ANALYSIS OF FLUID DYNAMICS MODELLING HAVING APPLICATIONS IN CIRCULATORY DISORDERS- A MATHEMATICAL STUDY

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Abstract

Fluid Mechanics riches with the different applications of Dynamical systems, having applications towards the flow patterns and flow characters. A wide variety of modelling and structural dynamics have been studied in the vicinity of research on stenosis or cardio-vascular disorders. Mainly the in-growth of tissues in the inner arterial wall causes for different fatal diseases like coronary thrombosis, Atherosclerosis. Here the flow characters are modelled with respect to the various phenomena in view of mathematical analysis. The axisymmetric flow patterns are considered with wide outline to estimate the flow patterns in the effected arteries on the basis of irregularity of plaque formation. The formation of three-dimensional mild casting is analyzed so that the problem becomes realistic in physiological approach. Here the theoretical investigation of fluid dynamical systems is based on Casson fluid model and Herschel Bulkley fluid Model in a comparative way.

Keywords: *Blood flow, Velocity, Overlapping Stenosis, Herschel Bulkley fluid Model, Casson fluid model.*

INTRODUCTION

Circulatory abnormalities continue to be a prominent cause of death among all human deadly diseases like cardio vascular disease. Many arterial illnesses include mechanical behaviour of the inner wall of the artery and blood flow characteristics, as its root causes and growth. Stenosis is the medical word for the abnormal and unnatural increase in the artery wall thickness at various inner lining of the arterial wall of the cardio-vascular system. Many cardiovascular disorders are brought on by geometric distortion in the inner artery wall, which are to responsible for the inadequate blood flow from the coronary arteries into the heart. The flow resistance in arteries is increased by high grade stenosis. The body is compelled to elevate blood pressure, and arterial constriction together with increased flow velocity, shear stress, and significantly decreased pressure at the stenosis neck all contribute to thrombus development. If the condition worsens, it could result in serious circulation problems, morbidity, or even death. Several researchers have been drawn to investigate current approaches and ever-more complex mathematical models for examination on flow through stenotic arteries owing to the important significance that haemodynamic parameters play in the development and progression of the illness. The sophistication and practical utility of computational techniques for patient-specific blood flow modelling have substantially risen over the past few years. Understanding the hemodynamic effects of simulations has been crucial for analyzing the status of the patient. (Young, et al.,1986); (Chaturani and Samy, 1985) dealt with the problems of blood flow through the arterial segments having the stenosis or multiple stenoses (regular or irregular shape) based upon the assumption that fluid representing blood is Newtonian and the stenotic geometry to be a smooth cosine function. (Srivastava & Saxena, 1994); (Chaturani & Biswas, 1984) considered the blood flow through a composite stenosis in catheterized arteries assuming that the flowing blood behaves like a Newtonian fluid. The assumption of the Newtonian behaviour of the blood is acceptable for a high shear rate flow through larger arteries.

Many experimental observations reveal that blood, being predominantly a suspension of erythrocytes in plasma behaves like a non-Newtonian fluid at low shear rates (Srivastava & Saxena, 1994). During its flow through micro vessels especially in diseased states clotting affects small arteries.

Theoretical investigations of non-Newtonian blood flow in constrictive arteries use the Herschel-Bulkley fluid model and the Casson fluid model. The impacts of an overlapping stenosis of flow behaviour of the flowing blood were shown in the mathematical analysis by Nanda and Basu Mallik (2012), which was considered as a microscopic two-phase fluid (i.e., suspension of erythrocytes in plasma). Blood was treated as H-B fluid in (Sankar and Lee, 2009) analysis of the pulsatile blood flow via stenosed narrow arteries with body acceleration.

(Sarojamma and Nagarani, 2000) investigated the Casson fluid flow in a porous medium-filled tube under periodic acceleration In a published paper by (Mandal et al., 2007), where a two-dimensional mathematical model was developed to investigate the impact of externally imposed periodic body acceleration on non-Newtonian blood flow through an elastic stenosed artery. The generalized power law model of the blood was used to describe the blood flow. The problem of blood flow through a stenosed portion of an artery, where the rheology of blood is characterized by the Herschel-Bulkley model and the Casson fluid model, is the focus of the current work. The desirability of an artery wall has been calculated using regional fluid mechanics. In order to solve the unsteady non-Newtonian blood flow with various boundary conditions in a cylindrical coordinate system, an appropriate finite difference technique will be taken into account (Misra & Chakravarty, 1986; Ponalagusamy, 2007).

The varying values of the material constants and other factors will be taken into consideration for a quantitative analysis based on numerical calculations. In relation to the blood flow velocity in the arterial segment, the variation of skin friction with axial distance and impedance in the location of the stenosis is visually displayed (Bose & Nanda, 2012a; Bose & Nanda, 2012b). It is noted that blood only complies with Casson's equation for moderate shear rates, and the Herschel-Bulkley equation largely captures the behaviour of blood. Blood behaves more like H-B fluid for tube diameters of 0.095mm than like power-law and Bingham fluids, according to Chaturani and Ponnalagar Samy, (Chaturani & Samy, 1985; Chaturani & Palanisamy, 1990). (Iida, 1978) reports "That velocity profile in the arterioles having diameter less than 0.1mm are generally explained fairly by the two models. However, velocity profiles in the arterioles whose diameters are less than 0.065mm does not conform to the Casson model but can still be explained by H-B fluid model".

