A 2-connected graph in two dimensions containing 25 vertices with Gallai's Property

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Abstract

Graph theory recently considers as a modern field of mathematics and it was introduced by a great mathematician Leonhard Euler in 1735. Late then it is a flowered in the strong tool used in closely each area of science and nowadays it is most attractive and active area of mathematics research. Actually, graph theory is the study of graph, structure and molecules which have association in each other. It is very essential part of discrete mathematics which has been created with collaboration of nodes (vertices)and edges (lines or links), if a graph is connected with vertices directly by edges it is direct graph or a graph having symmetrically lines is also called direct graph or diagraph. In the studies of graph, we have investigated so many kinds of graph, Hypo-Hamiltonian is one of them. It is most important and it can be defined as the graph which hasn't Hamiltonian cycle but can be developed with the removing of single vertex form the developed graph. Lot of researchers have given the contribution in this research just like $Naeem\ et\ al$ has worked on "*A Two-Connected Graph with* $Gallai's$ *Property*" containing 12,18 and 25 vertices.In this research paper we have developed two different graphs consisting 25 Eulerian graph which has highest cycle and path order $C(G) = 24$ and $P(G) = 25$.Other graph is in 3D under cycle which contains 23 nodes.

Keywords: *Group theory, longest path and longest Cycle, Gallai's Property*

Introduction

Graph theory is an important field of mathematics and having very huge usages in the field of science and engineering particularly in switching theory and logical design, artificial intelligence, formal languages, computer graphics, operating systems, compiler writing, information organization and retrieval. Graph or molecule consists of nodes, edges, incident point, vertices adjacent, loops and also isolated points.

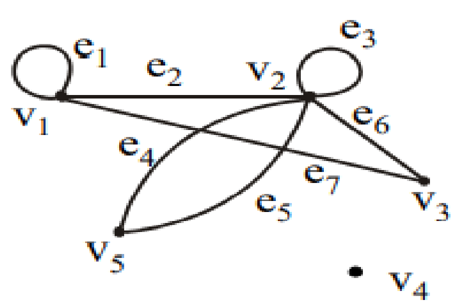
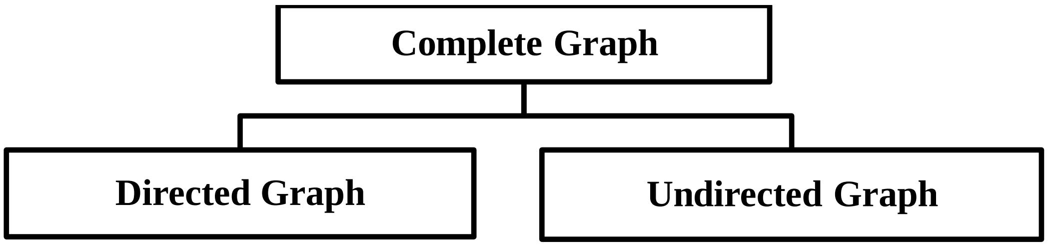
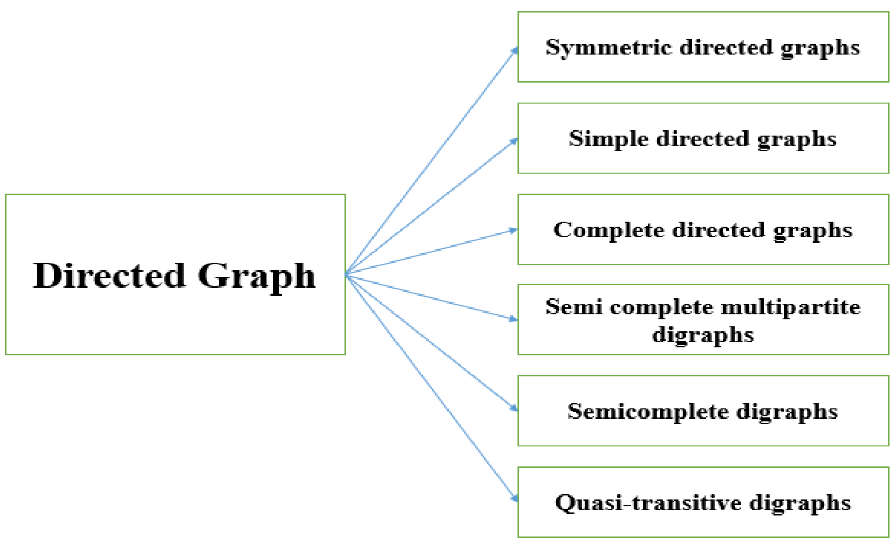


