	07 77	63	07	<u> </u>	66	73
40	64	59	54	52	65	15
49 50	73	59 67	54	<u> </u>	58	68
51	67	63	66	60	<u> </u>	64
52	70	55	00 77	61	65	68
52	70 60	<u> </u>	54	54	70	65
55	69	66		50	70	67
55	65	64	70	50	70	66
55	03	04		30	70	00
50	60	66	60	62	72	60
50	67	65	67	60	75	09
58	69	<u> </u>	67	62	13	60
59	70	59	60	67	60 60	69
60	65	50	72	50	65	08
61	60	54	75	50	67	
62	69	54	/3	59	50	
63	67	<u> </u>	69	<u> </u>		00
64	07	59	<u> </u>	<u> </u>	70	66
65	/9	<u>69</u>	12	58	69	00
66	68	58	68	62	04 70	(5
6/	65	67	95	<u> </u>	79	03
68	54	60	65	50	/3	(0)
69	<u> </u>	04 50	69	50	89	69
/0	59	59	64	<u> </u>	69 79	69
/1	67	63	15	51	/8	
72	64	69	65	01	88	00
13	62	54	69 57	49	50	
/4	03	65	<u> </u>	50	/4	67
15	64	50	/9	54	03	<i>c</i> 0
/6	69 5 A	69	<i>c</i> 0	<u> </u>	94 54	69
//	54	64	69	54	54	(7
/8	69	60	00	43	69	0/
/9	64	61	/6	60	80	64
80	/9	59	68	50	/9	<u></u>
81	65	59	64	/1	62	68
82	/5	65	67	63	6/	/9 75
83	68	62		52	59	/5
84	61	63	70	53	90	69
85	69	69	79	59	/8	69

86	59	57	70	62	69	
87	66	65	70	02	74	67
07	00	05	11		/4	07
80	68	57		40	65	65
00	64	62	60	4 <i>9</i> 51	60	60
90	60	63	03	56	60	00
91	65	60	92	50	65	
92	67	54	00		64	69
93	60	54	77	49	60	08
94	69	64	11	69	09	<u> </u>
95	49	<u> </u>	69	00 (7	00 (5	59
96	68	53	64	0/ 5(03	00
97	65	<u> </u>	67	50	93	68
98	50	59	90	50	03	03
99	<u> </u>	<u>60</u>	69	59	09	(7
100	/0	57	0/	51	91	67
101	65	64	/1	56	69	69
102	50	50	11	50	65	65
103	70	59	63	54	68	66
104	53	64	69	52	/9	(0)
105	66	55	65	64	6/	68
106	59	57	67	49	66	(0)
107	64	58	69	59	/0	60
108	73	69	66	66	64	. .
109	65	65	65	69	11	67
110	51	60	61	65	68	
111	66	58	79	59	89	71
112	69	58	79	59	89	71
113	67	65	64	50	63	65
114	72	56	67	51	78	69
115	66	63	65	63	70	
116	60	55	60	54	75	67
117	68	49	85	69	66	
118	69	69	69	60	68	
119	67	60	90	63	90	
120	64	66	65	55	69	
121	58	54	67	54	65	61
122	57	48	62	39	49	
123	95	67	60	65	93	76
124	79	66	69	59	73	
125	70	64	65	64	69	
126	79	64	67	69	55	63
127	65	60	59	61	67	69
128	64	61	55		69	64
129	69	69	69	54	85	
130	67	65	59		69	66
131	60	59	57		38	59
132	68	63	68	59	60	65
133	66	67	67	52	68	

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134	54	49	56	49	49	
135	67	63	65	51	54	
136	69	61	69	59	59	68
137	65	62	66	53	65	66
138	92	69	79	65	89	70
139	67	65	69		84	
140	90	66		69	70	73
141	59	64	50	44	44	
142	50	63	64	47	40	
143	68	48	57	53	52	
144	66	61	60	55	68	67
145	59	64	68	40	54	
146	64	50	65	50	41	
147	60	54	69	54	66	
148	69	56	64	60	51	
149	50	60	67	64	53	60
150	62	53	59	51	59	
151	69	64	74	45	60	67
152	65	60	70	64	60	74
153	66	59		59	49	69
154	67	63	79	59	54	65
155	69	69		54		
156	66	61	69	50	59	70
157	68	62		51	64	66
158	54	55	73	62	45	
159	50	63	/1	56	50	(0)
160	63	66		51	55	69 75
101	59	<u> </u>	00 00	52	65	75
162	54	57		50	54	79 65
164	57	64	54	50	54	78
165	69	55	04 67	55	66	78 67
165	65	63	07	51	84	68
167	68	62	71		62	66
168	49	50	64	44	68	00
169	 66	64	79	53	65	69
170	70	60	65	50	79	65
171	60	62	58	49	69	
172	54	63	69	54	66	
173	69	54	66	52	68	
174	59	68	78	55	90	66
175	65	65	77	53	65	68
176	54	59		50	69	
177	59	62	74	53	66	
178	64	57	50	56	73	69
179	90	66	69	67	76	
180	94	69	75	50	67	76
181	89	67	68	54	65	65