Based on experimental findings, (Whitemore, 1968) hypothesized that blood, which is primarily a suspension of erythrocytes in plasma, exhibits remarkable non-Newtonian behaviour when it flows through small, low-shear arteries, especially in diseased states where small artery clotting effects are present. Blood flow via small arteries is theoretically investigated using the H-B fluid model and Casson fluid models. Studies on blood with varied hematocrit, anticoagulants, temperature, etc. indicate that the Casson model can better capture the behaviour of blood at low shear rates (Cokelet, et al., 1963; Merrill, et al., 1965; Blair, 1959). It is widely known that the gradient in the heart's pulse pressure causes the blood to flow through arteries to be extremely pulsatile. The suggested inquiry examines the periodic body acceleration-induced pulsatile flow of blood while treating it as a Casson fluid. An analysis of blood acceleration with account of velocity slip at the stenosed vessel wall was presented by Biswas and Chakraborty in 2009. (Biswas & Chakraborty, 2009). A Newtonian fluid has been used to represent blood. (Verma et al., 2011) studied the effect of body acceleration on the pulsatile flow of blood through some kind of stenosed artery while taking the Casson model for blood under the no slip condition into consideration.

When the Casson fluid model is used to represent the rheology of blood, it opens up the possibility of thinking about the issue of blood flow via a stenotic portion of an artery while subjected to body acceleration. Moreover, the flow variations can be measured when the study takes slip velocity into account. The problem's analytical solution was obtained using the perturbation approach. The present investigation takes into account the shape of the stenosis that will appear in the arterial segment. By creating computer codes, large-scale numerical simulations of the measured flow variables with greater physiological significance are performed as part of an exhaustive quantitative analysis. Towards the conclusion of the publication, their graphical depictions are shown along with the pertinent scientific analysis. In order to support the usefulness of the current model, comparisons are then done utilizing the various existing results.

MATHEMATICAL FORMULATION OF THE PROBLEM

1. Herschel–Bulkley fluid model:

Blood is assumed to be represented by a non-Newtonian fluid and considering the axisymmetric, Laminar, fully developed and steady one-dimensional flow of blood in an artery, the governing equation of motion of blood flow, under the conditions may be stated as

$$-\frac{dp}{dz}=\frac{1}{r}\frac{d(r\tau)}{dr} \tag{1.1}$$

in which τ represents the shear stress of blood considered as Herschel–Bulkley fluid and p the pressure at any point. The constitutive equation may be put as

$$-\frac{du}{dr}=f(\tau)=\frac{1}{k}(\tau-\tau_H)^n;\tau\geq\tau_H \tag{1.2}$$

$$=0 \qquad ;\tau\leq\tau_H$$

where u stands for the axial velocity of blood and τ_H is the yield shear stress and k, n are parameters which represent non-Newtonian effects.

Let us consider a bell-shaped stenosis geometry given by

$$R(z)=R_0[1-\frac{\delta}{R_0}\exp(-\frac{m^2\varepsilon^2z^2}{R_0{}^2})] \tag{1.3}$$

where R_0 stands for the radius of the arterial segment outside the stenosis, R (z) is the radius of the stenosed portion of the arterial segment under consideration at a longitudinal distance z from the left-end of the segment; δ is the depth of the stenosis at the throat and m is a parametric constant; ε characterizes the relative length of the constriction, defined as the ratio of the radius to half-length of stenosis, i.e. $\varepsilon=\frac{R}{L_0}$.

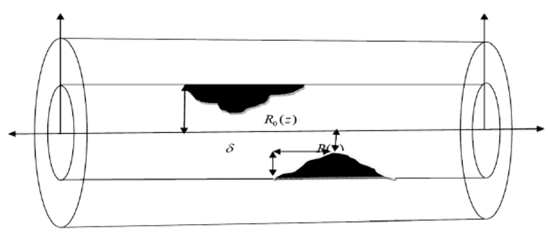

Figure: 1 (Picture of arterial stenosed segments & stenosis throat)

Considering the stenosis geometry to be of the form (cf. Fig. 1)

$$\frac{R(z)}{R_0}=1-ae^{-bz^2} \tag{1.4}$$

with $a=\frac{\delta}{R_0}$ and $b=\frac{m^2\varepsilon^2}{R_0{}^2}$ equations (1.1) and (1.2) are to be solved subject to the boundary conditions

u=0 at r=R (z) (no slip condition) **(1.5)** τ is finite at r = 0 (regularity condition) **(1.6)**

Integrating Equation (1.1) and using (1.6) we get

$$\tau = -\frac{r}{2}\frac{dp}{dz} \quad \textbf{(1.7)}$$

The skin-friction $\tau_R = -\frac{R}{2}\frac{dp}{dz}$ where R=R (z) **(1.8)**

The volumetric flow rate Q is given by the Rabinowitsch equation

$$Q=\frac{\pi R^3}{\tau_R^3}\int_0^{\tau_R}\tau^2 f(\tau)d\tau \quad \textbf{(1.9)}$$

where τ and τ_R given by the equations (1.7) and (1.8) respectively.