Figure no 1: Graph

In this graph we can show that $(i)\ v_1$ is incident with edges $e_1, e_2\ and\ e_7 (ii)$ vertices adjacent to $v_3\ are\ v_1$ and $v_2 (iii)$ loops are $e_1\ and\ e_3$ (iv) only edges $e_4\ and\ e_5$are parallel (v) The only isolated vertex is v_4 in this Graph see in figure no 1.Basically graph theory consists directed and undirected graph and so it has so many other types complete graph is one of them,



In the studies of graph theory especially complete graph it has two ways to explain, one it may be the simple and undirected graph which can be define as a unique edge may connect the pair of different nodes or vertices and other one is directed graph which is being defined as pair of the unique edges may connect with each pair of different nodes here edges mean one in each direction.



Symmetric directed graphs can be defined as a directed graph whose all links or lines are bi directed, a graph is said to be simple directed graphs if it hasn't any loop, Complete directed graphs define as the pair of the unique edges may connect with each pair of different nodes, semi complete multipartite digraphs are the simple graph in which the nodes set is discrete into partite set, semi complete digraphs are simple digraphs where there is an arc between each pair of vertices. Every semi complete digraph is a semi complete multipartite digraph, where the number of vertices equals the number of partite sets. Quasi-transitive digraphs are simple digraphs where for every triple x, y, z of distinct vertices with arcs from x to y and from y to z, there is an arc between x and z. Note that there can be just one arc between x and z or two arcs in opposite directions. A semi complete digraph is a quasi-transitive digraph. There are extensions of quasi-transitive digraphs called k-quasi-transitive digraphs. The Peterson graph is well known type of Hypo-Hamiltonian graph and it is undirected graph. In 1898 a great mathematician Julius Peterson developed a graph called as Peterson graph which is the minimum bridgeless cubic graph having 3 edges coloring. It is commonly made in pentagon with as a pentagon exclusive with five spokes. Before the development of Hypo-Hamiltonian graph, in 1966 Tiber Gallai′sprovided the impression of property about missing nodes in the graph and by his thoughts and struggle this property becomes famous as a Gallai′s Property. In 1969 H. Walther [1-2] was the mathematician who compliance the question on Gallai′s Property and developed a planar graph contains 25 nodes and satisfying the measurement of Gallai′s Property. In the contribution of research education lot of mathematicians have given distant ideas specially H. Walther and H. Voss [3] and Tudor Zamfirescu [4] established a graph containing 12 nodes or vertices and 12 vertices graph is the shortest possibility of the developed graph. After then Tudor Zamfirescuimproved the raised question and queried about the cycle and path of established graph. Several best examples are the evidence of Tudor Zamfirescuquestions which has been published in different article. A Hamiltonian cycle or circuit is a graph in which a cycle which passes each node or vertex of the graph only single time and will reach at starting nodes or vertex is called Hamiltonian cycle or circuit beside of this Hamiltonian path is the way or road map which passes each vertex but doesn't end at initial vertex. Furthermore, A traceable graph is a graph that owns a Hamiltonian path and if a graph G is called hypo Hamiltonian graph if G itself doesn't have a Hamiltonian cycle but each graph developed by removing a single node from G is Hamiltonian. In 1966 a Hungarian mathematician has contributed in the history of graph theory and was all-time associate and collaborator of Paul Erdős. He was the student of Dénes Kőnig and instructor of László Lovász [1]. A planar graph contains lowest 17 vertices has been developed by W. Schmitz and Tudor Zamfirescu was the first who developed the 2-connected planar graph on 82 vertices [6].In this time 26 vertex graphs are very famous [7],inversely nowadays the lowermost planar sample order is 32 [6]. In 1972 $Tudor\ Zamfirescu$had developed the idea on $Gallai's$ property he answered that if $p_j^i = \infty\ (p_i^{-j} = \infty)$ and thegraph is not planar or $i - connected$ graph such independently set of j points persist distinct from the lengthiest path condition $p_j^i \neq \infty\ (p_i^{-j} \neq \infty)$. For example, $p_j^i p_i^{-j}$displays shortest number of nodes of a $j - connected$ graph or planar graph such that autonomously the set of j selected nodes be the disjoint from the lengthiest path. Similarly, two cases $C_i^j\ and\ C_i^{-j}$are normally applied for the longest circuits as the conversion of longest path. For searching the suitable reply of the assessed questions about the problem the developed work of the Zamfirescu is being carried out by another mathematicians W. Schmitz and H. Walther [5,8]. H. Walther developed an example $C_2^2 \leq 220\ \&C_2^1 \leq 105$ [2] and B. Grunbaum, W. Hatzel and Tudor Zamfirescu have work on it [9-10] and have developed their consequences in advanced technologies [11, 12]. Some Pakistani researcher have given contribution among those Dr Ali Dino *et al* and so many other are there have worked on the Gallai Property by choosing 18 and 22 vertices in his research paper 2019 and another research paper they have taken 20 vertices and have found out the longest path as well as longest cycle [13-17]. The graph is widely applicable to the modern models of social structures which is based on several types of relationships among peoples, networks and group of the peoples. These types of graph which are being used in social structure play an important role in human life through social network. In these projected graph models, organization and federation are represented by vertices or nodes, relationship among them are edges or sides.

