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LABELING OF 2-REGULAR GRAPHS BY ODD EDGE MAGIC

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Abstract

An edge magic total labeling of a graph G(p,q) is said to be an Odd edge magic total labeling if $\lambda(E) = \{1,3, \dots 2q-1\}$ with the condition that for each edge $uv \epsilon E, \lambda(u) + \lambda(uv) + \lambda(v) = k_e$,where k_e is known as the magic constant. In this paper we resolute cycles of odd length, disjoint union of $C_3 \cup C_{4r+2}$ $(r \geq 1)$, disjoint union of $C_4 \cup C_{4r-1}$ $(r > 1)$,disjoint union of $C_3 \cup C_{4r}$ $(r > 1)$, disjoint union of $C_4 \cup C_{4r-3}$ $(r > 1)$ are Odd edge magic graphs.

Keywords: *Labeling, Edge magic total labeling, Odd edge magic total labeling, Odd edge magic graphs.*

AMS subject Classification Code: 05C78

1. Introduction

Graph Labeling is one of the most growing areas in graph theory. In graph theory, the labeling of graphs noticed to be a theoretical topic. It is used in countless applications like coding theory, X-Ray crystallography and astronomy etc. Design of graph labeling is helpful to network security, network addressing and social network in communication network. An edge-magic total labeling of a graph is a motivating research area.

In this paper, we mean only finite, simple and undirected 2-regular graphs. A graph G has vertex set $V(G)$,edge set $E(G)$ and the number of vertices equals the number of edges.

A labeling of a graph G is a mapping from a set of vertices (edges) into a set of numbers, usually integers. Many kinds of labelings have been studied and an excellent survey of graph labelings can be found in [3].

In 1963, Sedl$\grave{a}\check{c}$ek[7] introduced the concept of magic labeling in graphs. A graph G is *magic* if the edges of G can be labelled by a set of numbers$\{1,2, \dots q\}$ so that the sum of labels of all the edges incident with any vertex is the same.In 1970,Kotzig and Rosa [1]defined a magic valuation of a graph $G(V, E)$ as a bijection f from $V \cup E$ to $\{1,2,3 \dots |V \cup E|\}$ such that for all xy,$f(x) + f(y) + f(xy)$ is a constant.

H. Enomota et al., [2] proved that A cycle C_n is super edge magic iff n is odd, $K_{m,n}$

is super edge magic if $m = 1$ or $n = 1$.A wheel graph W_n of order n is not super edge magic. Moreover, W_n is not edge-magic if $n \equiv 0(mod)4$. They also proved that every tree is edge magic and super edge-magic.

CT.Nagaraj,C.Y.Ponnappan, G.Prabakaran [4] in introduced the concept of an odd vertex magic total labeling. A vertex magic total labeling is odd if $f\big(V(G)\big) = \{1,3, \dots ,2n - 1\}$.

The authors of [5] studied the basic properties of odd vertex graphs and showed among other things that C_n and P_n have an odd vertex magic total labelings for $n \geq 3$,that rc_s is an odd vertex magic graph and that (s, t) −kite graph is an odd vertex graph iff $s + t$ is an odd.

In [5] they proved the following results. A star graph $k_{1,r}$ is an odd vertex magic iff $r = 2$. If a tree T is an odd vertex magic then n is odd.

C.T Nagaraj,C.Y.Ponnappan,G.Prabakaran[6] studied Odd vertex magic total labeling of some 2-regular graphs.

2. Odd edge magic total labeling on cycles

Definition
An edge magic total labeling λ of a graph G is called an Odd edge magic total labeling if $\lambda(E) = \{1,3, \ldots 2q - 1\}$ and that graph G is called an Odd edge magic total graph.