Therefore substituting the value of f (τ) from equation (1.2) we get.

$$Q=\frac{\pi R^3}{\tau_R^3}\int_0^{\tau_R}\tau^2\frac{1}{k}(\tau-\tau_H)^n d\tau \quad \textbf{(1.10)}$$

(Where n=fluid index parameter)

$$=\frac{\pi R^3\tau_R^n}{k(n+3)}(1-\frac{\tau_H}{\tau_R})^{n+1}\left[1+(\frac{2}{n+2})\frac{\tau_H}{\tau_R}+\frac{2}{(n+1)(n+2)}(\frac{\tau_H}{\tau_R})^2\right] \quad \textbf{(1.11)}$$

When $(\frac{\tau_H}{\tau_R}) \leq 1$ the above equation reduces to

$$Q=\frac{\pi R^3}{k(n+3)}\left\{\tau_R-\left(\frac{n+3}{n+2}\right)\tau_H\right\}^n \quad \textbf{(1.12)}$$

Again, assuming that the flowing blood is representing a non- Newtonian fluid

$$\frac{dp}{dz}=\frac{1}{r}\frac{\partial}{\partial r}\left(r\mu\frac{\partial u}{\partial r}\right) \quad \textbf{(1.13)}$$

where u is the flow. velocity

The volumetric flow flux Q is thus calculated as

$$Q=2\pi\int_0^{R(Z)} rudr \quad \textbf{(1.14)}$$

$$Q=\pi\{R(z)\}^2 u$$

Now from (1.12) and (1.14) we have

$$Q=\frac{\pi R^3}{k(n+3)}\left[\tau_R-\left(\frac{n+3}{n+2}\right)\tau_H\right]^n=\pi\left\{R(z)\right\}^2 u \tag{1.15}$$

$$u=\frac{R^3\left\{\tau_R-\left(\frac{n+3}{n+2}\right)\tau_H\right\}^n}{k(n+3)R^2(z)} \tag{1.16}$$

Again, resistance to flow λ is defined by

$$\lambda=\frac{P_1-P_2}{Q}$$

Using $\tau_R=-\frac{R}{2}\frac{dp}{dz}$

Where R=R (z) in equation

$$Q=\frac{\pi R^3}{k(n+3)}\left[-\frac{R}{2}\frac{dp}{dz}-\left(\frac{n+3}{n+2}\right)\tau_H\right]^n \tag{1.17}$$

$$-\frac{dp}{dz}=\left\{\frac{2^n kQ(n+3)}{\pi R^{n+3}}\right\}^{\frac{1}{n}}+\frac{2(n+3)}{(n+2)}\frac{\tau_H}{R} \tag{1.18}$$

Therefore from (1.3) we have, substituting the value of $\varepsilon=\frac{R_0}{L_0}$

$$\left(R(z)/R_0\right)=\left[1-\left(\frac{\delta}{R_0}\right)e^{\left(-\frac{m^2z^2}{L_0^{\,2}}\right)}\right] \tag{1.19}$$

2. Casson fluid model:

Considering a one-dimensional pulsatile flow of blood in an artery in the presence of externally imposed periodic body acceleration with mild stenosis. We consider the flow to be axially symmetric, laminar, fully developed by considering blood as a Casson fluid. The geometry of the flow is shown in *fig.1* and is given by:

$$\bar{R}(\bar{z})=\begin{cases}\bar{R}_0-\frac{\bar{\delta}}{2}\left(1+cos\frac{\pi\bar{z}}{\bar{z}_0}\right); & for\ |\bar{z}|\le\bar{z}_0\\ \bar{R}_0; & for|\bar{z}|>\bar{z}_0\end{cases} \tag{2.1}$$

where R (z) is the radius of the obstructed artery, R_0 is the constant radius of the normal artery, L_0 is the length of the stenosis, L is the length of the artery, d is the location of the stenosis and δ is the maximum height of the stenosis. The periodic body acceleration $F(\bar{t})$ in the axial direction is given by:

$$F(\bar{t}) = a_0 \cos(\omega_b \bar{t} + \emptyset) \tag{2.2}$$

Where a_0 is the amplitude, ω_b=$2\pi. f_b$, f_b is the frequency in Hz, $\emptyset$ is the lead angle of $F(\bar{t})$. The frequency of body acceleration f_b is assumed to be small, so that wave effects can be neglected. The pressure gradient at any $\bar{z}$ is given by

$$-\frac{\partial \bar{p}}{\partial \bar{z}} = A_0 + A_1 \cos\left(\omega_p \bar{t}\right) \tag{2.3}$$

