

# ON INVESTIGATION OF SOME ASSET RETURNS IN NIGERIA STOCK EXCHANGE USING FIGARCH APPROACH

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## ABSTRACT

Scholars over decades have argued and debated over the volatility behaviour of assets returns, but unilateral conclusion is yet to be reached as regard some unique features of some market assets. More so, the fact that prices of asset fluctuate more frequently over a period of time than the ones in the usual markets, as such, the present study investigated the asset return volatility in Nigerian Stock Exchange Market using five different firms which these include GUINNESS, UBA, UBN, CADBURY and FIRST BANK. The data used for this study were daily from 4th January, 2010 to 16<sup>th</sup> December, 2019 and sourced from the Nigerian Stock Exchange. The research employed Generalized Auto Regressive Conditional Heteroskedasticity (GARCH) model, and other developed members of its family-type, such as Autoregressive Fractionally Integrated Moving Average (ARFIMA), Fractionally Integrated Generalized Auto Regressive Conditionally Heteroskedastic (FIGARCH) models. These competing GARCH-type family models were tested and selection of the optimum model was carried out using the Log likelihood (LogL), Akaike information criterion (AIC), Schertz information criterion (SIC) and Hannan Quinn criterion (HQC). The results from the analysis revealed that FIGARCH (1,d,1) produced the best fit for GUINNESS, FBANK, UBN and CADBURY, where the fraction order are (0.34, 0.28, 0.32 and 0.24) respectively. While ARFIMA (1,1,1) produced the best fit for UBA. This clearly indicated that asymmetric FIGARCH model produced better fits in volatility models.

**Keywords:** *Garch, Figarch, Arfima, Arch Lm, Volatility*

## 1. INTRODUCTION

### 1.1 Background of the Study

The work of the like of Engle (1982) and Bollerslev (1986) were the pioneer research in this area of financial instrument volatility. Over the years their results have metamorphosized into various Generalized Autoregressive Conditional Heteroscedasticity (GARCH) type family of volatility models. These models have distinguished themselves among other models used for measuring financial risk in the stock markets. Engle and Patton (2001) extensively buttressed on the importance and applicability of these models in finance. More of the works in this area

include: (Hsieh, 1991; Babayemi and Asare, 2010; Yaya, 2013; Hojatallah and Ramanarayanan, 2011; Eric, 2008; and Hansen and Lunda, 2004).

Many studies attempted to review the theory and applications of autoregressive fractionally integrated moving average (ARFIMA) and fractionally integrated generalized autoregressive conditional heteroskedasticity (FIGARCH) models, mainly for the purpose of the description of the observed persistence in the mean and volatility of a time series. The power of these models in through their feature were over emphasized, many studies commenting and concluding that FIGARCH models in asset return outperform other models. Nevertheless, Rafik *et al.* (2014) proved FIGARCH as better candidate than other volatility models.

Hammoudeh and Li (2017) investigated volatility in Gulf Arab countries stock markets and their result showed significant reduction in volatility shock persistence. Kang *et al.* (2018) investigated the impact of structural breaks in conditional volatility on variance persistence of asymmetric effects in volatility models, the result from the study revealed persistency in reduction of variance and was statistically significant. Others in this regard include (Kang *et al.* 2019; and Ewing and Malik 2013).

The recurring or fluctuation of unstable price of assets return in Nigerian stock market has always remain a problem that researchers to date do not overcome and such give a room to different scholars creating different models to partially overcome some of these challenges. Mankiw *et al.* (2015) expressed that over decades up till present day research, there are interesting and challenging argument and debate about modelling problems in Nigerian stock exchange. One fact is that so far there were hundreds and thousands of estimated models flinging around the literatures of stock exchange market in relation to asset returns. The need for more powerful statistical techniques such as ARFIMA, FIGARCH, and Artificial Neural Network should be inquired to overcome some of these problems.