182	69	65	70	51	68	67
183	60	68	67	52	67	64
184	63	68		50	70	66
185	66	59	69	69	79	
186	59	64	65	65	66	79
187	60	60		44	68	65
188	55	61	68	64	39	
189	64	64		54	59	68
190	62	60		51	50	
191	60	63	66	59	55	67
192	63	61	69	56	68	70
193	59	65	90	55	65	69
194	65	55	73	49	69	
195	69	54	66	45	38	
196	54	68	67	52	58	66
197	66	64	79	50	67	
198	69	66	90	54	70	
199	67	60	75	59	66	64

Source: Office of the Principal, CCS Abonima

APPENDIX V

Chi-Square Test for the Data Collected

2018 RESULT

Descriptive Statistics Ν Minimum Mean Std. Deviation Maximum MATHS 110 61.1273 11.68871 39.00 99.00 ENGLISH 110 55.0818 7.77921 39.00 97.00 MARKETING 105 63.7619 7.65547 49.00 93.00 **ECONS** 80.00 104 60.7115 6.08782 48.00 C.EDU 76.1009 53.00 98.00 109 10.81832 38.00 BIOLOGY 78 53.6538 6.95781 68.00

Test Statistics

	MATHS	ENGLISH	MARKETING	ECONS	C.EDU	BIOLOGY
Chi-Square	88.400ª	88.545 ^b	97.829 ^c	40.192 ^d	60.239 ^e	75.179 ^f
df	31	29	30	22	32	28
Asymp. Sig.	.000	.000	.000	.010	.002	.000

a. 32 cells (100.0%) have expected frequencies less than 5. The minimum expected cell frequency is 3.4.

b. 30 cells (100.0%) have expected frequencies less than 5. The minimum expected cell frequency is 3.7.

c. 31 cells (100.0%) have expected frequencies less than 5. The minimum expected cell frequency is 3.4.

d. 23 cells (100.0%) have expected frequencies less than 5. The minimum expected cell frequency is 4.5.

e. 33 cells (100.0%) have expected frequencies less than 5. The minimum expected cell frequency is 3.3.

f. 29 cells (100.0%) have expected frequencies less than 5. The minimum expected cell frequency is 2.7.

2019 RESULT

Descriptive Statistics									
	N	Mean	Std. Deviation	Minimum	Maximum				
MATHS	86	67.5349	4.25011	53.00	79.00				
ENGLISH	86	54.2093	5.44300	38.00	68.00				
MARKETING	81	64.7284	11.47172	38.00	97.00				
ECONS	69	70.5217	9.10496	57.00	97.00				
C.EDU	85	73.5176	13.76643	6.00	98.00				
BIOLOGY	65	55.7077	4.87251	45.00	69.00				

Test Statistics									
	MATHS	ENGLISH	MARKETING	ECONS	C.EDU	BIOLOGY			
Chi-Square	99.581ª	90.512 ^b	69.407 ^c	57.391 ^d	68.424 ^e	47.200 ^f			
df	13	21	30	18	26	16			
Asymp. Sig.	.000	.000	.000	.000	.000	.000			

a. 0 cells (0.0%) have expected frequencies less than 5. The minimum expected cell frequency is 6.1.

b. 22 cells (100.0%) have expected frequencies less than 5. The minimum expected cell frequency is 3.9.

c. 31 cells (100.0%) have expected frequencies less than 5. The minimum expected cell frequency is 2.6.

d. 19 cells (100.0%) have expected frequencies less than 5. The minimum expected cell frequency is 3.6.

e. 27 cells (100.0%) have expected frequencies less than 5. The minimum expected cell frequency is 3.1.

f. 17 cells (100.0%) have expected frequencies less than 5. The minimum expected cell frequency is 3.8.

2020 RESULT

Descriptive Statistics									
	N	Mean	Std. Deviation	Minimum	Maximum				
MATHS	196	65.9541	8.16201	44.00	99.00				
ENGLISH	196	61.1020	5.47346	44.00	69.00				
MARKETING	175	68.1943	7.86941	50.00	96.00				
ECONS	184	56.3696	6.73417	39.00	71.00				
C.EDU	195	68.0256	11.57692	38.00	94.00				
BIOLOGY	123	67.7805	4.47678	58.00	79.00				

Test Statistics							
	MATHS	ENGLISH	MARKETING	ECONS	C.EDU	BIOLOGY	

Chi-Square	306.714 ^a	82.571 ^b	259.280 ^c	133.109 ^d	290.169 ^e	132.650 ^f
df	33	20	32	28	47	18
Asymp. Sig.	.000	.000	.000	.000	.000	.000

a. 0 cells (0.0%) have expected frequencies less than 5. The minimum expected cell frequency is 5.8.

b. 0 cells (0.0%) have expected frequencies less than 5. The minimum expected cell frequency is 9.3.

c. 0 cells (0.0%) have expected frequencies less than 5. The minimum expected cell frequency is 5.3.

d. 0 cells (0.0%) have expected frequencies less than 5. The minimum expected cell frequency is 6.3.

e. 48 cells (100.0%) have expected frequencies less than 5. The minimum expected cell frequency is 4.1.

f. 0 cells (0.0%) have expected frequencies less than 5. The minimum expected cell frequency is 6.5.