Results and Conclusion

In this investigation we have taken Eulerian Graph which contains 25th nodes and the quality of this graph is, it has equal number of order degree. During the investigation we have adopted four loops on different nodes $\boldsymbol{l_1\ on\ M, l_2 on\ P, l_3 on\ S\ and\ l_4\ on\ V}$ and developed 25 vertices connected graph and we have to satisfy the $Gallai's$ property. In the developed graph first, we will be discussed on vertices, degree of the graph and incident lines of the graph and their detail can see in Table no 1.

Secondly, we have to investigate the vertex order with adjacent vertices and the detail of said topics can be seen in Table no 2. Later then we have investigated the longest cycle and longest path from the developed graph.

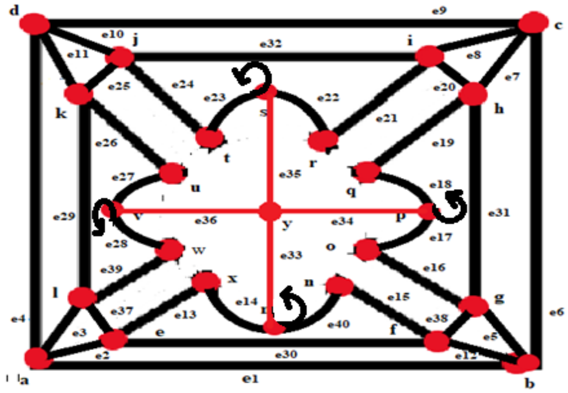


Figure no 2 Eulerian Graph

Table no 1: Vertex. Degree and incident of graph

V	D	incident	V	D	Incident
A	4	e_1, e_2, e_3, e_4	M	3	$e_{14}, e_{40}, e_{33}, l_1$
B	4	e_1, e_{12}, e_5, e_6	N	2	e_{15}, e_{40}
C	4	e_6, e_7, e_8, e_9	O	2	e_{16}, e_{17}
D	4	$e_9, e_{10}, e_{11}, e_{12}$	P	3	$e_{17}, e_{18}, e_{34}, l_2$
E	4	e_2, e_3, e_{30}, e_{37}	Q	2	e_{18}, e_{19}
F	4	$e_{12}, e_{15}, e_{30}, e_{38}$	R	2	e_{21}, e_{22}
G	4	$e_5, e_{16}, e_{31}, e_{38}$	S	3	$e_{22}, e_{23}, e_{35}, l_3$
H	4	$e_7, e_{19}, e_{20}, e_{31}$	T	2	e_{23}, e_{24}
I	4	$e_8, e_{20}, e_{21}, e_{32}$	U	2	e_{26}, e_{27}
J	4	$e_{10}, e_{24}, e_{25}, e_{32}$	V	3	$e_{27}, e_{28}, e_{36}, l_4$
K	4	$e_{11}, e_{25}, e_{26}, e_{29}$	W	2	e_{28}, e_{39}
L	4	$e_3, e_{29}, e_{37}, e_{39}$	X	2	e_{13}, e_{14}
			Y	4	$e_{33}, e_{34}, e_{35}, e_{36}$