Furthermore, the demand for good estimation procedure of volatility models will ever remain in vogue because of their importance and application in financial risk measurement. Some of the volatility models that are still within ambient of modern research include:

- i. Asymmetric and power GARCH models (APARCH)
- ii. Fractionally Integrated GARCH models (FIGARCH) and
- iii. Autoregressive Fractionally Integrated Moving Average (ARFIMA)

Some of the stylized facts about asset returns that are conspicuously identifiable in modern research are:

- i. Volatility clusters
- ii. Volatility persistence
- iii. Fat tails
- iv. Nonlinear dependence

This research work aimed at investigating the asset return volatility in Nigerian stock exchange market using five Nigerian firms' which include GUINNESS, UBA, UBN, CADBURY and FIRST BANK using three different financial statistical techniques, which includes GARCH, ARFIMA and FIGARCH.

## 2 MATERIALS AND METHODS

### 2.1 Augmented Dickey Fuller (ADF) Test

This was an improvement over Dickey and Fuller (1979) test in that it incorporated term that adequately takes care of highly and serially correlated data. It was augmented for serial correlation in Dickey- Fuller test. ADF test was conceived by Said and Dickey (1984) to test the null of the presence of unit root(s) in time series dataset. The data series regression is given as:

$$\Delta Z_t = \beta' B_t + \partial Z_{t-1} + \sum_{j=1}^p \varphi_j \Delta Z_{t-j} + \varepsilon_t \quad \dots 2.1$$

where  $B_t$  is a vector of deterministic terms such as constant and trend. The  $p$  lagged difference terms,  $\sum_{j=1}^p \varphi_j \Delta Z_{t-j}$ , the incorporated term to take care of highly and serially correlated data series.  $\varepsilon_t$  is the error term and assumed to be homoscedastic. Under the null hypothesis,  $\Delta Z_t$  is  $I(0)$  which implies that  $\partial = 0$ . The ADF t-statistic is given as:  $ADF_{statistic} = \frac{\hat{\partial}}{se(\hat{\partial})}$  and null is rejected if p-value is less than 5% of significance.

### 2.2 Phillips-Perron (PP) Test

PP test by Phillips-Perron (1988) is also a unit root test that builds on Dickey-Fuller test to test for the null of presence of unit root in a data series. It in its case addresses issue that data generating process might have a higher order of serial correlation than admitted in the test regression equation by making immediate past process being endogenous. It equally makes a non-parametric correction to the t-test statistic which paves ways for its robusticity with respective to autocorrelation and heteroskedasticity in error process test equation. The test regression equation is given as:

$$\Delta Z_t = \beta' B_t + \partial Z_{t-1} + \varepsilon_t \quad \dots 2.2$$

### 2.3 ARCH (AUTOREGRESSIVE CONDITIONAL HETROSCEDASTICITY) TEST

The arch test of the residuals is performed to check if the residuals are consistent with a standard normal distribution. The **ARCH** test checks the pair of hypothesis

$H_0: \alpha = 0$  (The distribution is symmetry)

$H_1: \alpha \neq 0$  (The distribution is asymmetry)

The test statistics has an asymptotic  $\chi^2$ - distribution. The null hypothesis is rejected if the test statistics is greater than the significant level  $\alpha$ .

## 2.4 GARCH MODELING

The importance and genuineness of the assumption on heteroskedastic variance for volatility models has been extensively debated and argued out by (Engle, 1982; Engle and Patton, 2001; and Bollerslav, 1986). The prototype of the equations from most of the literature emanated from seasonal and non-seasonal ARIMA, given as:

$$Y_t = C_0 + \sum_{i=0}^{\ell} \omega_i X_{t-i} + \sum_{j=1}^p \phi_j Y_{t-j} + \theta(B)a_t \quad \dots 2.3$$

## 2.5 FIGARCH Model

The Autoregressive Conditional Heteroskedastic (ARCH) processes were presented by Engle (1982), where he used this model to estimate the means and variances of inflation in the U.K.. These are mean zero, serially uncorrelated processes with non-constant variances conditional on the past, but constant unconditional variances. Accordingly with Engle (1982), the time-series  $y_t$  and the associated prediction error  $\varepsilon_t = y_t - E_{t-1}y_t$  are considered, where  $E_{t-1}$  is the expectation of the conditional mean on the information set at  $t - 1$ .