Where A_0 the steady component of pressure gradient is, A_1 is the amplitude of the fluctuating component ω_p=$2\pi. f_p$, $f_p$is the pulse frequency. The momentum equation governing the flow in cylindrical co-ordinate system is given by

$$\bar{\rho}.\frac{\partial \bar{u}}{\partial \bar{t}} = -\frac{\partial \bar{p}}{\partial \bar{z}} + \frac{1}{\bar{r}}.\frac{\partial(\bar{r}.\bar{\tau}_{\bar{r}.\bar{z}})}{\partial \bar{r}} + F(\bar{t}) \tag{2.4}$$

$$\frac{\partial \bar{p}}{\partial \bar{r}} = 0 \tag{2.5}$$

Where $\bar{r}, \bar{z}$ denotes the radial and axial co-ordinates respectively, $\bar{\rho}$ denotes density, $\bar{u}$ is the axial velocity of blood, $\bar{t}$ is time, $\bar{p}$ is pressure and $\bar{\tau}$ the shear stress.
For Casson fluid the relation between shear stress and shear rate is given by

$$\sqrt{\bar{\tau}} = \sqrt{\bar{\tau}_y} + \sqrt{\mu.\left(-\frac{\partial \bar{u}}{\partial \bar{r}}\right)} \quad ; \qquad if\, \bar{\tau} \geq \bar{\tau}_y, \frac{\partial \bar{u}}{\partial \bar{r}} = 0 \tag{2.6}$$

Where $\bar{\tau}$ denotes yield stress and $\bar{\mu}$ denotes the viscosity of blood. The boundary conditions are:

$$\bar{u} = \bar{u}_s \quad \text{at} \quad \bar{r} = \bar{R}(\bar{z}) \tag{2.7}$$

$$\text{And} \quad \bar{\tau} \quad \text{is finite at} \quad \bar{r} = 0 \tag{2.8}$$

Where $\bar{u}_s$ is the slip velocity at the stenotic wall.

Introducing the non-dimensional variables: $u = \frac{\bar{u}}{A_0 R_0^2 / 4\mu}$; $z = \frac{\bar{z}}{R_0}$; $z_0 = \frac{\bar{z}_0}{R_0}$; $t = \omega_p \bar{t}$; $\delta = \frac{\bar{\delta}}{R_0}$; $\tau = \frac{\bar{\tau}}{A_0 R_0 / 2}$; $\theta = \frac{\bar{\tau}_y}{A_0 R_0 / 2}$;

$$R(z) = \frac{\bar{R}(\bar{z})}{R_0}; \quad r = \frac{\bar{r}}{R_0}; \quad e = \frac{A_1}{A_0}; \quad B = \frac{a_0}{A_0}; \quad \omega = \frac{\omega_b}{\omega_p}; \quad u_s = \frac{\bar{u}_s}{A_0 R_0^2 / 4\mu} \tag{2.9}$$

The non-dimensional equation (4) becomes:

$$\alpha^2 . \frac{\partial u}{\partial t} = 4(1 + e\cos(t)) + 4B\cos(\omega t + \emptyset) + \frac{2}{r}\frac{\partial}{\partial r}(r\tau_{rz}) \tag{2.10}$$

Where$\alpha^2 = \frac{\omega_p R_0^2}{\mu / \rho}$, α is called Womersley frequency parameter.

Equation (2.6) changes to:

$$\sqrt{\tau} = \sqrt{\theta} + \frac{1}{\sqrt{2}}\sqrt{-\frac{\partial u}{\partial r}}; \quad if \tau \geq \theta, \ and \quad \frac{\partial u}{\partial r} = 0; \quad if \quad \tau \leq \theta \tag{2.11}$$

The boundary conditions (equations (2.7), (2.8)) reduce to

$$\mathrm{u = u_s \quad at \quad r = R(z)} \tag{2.12}$$
$$\mathrm{And}\ \tau\ \mathrm{is\ \ finite\ \ at\ \ r = 0} \tag{2.13}$$

The geometry of the stenosis in the non-dimensional form is given by:

$$R(z) = \begin{cases} 1 - \frac{\delta}{2}\left(1 + cos\frac{\pi z}{z_0}\right); & for\ |z| \leq z_0 \\ 1; & for |z| > z_0 \end{cases} \tag{2.14}$$

On using perturbation method, the velocity u, and shear stress, τ are expressed in terms of Womersley parameter, α^2 (where $\alpha \ll 1$)

$$u(z, r, t) = u_0(z, r, t) + \alpha^2 u_1(z, r, t) + .. \ .. \ .. \tag{2.15}$$

$$\tau(z, r, t) = \tau_0(z, r, t) + \alpha^2 \tau_1(z, r, t) + .. \ .. \ .. \tag{2.16}$$

Substituting (2.15) and (2.16) in equation (2.10) and equating the constant terms and α^2, we get:

$$\frac{\partial}{\partial r}(r\tau_0) = -2r[(1 + e\cos t) + B\cos(\omega t + \emptyset)] \tag{2.17}$$

$$\frac{\partial u_0}{\partial t} = \frac{2}{r} . \frac{\partial}{\partial r}(r\tau_1) \tag{2.18}$$