Example 1

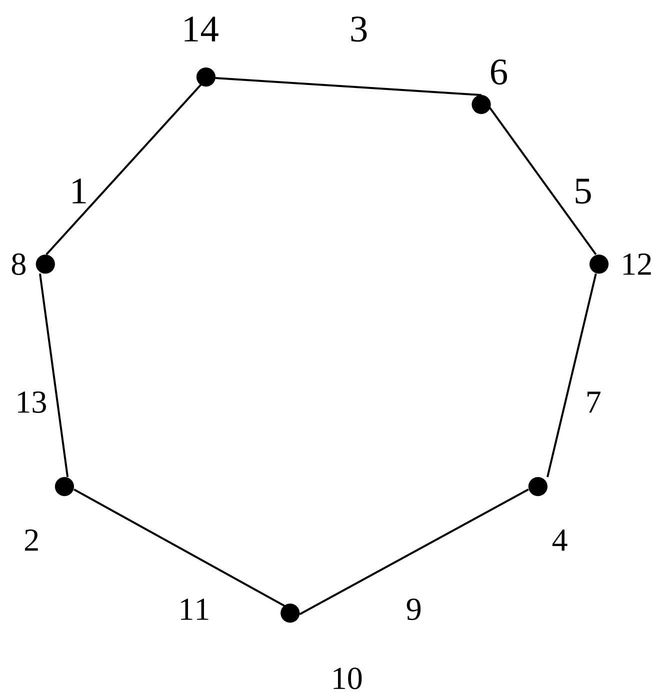

Figure 1: $p = 7, q = 7, k_e = 23$

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Theorem 2.1

The graph $C_3 \cup C_{4r}$, $r > 1$ is an Odd edge magic total graph.

Proof:

The Vertices of C_3 are labelled as $2r + 2$, $2r + 4$,$2r + 6$.

The Edges of C_3 are labelled as $8r + 3$, $8r + 5$,$8r + 1$.

Labeling of the vertices and edges of C_{4r} by

$$\lambda(v_i) = \begin{cases} i + r\ ,\ i\ odd,\ \ 1 \le i \le 2r - 3 \\ i + 9, \quad i\ odd,\ 2r - 1 \le i \le 4r - 3 \\ 2\ , i = 4r - 1 \\ 4r + i + 6,\ i\ even \end{cases}$$

$$\lambda(e_i) = \begin{cases} 8r + 1 - 2i\ , \quad i = 1,2, \dots ..2r - 3 \\ 8r - 5 - 2i \quad ,\ 2r - 2 \le i \le 4r - 3 \\ 4r + 5\ , i = 4r - 2 \\ 4r + 3\ , i = 4r - 1 \\ 4r + 1\ , i = 4r \end{cases}$$

Therefore, the graph $C_3 \cup C_{4r}$, $r > 1$ is an Odd edge magic total graph.

Example 2

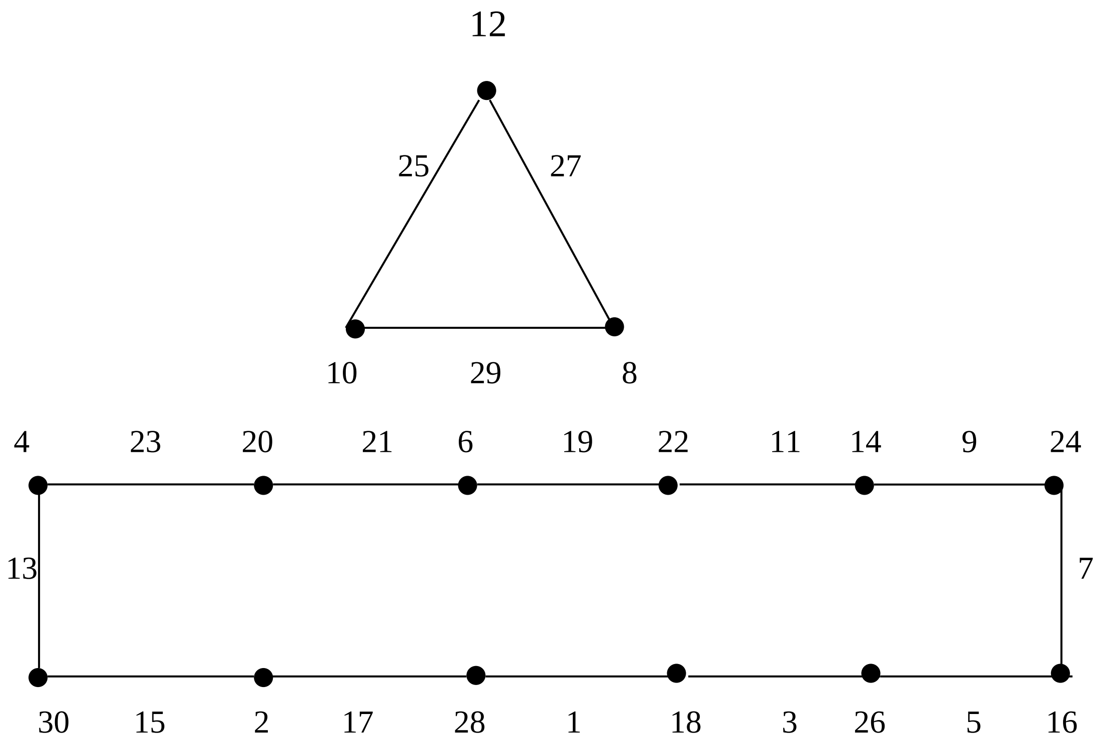

Figure 2: $C_3 \cup C_{12}, k_e = 47$

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Theorem 2.2

The graph $C_3 \cup C_{4r+2}$, $r \geq 1$ is an Odd edge magic total graph.