A Generalized Autoregressive Conditional Heteroskedasticity (GARCH) model was proposed by Bollerslev (1986) and is as follows:

$$\varepsilon_t = z_t \sigma_t, z_t \sim N(0, 1), \sigma_t^2 = \omega + \alpha(L)\varepsilon_t^2 + \beta(L)\sigma_t^2, \quad \dots 2.4$$

where  $\omega > 0$ ,  $\alpha(L)$  and  $\beta(L)$  are polynomials in the lag operator  $L(L^i x_i = x_{t-i})$  of order  $q$  and  $p$ , respectively. Assuming that  $\alpha_i \geq 0$  and  $\beta_i \geq 0$  for all  $i$ , the GARCH ( $p, q$ ) model in Eq. (10) can be rewritten in the form of an ARMA( $m, p$ ) process:

$$\phi(L)\varepsilon_t^2 = \omega + [1 - \beta(L)]v_t \quad \dots 2.5$$

where  $v_t \equiv \varepsilon_t^2 - \sigma_t^2$ , and  $\phi(L) = [1 - \alpha(L) - \beta(L)]$ . The  $v_t$  process is interpreted as an innovation for the conditional variance, has a zero mean serially uncorrelated. In the GARCH model, the effect upon the past squared innovations on the current conditional variance decays exponentially with the lag length. This model presents some limitations since it assumes that the shocks decay at a fast geometric rate, thus only has short term persistence.

To overcome this problem it was developed the Integrated GARCH (IGARCH), by Engle and Bollerslev (1986) and can be written as follows:

$$\phi(L)(1-L)\varepsilon_t^2 = \omega + [1 - \beta(L)]v_t, \quad \dots 2.6$$

This model is characterized by having infinite memory. That is, the occurrence of a shock to the IGARCH volatility process will never die out. This feature may reduce its appeal to be used in asset pricing purposes, because this assumption would make the pricing functions for long-term contracts particularly prone to the initial conditions. To overcome this Baillie *et al.* (1996) introduced the Fractionally

Integrated Generalized Autoregressive Conditionally Heteroskedastic (FIGARCH). The FIGARCH ( $p, d, q$ ) model is given by:

$$\phi(L)(1-L)^d\varepsilon_t^2 = \omega + [1 - \beta(L)]v_t,$$

where  $0 \leq d \leq 1$  is the fractional differencing parameter which measures the degree of long memory.

This model imply a slow hyperbolic rate of decay for lagged squared innovations in the conditional variance function, although the cumulative impulse response weights associated with the influence of a volatility shock on the optimal forecasts of the future conditional variance eventually tend to zero, this is a feature that the model shares with the weak stationary GARCH process.

This model has greater flexibility for modeling the conditional variance since it accommodates the covariance stationary GARCH model when  $d = 0$  and the IGARCH model when  $d = 1$ , as special cases. The advantage of the FIGARCH model is that, for  $0 < d < 1$ , it is a lot more flexible to allow for an intermediate range of persistence. One of the disadvantages of the FIGARCH model is that it assumes strict stationarity but not weak stationarity.

Chung (1999) argues that Baillie *et al.* (1996) parameterization of the FIGARCH model may have a specification problem. He argues that the relations of BBM FIGARCH model with the ARFIMA models for the conditional mean are not perfect. The constant  $\omega$  it is different than the constant  $\mu$  in the ARFIMA models. Chung (1999) redefines the FIGARCH model as:

$$\phi(L)(1-L)^d(\varepsilon_t^2 - \sigma^2) = [1 - \beta(L)]v_t, \quad \dots 2.7$$

the relationship between the parameter  $\omega$  and the  $\sigma^2$  parameter is:

$$\omega = \phi(L)(1-L)^d\sigma^2. \quad \dots 2.8$$

Parameters descriptions

$\theta$  = Vector that contains the parameter of the GARCH process

$\beta$  = Vector that contains the parameter of the GARCH process

$X_t$  = Explanatory variables, which determine the value of conditional, mean of the series

$\gamma_t$  = The return at day

$h_t$  = Conditional volatility at day t

### 3. DATA ANALYSIS AND DISCUSSION OF RESULTS

The analysis was carried out using two different statistical packages R and Gretl where the data from Nigerian Stock Exchange market using asset returns in GUINNESS, UBA, UBN, CADBURY and FIRST BANK were modelled using GARCH family including ARFIMA and FIGARCH.