Integrating equation (2.17) and using boundary condition (13), we get: $\tau_0 = -f(t) . r$ (2.19)

where,$f(t) = [(1 + e\cos t) + B\cos(\omega t + \emptyset)]$ (2.20)

Substituting u from equation (2.15) into condition (2.12), we get

$$u_0 = u_s, u_1 = 0 \ \ at\, r = R(z) \tag{2.21}$$

Substituting (15) and (16) in (11), we get

$$-\frac{\partial u_0}{\partial r} = 2[\theta + |\tau| - 2.\sqrt{\theta\tau_0}] \tag{2.22}$$

$$-\frac{\partial u_1}{\partial r} = 2|\tau_1|.\left[1 - \sqrt{\theta/\tau_0}\right] \tag{2.23}$$

Integrating equation (2.22), and using the relation (2.19) and the boundary condition (2.21), we obtain

$$u_0 = f(t)R^2\left[1 - \left(\frac{r}{R}\right)^2 - \frac{8}{3}\frac{k}{\sqrt{R}}\left\{1 - \left(\frac{r}{R}\right)^{3/2}\right\}\frac{2.k^2}{R}\left\{1 - \left(\frac{r}{R}\right)\right\}\right] + u_s \tag{2.24}$$

Where $k^2 = \theta/_{f(t)}$.

Similarly, the solutions for τ_1 and u_1 can be obtained using equations (2.18), (2.23), and (2.24) as:

$$\tau_1 = \frac{f'(t).R^3}{8}\left[22\left(\frac{r}{R}\right) - \left(\frac{r}{R}\right)^3 - \frac{8}{21}\frac{k}{\sqrt{R}}\left\{7\left(\frac{r}{R}\right) - 4\left(\frac{r}{R}\right)^{5/2}\right\}\right] \tag{2.25}$$

$$u_1 = \frac{f'(t)R^4}{16}\left[\left(\frac{r}{R}\right)^4 + 4\left(\frac{r}{R}\right)^2 + 3 + \frac{k}{\sqrt{R}}\left\{\frac{16}{3}\left(\frac{r}{R}\right)^2 - \frac{424}{147}\left(\frac{r}{R}\right)^{7/2} + \frac{16}{3}\left(\frac{r}{R}\right)^{3/2} - \frac{1144}{147}\right\} + \frac{k^2}{R}\left\{\frac{128}{63}\left(\frac{r}{R}\right)^3 - \frac{64}{9}\left(\frac{r}{R}\right)^{3/2} + \frac{320}{63}\right\}\right] \tag{2.26}$$

Using equations (2.15) and (2.16), the total velocity distribution can be written as

$$u = f(t).R^2\left[1 - \left(\frac{r}{R}\right)^2 - \frac{8}{3}\frac{k}{\sqrt{R}}\left\{1 - \left(\frac{r}{R}\right)^{3/2}\right\} + \frac{2k^2}{R}\left\{1 - \frac{r}{R}\right\} + \frac{\alpha^2R^2C}{16}\left\{\left(\frac{r}{R}\right)^4 - 4\left(\frac{r}{R}\right)^2 + 3 \quad + \frac{k}{\sqrt{R}}\left\{\frac{16}{3}\left(\frac{r}{R}\right)^2 - \frac{424}{147}\left(\frac{r}{R}\right)^{7/2} + \frac{16}{3}\left(\frac{r}{R}\right)^{3/2} - \frac{1144}{147}\right\} + \frac{k^2}{R}\left\{\frac{128}{63}\left(\frac{r}{R}\right)^3 - \frac{64}{9}\left(\frac{r}{R}\right)^{3/2} + \frac{320}{63}\right\}\right\}\right] + u_s \tag{2.27}$$

The volumetric flow rate flow rate$Q$ is given by:

$$Q(z,t) = 4\int_0^{R(z)} r.u(z,r,t)dr$$

Where $Q(z,t) = \frac{\bar{Q}(\bar{z},\bar{t})}{\pi \bar{R}_0 A_0 /_{8\bar{\mu}}}$

$$= f(t)R^4\left[\frac{1}{4}+\frac{4}{7}\frac{k}{\sqrt{R}}+\frac{1}{3}\left(\frac{k}{\sqrt{R}}\right)^2+\frac{\alpha^2R^2C}{16}\left\{\frac{2}{3}+\frac{120}{177}\frac{k}{\sqrt{R}}+\frac{32}{35}\left(\frac{k}{\sqrt{R}}\right)^2\right\}\right]+2u_sR^2 \quad (2.28)$$

NUMERICAL RESULTS AND DISCUSSION:

It is vital to assess the analytical results produced for dimensionless shear stress to flow(τ) in order to have an assessment of the quantitative impact of the different characteristics included in the analysis. It is based on area-axial average velocity of flow of H-B fluid model on constant tube diameter, where the constitutive co-efficient m=0.1260 g/cm .s in Power law fluid model (Razavi, et al., 2006). The value of the power law index 'n' for blood flow problems are generally taken to lie between 0.9 and 1.1 (Sankar, & Lee, 2009) and in this analysis, we have used the value 0.95 for n < 1 and 1.05 for n > 1. u=(0.5,2.5,4.5,6.5,8.5), =(0.1,0.2,0.3,0.4,0.5), =0.05 then k=3 ,and when =0.10 then k=4, because viscosity of blood at 370 C is (3-4) × Pa.S. (Sankar, 2010).