Table no 2: Adjacent Vertex

E	Adj	E	Adj	E	Adj	E	Adj

1	A,B	11	D,K	21	I,R	31	G,H
2	A,E	12	B,F	22	R,S	32	I,J
3	A,L	13	E,X	23	S,T	33	M,Y
4	A,D	14	M,X	24	J,T	34	P,Y
5	B,G	15	F,N	25	J,K	35	S,Y
6	B,C	16	G,O	26	K,U	36	V,Y
7	C,H	17	O,P	27	U,V	37	E,I
8	C,I	18	P,Q	28	V,W	38	F,G
9	C,D	19	H,Q	29	I,K	39	I,W
10	D,J	20	H,I	30	E,F	40	M,N

LONGEST CYCLE FROM DEVELOPED GRAPH

In this connected graph we have found 24th order longest cycle and missing single vertex and loop which have not derived in the graph. Every vertex has same order of cycle and if we choose the 25th vertices the we will not find the 24th order longest cycle. Some drawing of 24thcycle taken from the 25th order graph is given below in figure no 3 and the detail of cycle is given in Table no 3.

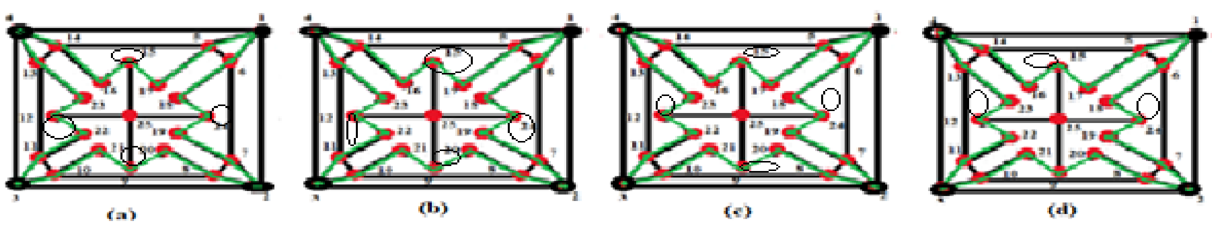


Figure no 3: Longest Cycle in the graph

Table no 3: Longest Cycle of the Graph

Cycle	Detail of Vertex in longest Cycle	Missed Vertex
A	1,6,13,14,19,7,2,5,20,9,21,10,3,11,22,12,23,13,4,14,16,15,17,5	25 and loops
B	2,7,19,24,18,6,1,5,17,15,16,14,4,13,23,12,22,11,3,10,21,9,20,8	25 and loops
C	3,10,21,9,20,8,2,7,19,24,18,6,1,5,17,15,16,14,4,13,23,12,22,11,	25 and loops
D	4,14,16,15,17,5,1,6,18,24,19,7,2,8,20,9,21,10,3,11,22,12,23,13	25 and loops

In Figure no 3, we have drawn four figure a, b, c and d for longest cycle from the graph. In figure (a, b, c and d) we have fixed the vertex and developed a cycle of 25th in graph which is shown in figure no 4.