#### 3.1 DATA ANALYSIS

**Table 3.1:** Descriptive statistics of Asset Return in Nigerian Stock Exchange Summary Statistics, using the observations 04<sup>th</sup>/01/2010 to 16<sup>th</sup>/12/2019

Firms	Mean	Median	Minimum	Maximum
GUINNESS	98.8892	103.530	29.0000	190.560
UBA	16.8146	10.9900	1.30000	63.9400
UBN	24.2670	24.9100	3.14000	50.3300
CADBURY	38.7356	33.2100	8.55000	101.000
FIRSTBANK	27.1684	23.7500	10.7000	69.3000

Firms	Std. Dev.	C.V.	Skewness	Ex. kurtosis	Jarque-Bera	P-value
GUINNESS	40.3953	0.408490	-0.141633	8.7632	56277.5	0.0000
UBA	13.6505	0.811823	1.80098	9.6071	13261.6	0.0000
UBN	10.2731	0.423337	-0.156001	7.9838	37008.9	0.0000
CADBURY	17.9376	0.463078	0.712177	6.2398	84194.8	0.0000
FIRSTBANK	10.3241	0.380004	0.887143	6.04342	85071.4	0.0000

The descriptive results of daily assets return for the five firms were shown in Table 3.1. The statistics results shows that the Jarque-Bera test fail to accept the null hypothesis of normality in all assets returns with highly significant p-values and further we discovered that the mean of all assets returns were positive, which served as a signed profits or gained over the period of study, but the finding shows that Guinness generates more profit with 98.8892 while UBA has the least return with 16.8146 average asset returns. The daily returns' standard deviations of all firms are high, indicating the levels of dispersals which may lead to the higher volatility of the market and served as a signed of risk. A reasonable wide range between the minimum and maximum of the firms' returns give supportive evidence to the high level of variability in assets returns of the firms. The return series for UBA, CADBURY and FIRST BANK, parade a positive skewness, in other word the three mention firms are skewed to the right, which indicates that the returns has non-symmetric behaviour, whereas the GUINNESS and UBN are

skewed to the left. In the other hand we can describe a positive skewness as an indication that the upper tail of the distribution is thicker than the lower tail meaning that the returns rises more often than it drops, as the time plot will detail more, and such reflect the sense of renewed confidence in the market for the investor. The firms' returns exhibit excess kurtosis and we discovered that all the return series have non-normal distributions with high kurtosis.

**Table 3.2:** Heteroskedasticity Test Results

Returns	F-statistic	P-value	nR <sup>2</sup>	P-value
GUINNESS	8.985869	0.0009	9.98630	0.0006
UBA	12.125789	0.0022	10.125965	0.0027
UBN	357.6080	0.0000	303.6446	0.0000
CADBURY	8.032574	0.0000	7.020754	0.0081
FIRSTBANK	14.81881	0.0000	15.74606	0.0001

Table 3.2 depicted test-statistics and p-value that are all statistically significant at 1% level of significance. This means that all the five firms stock returns exhibit heteroskedasticity and can be modelled using GARCH models.

### 3.3 GARCH MODELS

**Table 3.4: Symmetric GARCH (1,1) models**

Firms	$\mu$	$\omega$	$\alpha_1$	$\beta_1$	$\alpha_1 + \beta_1$	$v$
GUINNESS	-0.0010*	-0.095*	0.19042	0.79992	0.99034	1.0022*
UBA	-0.0310*	1.051*	0.2969*	0.59134*	0.8882*	1.0651*
UBN	-0.0041*	0.381*	0.2965*	0.63425*	0.93075*	0.9898*
CADBURY	0.0020*	0.941*	0.2955*	0.69234*	0.98784*	1.0343*
FIRST_BANK	0.0092*	1.212*	0.28045*	0.6976*	0.97805*	0.9881*

\* indicates a statistical significant result of 1% .