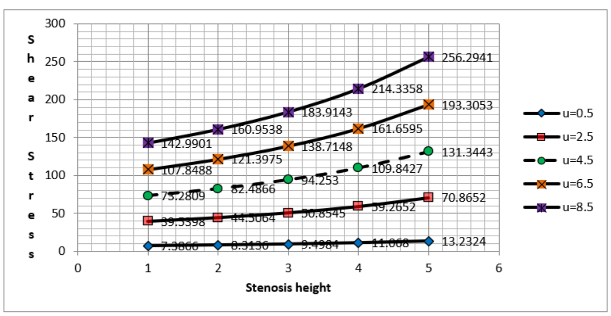

Figure .2: Variation of shear stress w.r.to stenosis height and axial velocity τ_H = 0.05 and k=3

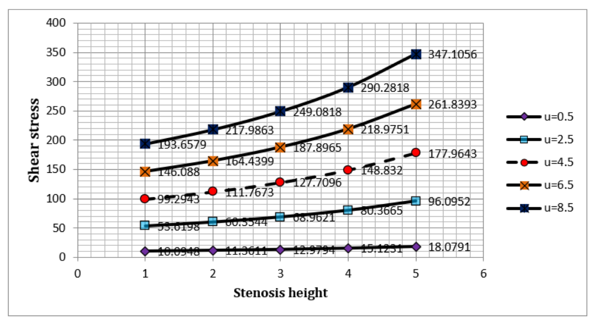

Figure .3: Variation of shear stress w.r.to stenosis height and Axial average velocity when τ_H=0.10 then k=4

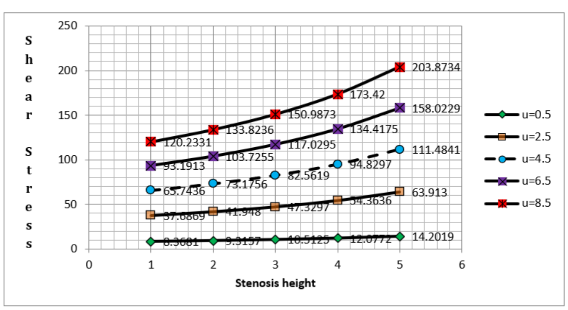

Figure .4: Variation of shear stress w.r.to stenosis height and axial average velocity τ_H = 0.05 and k=3, n=1.05

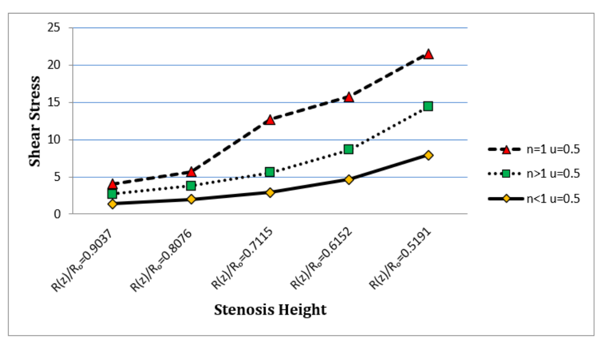

Figure .5: Variation of shear stress w.r.to stenosis height and average velocity in different flow models viz; Newtonian and Non-Newtonian depending on different fluid index n.

As the stenosis height increases, the shear stress increases for any set of values of the axial velocity (u). As u increases, the shear stress decreases sharply for any fixed value of $\frac{\delta}{R_0}$ .Figure 2 exhibits the results for shear stress with stenosis height for different values of axial velocity u for τ_H =0.05 and n = 0.95 (< 1). For higher values of u the trend is almost linear. The result is physically significant because stenosis height will enhance shear stress in the stenotic region. The increase of axial velocity in the stenotic region will naturally assume lower magnitude for higher stenosis height. The result is consistent with the observation of (Verma *et. al.*, 2011). Figure 3 and Figure 4 exhibits the higher values of n and τ_H of steaming blood and stenosis height with average axial velocity (when n=1.05). Using the experimental data that is currently accessible, the pertinent computational work has been carried out for some specific scenarios in order to determine the quantitative effects of the various factors involved in this investigation. By using the non-Newtonian (Casson fluid model) for blood, this numerical calculation aims to highlight the impacts of periodic body acceleration, slip velocity, and stenotic height on the pulsatile flow of blood via stenosed arteries. On using perturbation method, the velocity u is expanded in terms of the Womersley frequency parameter α^2 (where α^2<<1) (Womersley, 1955). The assumption of the small value of α is valid for physiological situations in small blood vessels. In most of the earlier investigations, the boundary condition is no-slip condition (velocity continuity). In this investigation two values of slip velocity u_s = 0(no slip) and u_s = 0.05 are taken. As velocity profiles play an important role in the flow field so the results for the axial velocity profiles of the streaming blood are studied under slip (u_s = 0.05) and no slip condition (u_s = 0) and body acceleration parameter B = 0, 1, 2. Fig.6 and Fig.7 illustrates that the axial velocity increase with radial distance and attains its maximum value at the axis (r = 0) and the minimum at the boundary (r = R (z)). In the presence of body acceleration, velocity increases rapidly. As the body acceleration increases, the plug region shrinks so more flow takes place. Fig.8 illustrates that the axial velocity increase with radial distance and attains its maximum value at the axis (r = 0) and the minimum at the boundary (r = R (z)). It also shows the variations of the axial velocity with B, δ and θ.Fig.9, Fig.10 and Fig.11 illustrates that the axial velocity increase with radial distance and attains its maximum value at the axis (r = 0) and the minimum at the boundary (r = R (z)). The axial velocity increases with the decrease in δ, the maximum height of the stenosis.