24THLONGEST PATH FROM DEVELOPED GRAPH

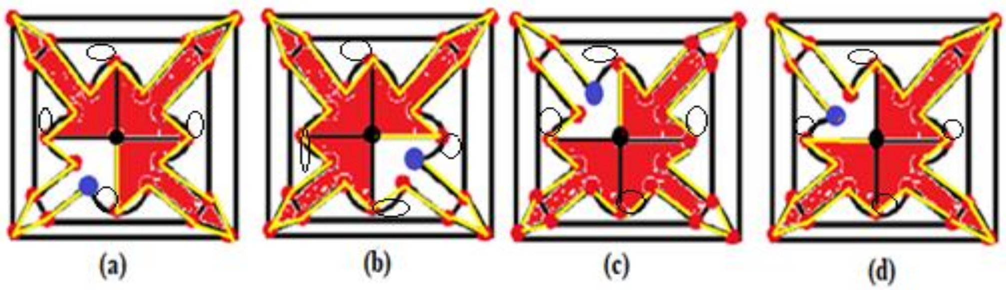


Figure No 4: Longest Path of the graph

In this investigation we have observed that there are longest path having order 24th and above four graphs in figure no 3 shows the longest path, in figure (a) staring vertex is 25th and end vertex is 21, In the figure (b) the initial or starting vertex is 25 but end vertex is 19. Furthermore, in graph (c and d) the starting vertex is 25 but end vertices are 16 and 23. In the figure no 4 black vertex shows the initial vertex in all the graph, yellow color shows the path and blue color shows the end vertex of the path of the developed graph. The detail of the longest path of the graph is given in Table no 4.

Table no 4: Longest Path of the Graph

Path	Detail of Vertex in longest Path	Missed Vertex
A	25,9,20,8,2,7,19,24,18,6,1,5,17,15,16,14,4,13,23,12,22,11,3,10,21	21
B	25,24,18,6,1,5,17,15,16,14,4,13,23,12,22,11,3,10,21,9,20,8,2,7,19	19
C	25,15,17,5,1,6,18,24,19,7,2,8,20,9,21,10,3,11,22,12,23,13,4,14,16	16
D	25,12,22,11,3,10,21,9,20,8,2,7,19,24,18,6,1,5,17,15,16,14,4,13,23	23

LONGEST CYCLE FROM DEVELOPED GRAPH 2

In this graph we have made three-dimensional figure under circle and containing 23 nodes and have found the longest cycle in the developed graph.

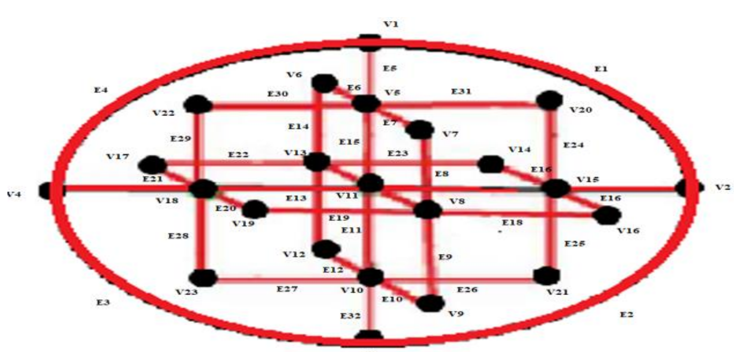


Figure no 5: Developed Graph

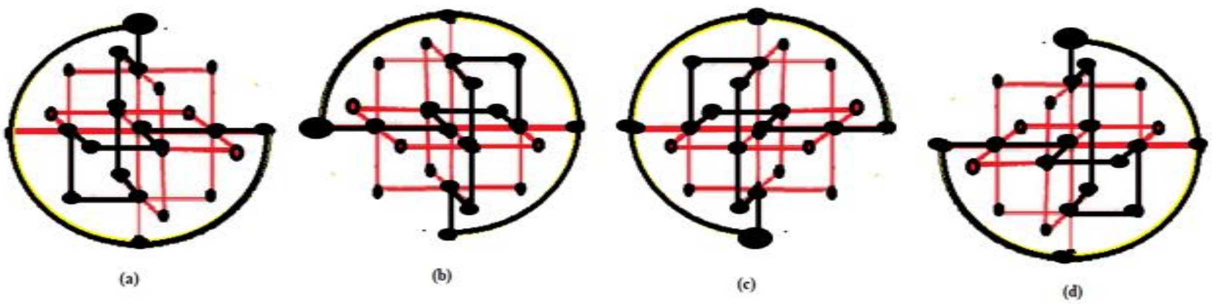


Figure No 6: Longest Cycle

In figure no 6 we have found some cycle from the graph in figure (a & d) we have taken v_1 the initial vertex and derived the cycle towards left and right, in figure no (b) taking v_3 and have derived the cycle in clockwise rotation and in (c) we have taken $v_4$and applied same kind of strategy to find the longest cycle in the graph.