Table 3.3 showed results of symmetric GARCH (1,1) of the five firms assets returns. The results presented depicted that the shock persistence parameter ( $\beta_1$ ) of all study's firms were reasonably high. GUINNESS have the most persistence value of (0.79992) and UBA with the least ( $\beta_1$ ) value of (0.59134). The mean reverting rates, designed to discovered volatility were

found stationary due to the fact that the sum of both GARCH and ARCH ( $\alpha_1 + \beta_1$ ) are  $<$  unity in the firm's assets return.

### 3.4 Model selection criteria and diagnostic checks

Table 3.04: MODEL SELECTON CRITERIA OF GARCH-type family

FIRMS	MODELS	MODEL SELECTION Criteria				ARCH LM TESTS	
		LogL	AIC	SIC	HQC	F_Stat.	P value
GUINNESS	GARCH (1,1)	-5395.25	3.3941	3.4065	3.3986	0.0055	0.941
	ARFIMA (1,1,1)	-5303.25	3.3167	3.2726	3.3721	0.0011	0.923
	<b>FIGARCH (1,0.34,1) *</b>	<b>-3163.25</b>	<b>2.0051</b>	<b>2.0306</b>	<b>2.0211</b>	<b>0.0005</b>	<b>0.915</b>
UBA	GARCH (1,1)	-400.25	3.6877	3.6971	3.6941	0.0041	0.911
	<b>ARFIMA (1,1,1) *</b>	<b>-2213.25</b>	<b>2.5511</b>	<b>2.0224</b>	<b>2.5424</b>	<b>0.0011</b>	<b>0.911</b>
	FIGARCH (1,0.34,1))	-5428.25	3.6715	3.6948	3.6802	0.0029	0.9491
UBN	GARCH (1,1)	-5328.25	3.2127	3.2248	3.2171	0.0004	0.41
	ARFIMA (1,1,1)	-5189.25	3.0974	3.1119	3.1027	0.004	0.938
	<b>FIGARCH ((1,0.32,1)) *</b>	<b>-3613.25</b>	<b>2.6302</b>	<b>2.6447</b>	<b>2.6355</b>	<b>0.0007</b>	<b>0.969</b>
CADBURY	GARCH (1,1)	-6148.25	2.8628	2.8728	2.8664	0.0304	0.892
	ARFIMA (1,1,1)	-6026.25	2.7817	2.7937	2.786	0.0004	0.943
	<b>FIGARCH ((1,0.24,1)) *</b>	<b>-4019.25</b>	<b>1.4431</b>	<b>1.452</b>	<b>1.4443</b>	<b>0.0018</b>	<b>0.979</b>
FBANK	GARCH (1,1)	-6415.25	3.0381	3.0481	3.0466	0.0037	0.71
	ARFIMA (1,1,1)	-6357.25	3.0005	3.0125	3.0048	0.0048	0.775
	<b>FIGARCH (1,0.28,1)</b>	<b>-5298.25</b>	<b>2.2922</b>	<b>2.3043</b>	<b>2.2966</b>	<b>0.0004</b>	<b>0.979</b>