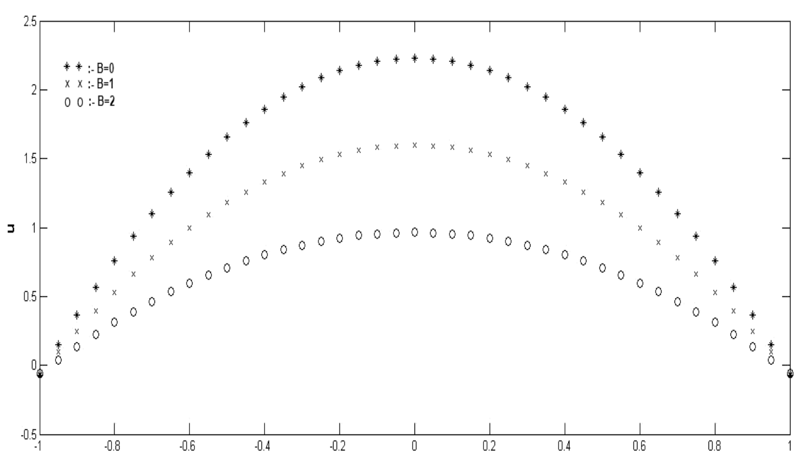

Figure 3.6 Axial velocity increase with radial distance at r=0

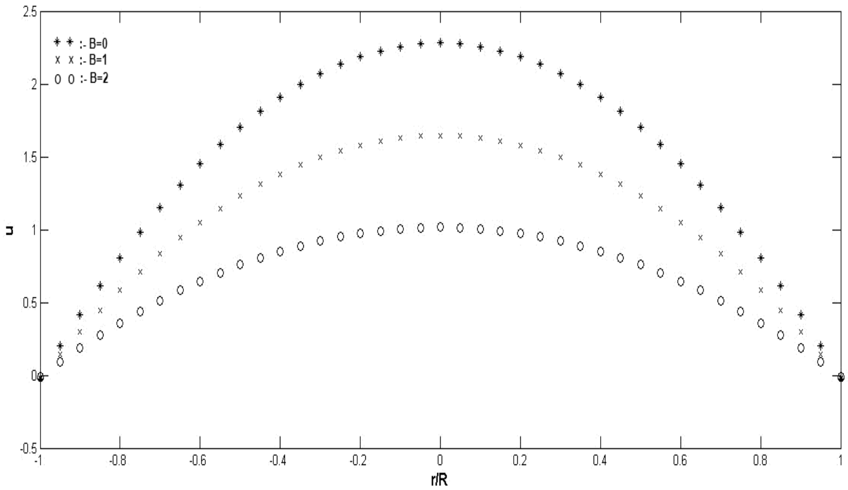

Figure 3.7 Axial velocity increase with radial distance at r=R(z)