Table no 5: Longest Cycle

Cycle	Detail of Vertex in longest Cycle	Missed Vertex
A	1,4,3,2,15,11,8,9,18,23,10,12,13,6,5	7,14,16,17,19,20,21,22
B	4,1,2,3,10,9,8,7,5,20,15,14,13,11,18	6,12,16,17,19,21,22,23
C	3,4,1,2,15,11,13,17,18,20,5,7,8,9,10	6,12,14,16,19,21,22,23
D	1,2,3,4,18,11,8,16,15,21,10,12,13,6,5	7,9,14,17,19,20,22,23

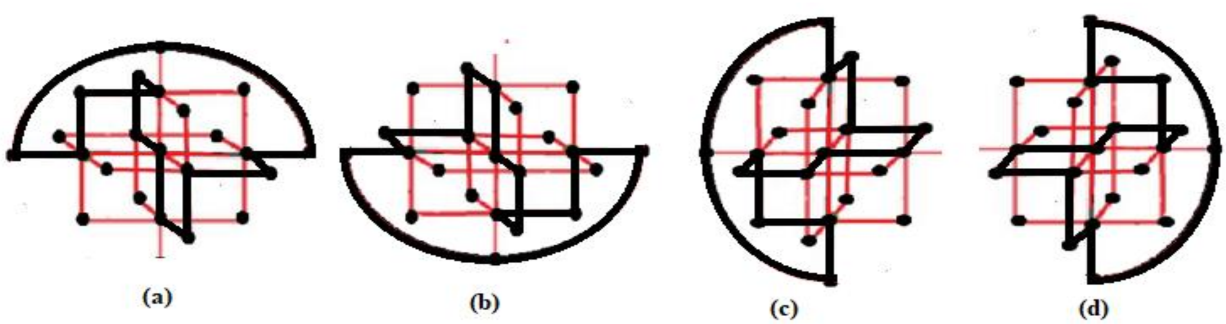
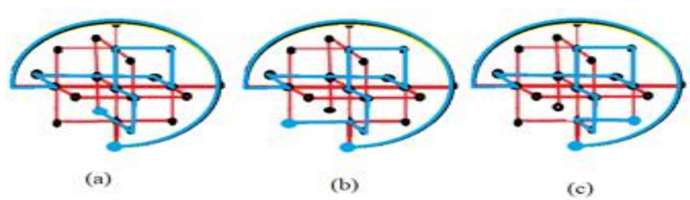


Figure no 7: 180^0 rotation

Here we have taken 180-degree plan for finding the longest cycle. So, in this regard we can apply counter clock wise direction for finding the longest cycle in the graph.

LONGEST PATH FROM DEVELOPED GRAPH 2



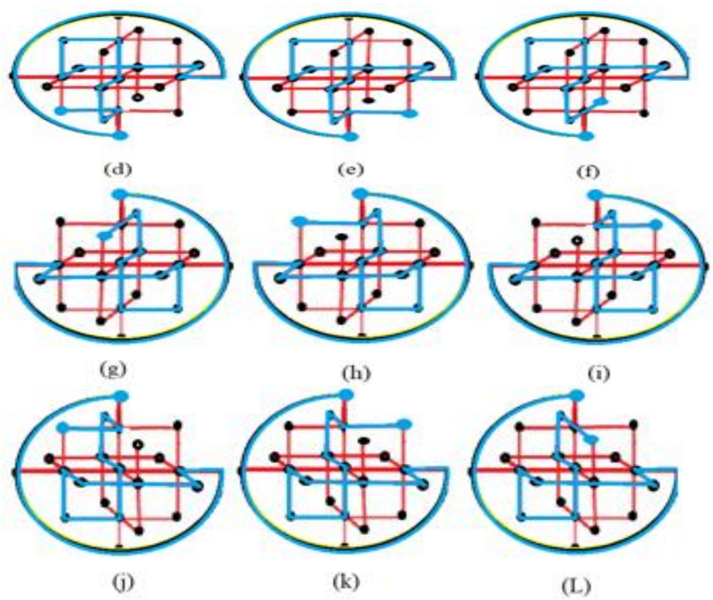


Figure no 8: Longest Path

In figure no 8 we have investigated that there are found that there are 18^{th} order highest path in the developed molecule and the order of missing vertices are 7^{th}. In the above figure no 8 labeled from $a\ to\ l$ all the graph has equal number of order$p(a\ to\ l) = 18$ and missing vertices order is $mv(a\ to\ l) = 7$