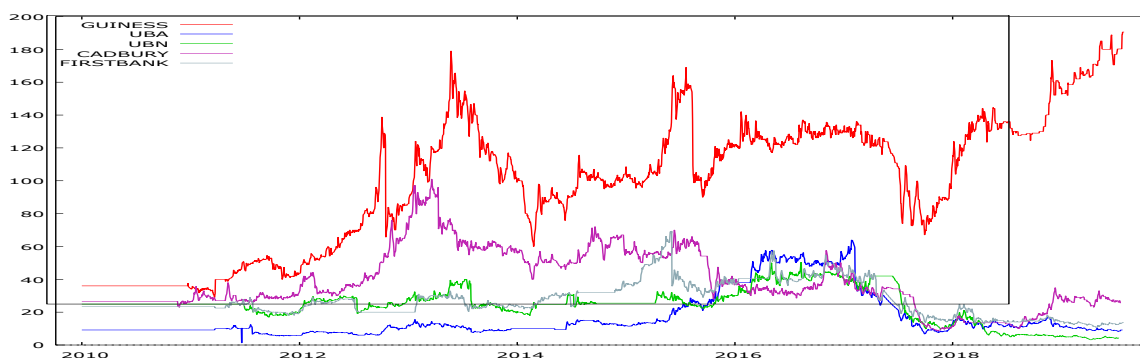
\* *Implies optimal Model selected*

The information criteria used to determine the best fitted model for the firms used in the study revealed that FIRGARCH (p,d,q) have the most suitable fitted models for First Bank (FBANK), DADBURY, Union Bank (UBN) and GUINNESS with  $d = ((1,0.28,1), (1,0.24,1),$



(1,0.32,1) and (1,0.34,1) respectively while ARFIMA (1,1,1) was found the most suited model for United Bank of Africa (UBA).

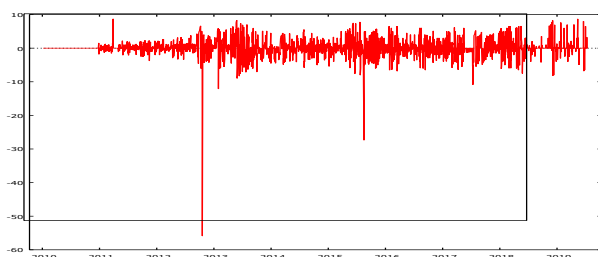
**Time plot of five firm’s asset return in Nigerian Stock Exchange 2010 to 2019**



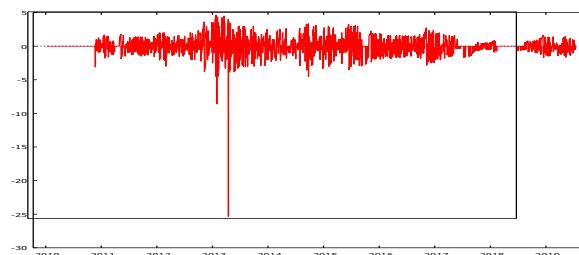
**Figure 1:** Daily asset return in original form

Figure 1 is a daily asset return in order of GUINNESS, UBA, UBN, CADBURY and FIRST BANK. The plot is the original displayed of the daily asset returns of the study’s’ firms. In Fig 1; we observed high volatility for Guinness and UBA in the between 2013 and early 2014, whereas Guinness witness the volatility increase in the late 2015 and keep increasing in the up to the present time of study while UBA decline significantly.