Proof:

For $r = 1$,the labeling the vertices and edges of C_3 by $\lambda(v_i) = 4{,}8{,}10$ for $i = 1{,}2{,}3$.

$\lambda(e_i) = 17{,}11{,}15$ for $i = 1{,}2{,}3$.

Labelling the vertices and edges of C_6 by $\lambda(v_i) = 2{,}14{,}12{,}16{,}6{,}18$ for $i = 1{,}2, \dots 6$ and $\lambda(e_i) = 13{,}3{,}1{,}7{,}5{,}9$ for $i = 1{,}2, \dots 6$ (magic constant $k_e = 29$).

For $r = 2$,the labeling the vertices and edges of C_3 by $\lambda(v_i) = 6{,}10{,}12$ for $i = 1{,}2{,}3$.

$\lambda(e_i) = 25{,}19{,}23$ for $i = 1{,}2{,}3$.

Labeling the vertices and edges of C_{10} by $\lambda(v_i) = 2{,}18{,}16{,}20{,}4{,}22{,}8{,}24{,}14{,}26$ for $i = 1{,}2, \dots 10$ and $\lambda(e_i) = 21{,}7{,}5{,}17{,}15{,}11{,}9{,}3{,}13$ for $i = 1{,}2, \dots 10$ (magic constant $k_e = 41$).

For $r \geq 3$, labeling the vertices and edges of C_3 by $2r + 2$, $2r + 4$,$2r + 6$ and $8r + 3$, $8r + 5{,}8r + 1$.

Labeling the vertices and edges of C_{4r+2} by

$$\lambda(v_i) = \begin{cases} 2\,, i = 1 \\ 4r + 8, i = 3 \\ i - 1, i\ odd\ 5 \leq j \leq 2r + 1 \\ 2r + 4, i\ odd, i = 2r + 3 \\ i + 5, i\ odd, 2r + 5 \leq i \leq 4r + 1 \\ i + 4r + 8, i\ even \end{cases}$$

$$\lambda(e_i) = \begin{cases} 8r + 5\,, i = 1 \\ 4r - 1, i = 2 \\ 4r - 3, i = 3 \\ 8r + 9 - 2j, 4 \leq i \leq 2r + 1 \\ 4r + 3, i = 2r + 2 \\ 4r + 1, i = 2r + 3 \\ 8r + 3 - 2i, 2r + 4 \leq i \leq 4r + 1 \\ 4r + 5, j = 4r + 2 \end{cases}$$

Therefore, the graph $C_3 \cup C_{4r+2}$, $r \geq 1$ is an Odd edge magic total graph.