References

[1] P. Erdos and G. Katona (eds.), Theory of Graphs, Proc. Colloq. Tihany, Hungary, Sept. 1966, Academic Press, New York (1968). 32 Abdul Naeem Kalhoro and Ali Dino Jumani: A Two-Connected Graph with Gallai's Property

[2] H. Walther, Uber die Nichtexistenzeines Knotenpunktes, durch den alle langstenWegeeinesGraphengehen, J. Comb. Theory 6 (1969) 1-6.

[3] H. Walther, H. J. Voss, Uber Kreise in Graphen, VEB Deutscher Verlag der Wissenschaften, Berlin, 1974.

[4] T. Zamfirescu, A two-Connected Planar Graph without Concurrent Longest Paths, J. Combin. Theory B13 (1972) 116-121.

[5] W. Schmitz, Uber LangsteWege und Kreise in Graphen, Rend. Sem. Mat. Univ. Padova 53 (1975) 97-103.

[6] T. Zmfirescu, on longest paths and circuits in graphs, Math. Scand. 38 (1976) 211-239.

[7] T. Zamfirescue, intersecting longest paths or cycles: A short survey, Analele Univ. Craiova, Seria Mat. Info. 28 (2001) 1-9.

[8] H. WALTHER, Uber die NichtexistenzzweierKnotenpunkteeinesGraphen, die alle llngstenKreisefassen, J. Combinatorial Theory 8 (1970), 330-333.

[9] B. Grunbaum, Vertices missed by longest paths or circuits, J. Comb. Theory, A 17 (1974), 31-38.

[10] W. Hatzel, Ein planarerhypohamiltonscher Graph mit 57 Knoten, Math. Ann. 243 (1979), 213-216.

[11] T. Zamfirescu, Graphen, in welchen je zweiEckpunktedurcheinenlangstenWegvermiedenwerden, Rend. Sem. Mat. Univ. Ferrara 21 (1975), 17–24.

[12] T. Zamfirescu, L'histoire et l'´etatpr´esent des bornesconnues pour Pk j , Ck j , Pk j et Ck j , Cahiers CERO 17 (1975), 427-439.

[13] Abdul Naeem & Ali Dino Jumani, ”Graph with non-concurrent Longest Paths”. IJCSIS International Journal of Computer Science and Information Security, VOL. 17 No. 4, April 2019.

[14] Abdul Naeem & Ali Dino Jumani. “A Planar Lattice Graph, with Empty Intersection of All Longest Path”. Engineering Mathematics. Vol. 3, No. 1, 2019, pp. 6-8. doi:10.11648/j.engmath.20190301.12.

[15] A. H JUNEJO, I AHMED, I. SOOMRO, A. N KALHORO, R MUHAMMAD, I ALI JOKHIO, R CHOHAN, A. D JUMANI. “A Connected Graph with set of empty Intersection of All Longest Cycles”. IJCSNS International Journal of Computer Science and Network Security, VOL. 19 No. 5, May 2019.

[16] A. H Junejo, A. N Kalhoro, Inayatullah soomro, Israr Ahmed, Raza Muhammad, Imdad Ali Jokhio, Rozina Chohan. “A Connected Graph with Non-concurrent Longest Cycles”. IJCSNS International Journal of Computer Science and Network Security, VOL. 19 No. 5, May 2019.

[17] A. H JUNEJO, A. N KALHORO, I AHMED, I SOOMRO, R MUHAMMAD, I ALI JOKHIO, R CHOHAN, A. D JUMANI, “A connected graph with non-concurrent Longest Paths”, IJCSNS International Journal of Computer Science and Network Security, VOL. 19 No. 4, April 2019.