**Fig.2:** Guinness: FIGARCH (1,0.34,1)

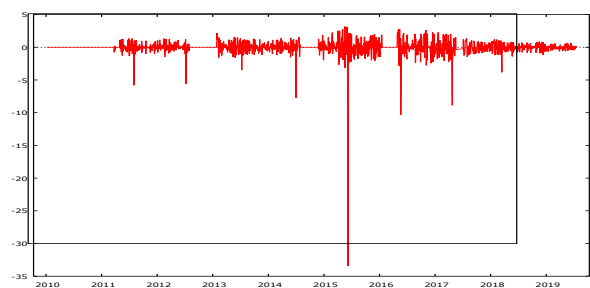


**Fig.3:** Cadbury: FIGARCH (1,0.24,1)

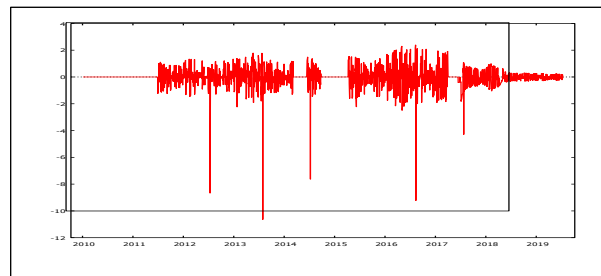


**Fig. 4:** FIRSTBANK FIGARCH (1,0.28,1),

**Fig. 5:** UBN: FIGARCH (1,0.32,1)



**Fig. 6: UBA ARFIMA (1,1,1)**



**Fig. 7: Combine plot of all assets returns**

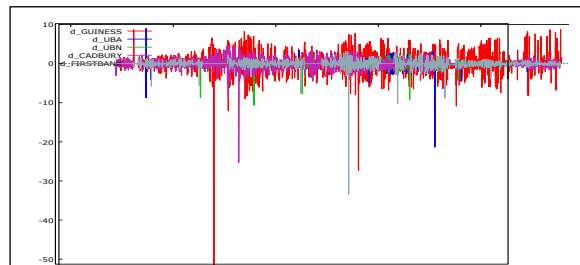
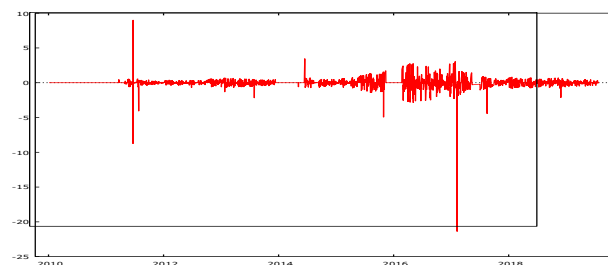


Figure 7 is combine time plots of the whole five firms' asset return in absolute returns form. Fig. 2 to 5 are a separate time plot in absolute returns of ARFIMA (1,1,1) in UBA as the best selected model and FIGARCH selected models in the other four firms.

## CONCLUSION

This study is designed to investigate five asset returns in Nigerian Stock Exchange and these include GUINNESS, UBA, UBN, CADBURY and FBN. We employed three different competing GARCH-type models which include GARCH, ARFIMA and FIGARCH and set as well as tested different orders of (p,d,q) along also was Fractionally Integrated part which ranges from 0 to 0.5 in case of FIGARCH. We estimate the models using three information criteria which also include AIC, SIC and HQC. Finally we discovered that the most suitable and well fitted models, that best fit First Bank (FBANK), DADBURY, Union Bank (UBN) and GUINNESS were **(1,0.28,1)**, **(1,0.24,1)**, **(1,0.32,1)** and **(1,0.34,1)** respectively while ARFIMA (1,1,1) was found the most suited model for United Bank of Africa (UBA). However, all models were performed good and passed the diagnostic test due to the fact that the mean reverting rates, designed to discovered volatility were found stationary and sum of both GARCH and ARCH ( $\alpha_1 + \beta_1$ ) are  $<$  unity in the firm's assets return.

## RECOMMENDATION

To date there are thousands of models offer in the literature of assets returns in Nigerian stock exchange. They are all important and always in demand. This is as a result of frequent changes in price, either decreases, and run to loss (negative mean) or increases (positive mean) and run to profit. Generally, there are hidden knowledge behind these abrupt changes in volatility such

as financial instability, economic downthrown and insecurity in a nation. In similar vein, the fluctuations may be as result of global financial crisis which most often at times cause by unforeseen circumstances such as war, breakout of diseases and political unrest among nations, thereby affecting other nations across the globe. Therefore, we recommend that a powerful statistical technique should be developed that can simultaneously study the crime and financial effects in the field of econometric.

## CONTRIBUTION TO KNOWLEDGE

This research work has been able to identify the volatility of five asset returns in Nigerian Stock Exchange market using GARCH-type family models with an up-to-date dataset. The present work demonstrated the use of specific statistical tools such as ARFIMA and FIGARCH. It was able to find that UBA bank can be modelled as ARFIMA (1,1,1) while the CADBURY, FIRST BANK, UBN AND GUINNESS exhibited best performance with FIGARCH (1,d,1).

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