Example 3

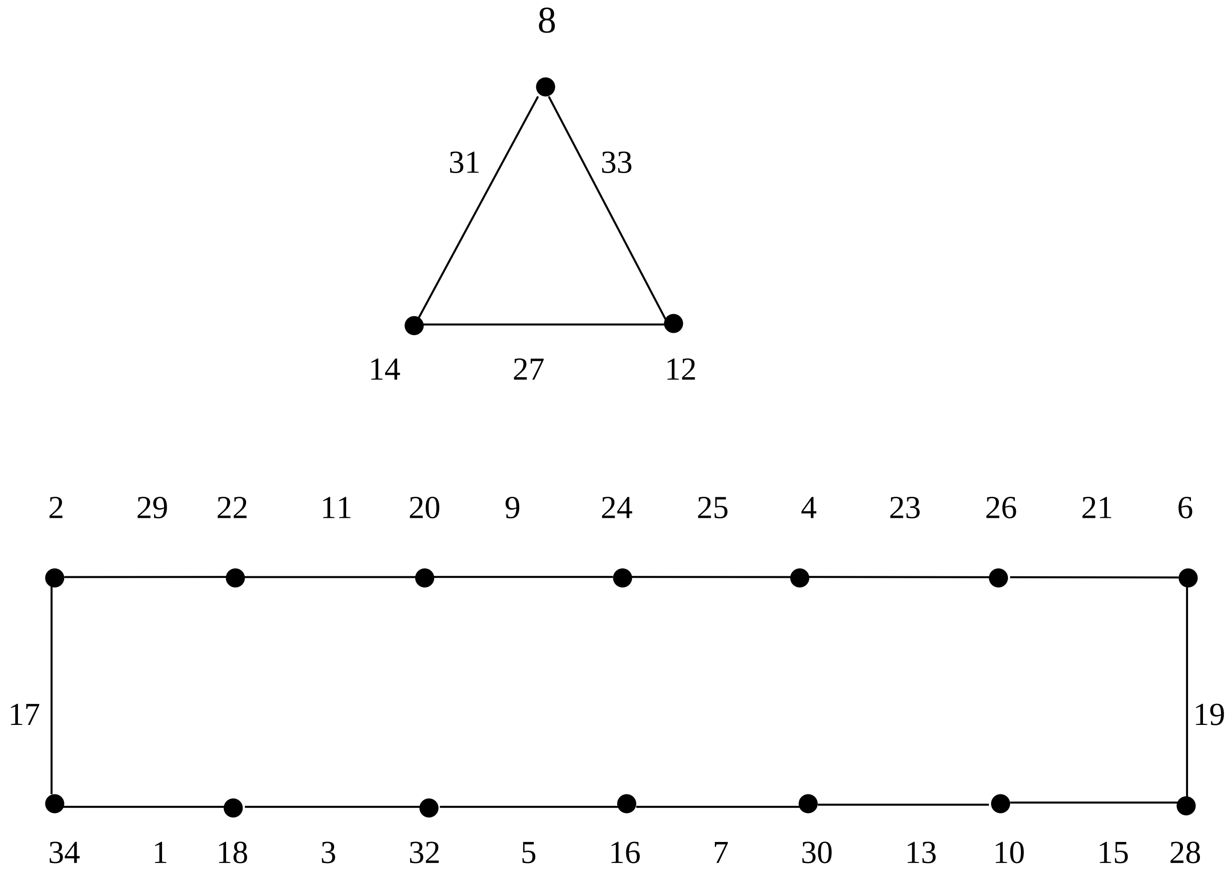

Figure 3: $C_3 \cup C_{12}, k_e = 53$

Theorem 2.3

The graph $C_4 \cup C_{4r-1}$, $r > 1$ is an Odd edge magic total graph.

Proof:

Labeling the vertices and edges of C_4 by $4r + 4, 8r + 4, 4r, 8r + 6$ and 3,7,5,1.

Labeling the vertices and edges of C_{4r-1} by

$$\lambda(v_i) = \begin{cases} 2r\ , i = 2r - 3 \\ r, i = r + 1 \\ 3r, i = 2r + 1 \\ 5r, i = 3r + 1 \\ 6r + 2, i = r \\ 9r, i = 2r \\ 8r, i = 3r \end{cases}$$

$$\lambda(e_i) = \begin{cases} 9r - i\,, i = 1,3,5 \\ 9r + 3, i = 3r + 1 \\ 9r + 1, i = r \\ 5r + 1, i = 2r \\ 3i + 3, i = 3r \\ 10r + 1, i = 4r \end{cases}$$

Therefore, the graph $C_4 \cup C_{4r-1}\,, r > 1$ is an Odd edge magic total graph.

Example 4

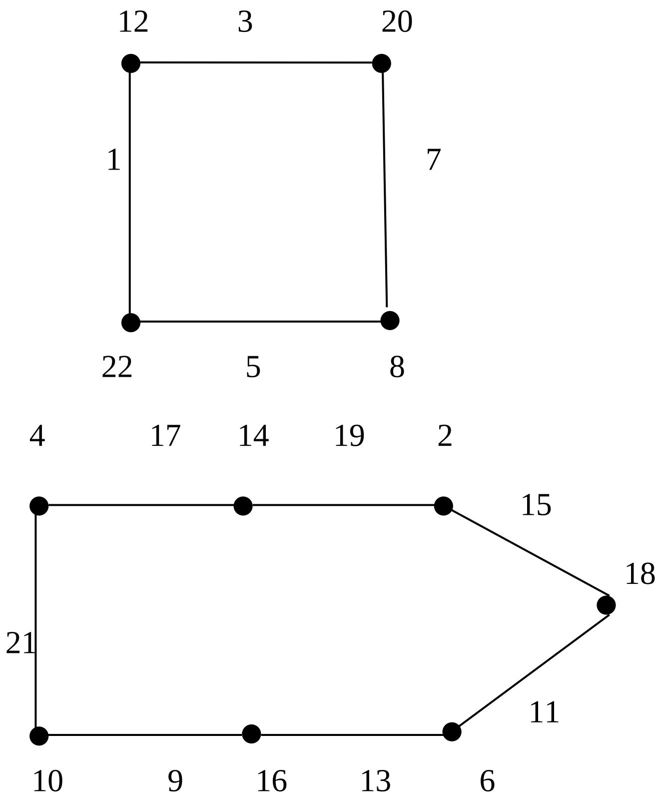

Figure 4: $C_4 \cup C_7, k_e = 35$

Theorem 2.4

The graph $C_4 \cup C_{4r-3}\,, r > 1$ is an Odd edge magic total graph.

Proof:

Labeling the vertices and edges of C_4 by $4r + 4, 8r + 4, 4r, 8r + 6$ and 3,7,5,1.

Labeling the vertices and edges of C_{4r-3} by

$$\lambda(v_i) = \begin{cases} r+1\,, i = r-2 \\ r-1, i = r-1 \\ 2r+2, i = r+2 \\ 2r, i = 2r+1 \\ 4r, i = 3r \\ 6r, i = r-1 \\ 5r+1, i = r+1 \\ 7r+1, i = 2r \\ 5r+5, i = 2r+2 \end{cases}$$

$$\lambda(e_i) = \begin{cases} 6r+1\,, i = r-2 \\ 6r+3, i = r-1 \\ 6r+5, i = r \\ 6r-1, i = r+1 \\ 3r+2, i = r+2 \\ 4r+1, i = 2r \\ 5r, i = 2r+1 \\ 3r, i = 2r+2 \\ 8r+1, i = 3r \end{cases}$$

Therefore, the graph $C_4 \cup C_{4r-3}\,, r > 1$ is an Odd edge magic total graph.

Example 5

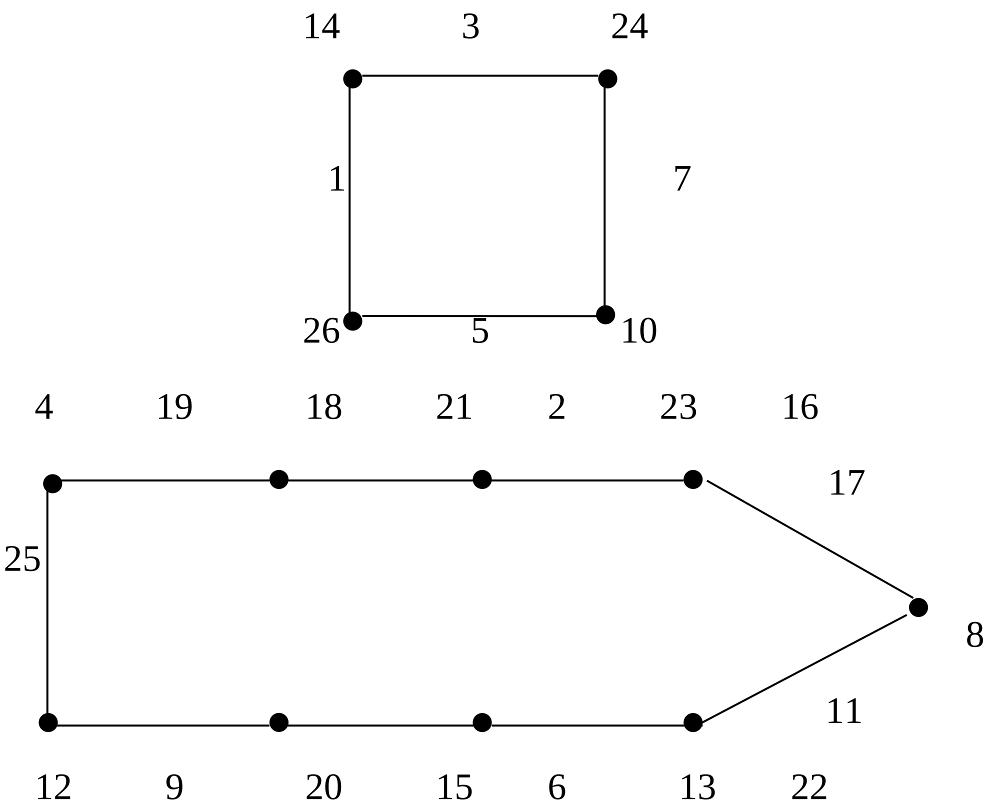

Figure 5: $C_4 \cup C_9, k_e = 41$

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3. Conclusion

In this paper we have discussed some cycles of graphs which admits Odd edge magic total labeling. In future we can prove different types of graphs which satisfy Odd edge magic total labeling.

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