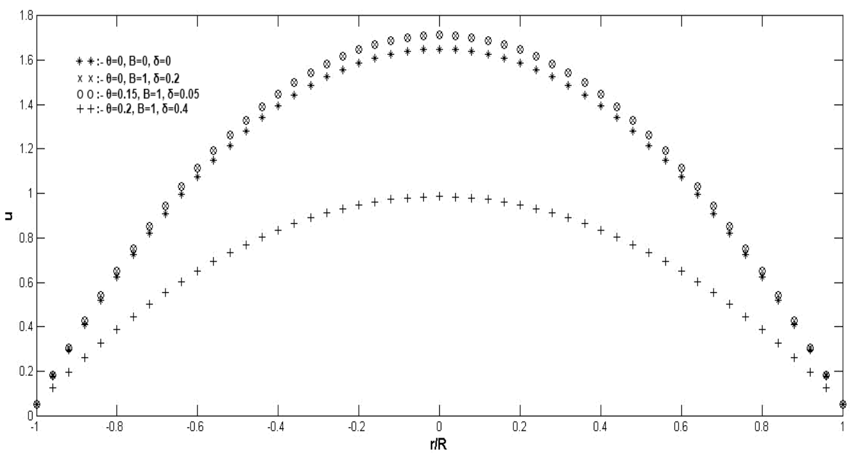

Figure 3.8. variations of the axial velocity with B and θ

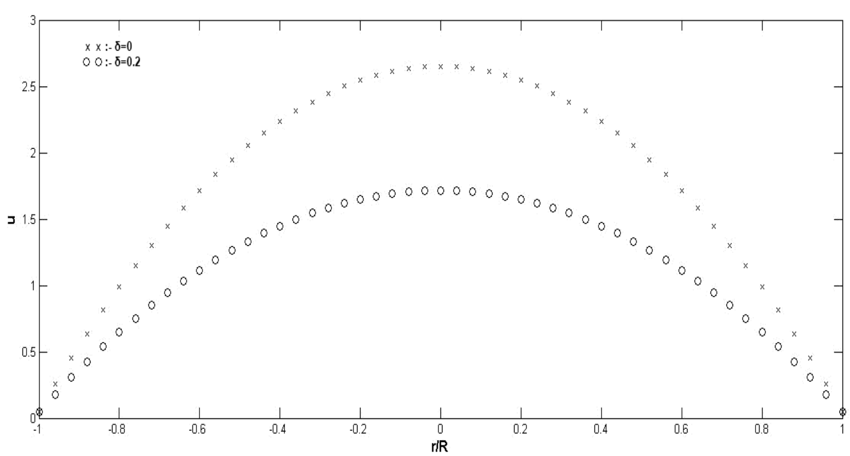

Figure 3.9. variations of the axial velocity with δ

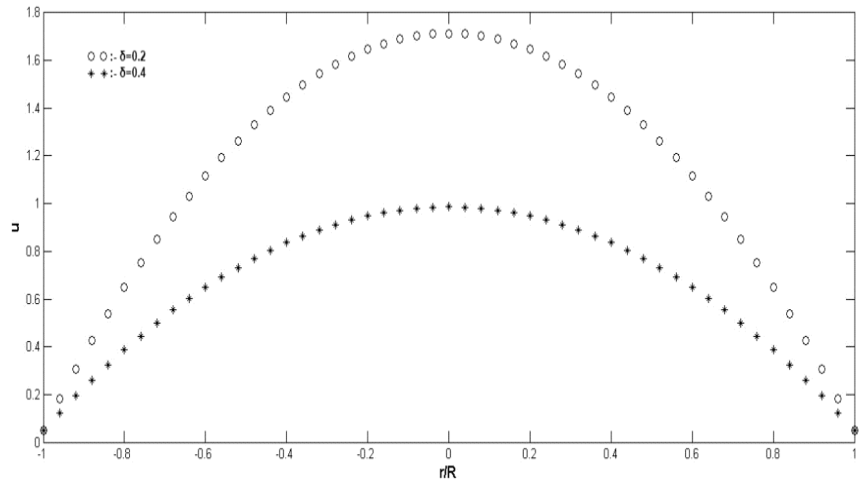

Figure 3.10. Axial velocity increase with radial distance and attains its maximum value at the axis (r = 0)

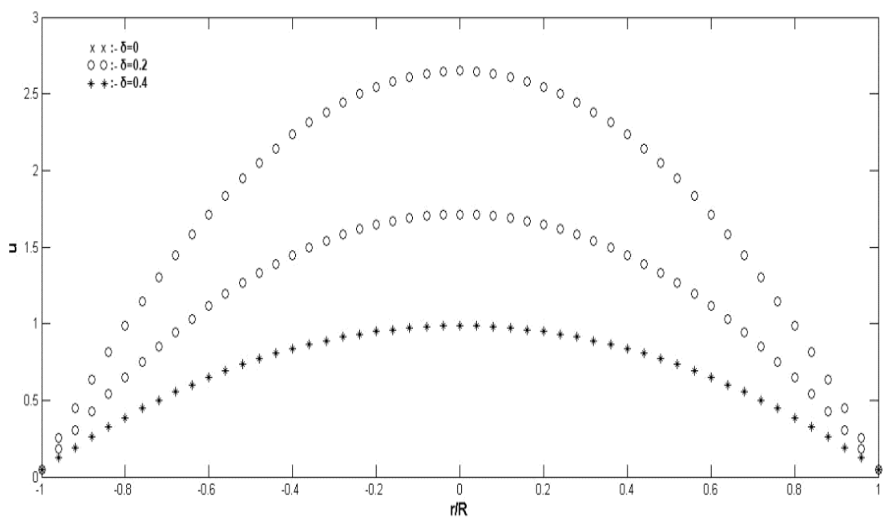

Figure 3.11 Axial velocity increase with radial distance and attains its minimum at the boundary (r = R (z))

CONCLUSION

The flow of blood through narrow stenosed arterial segments with periodic body acceleration has been studied in this analysis, treating blood as a Casson fluid model and flow of blood with multiple stenoses have been studied by modeling blood as a Herschel – Bulkley fluid. The numerical simulation shows that the rheological parameters, height of stenosis and yield stress of the fluid strongly influence the flow characteristics qualitatively and quantitatively. It is also observed in the present investigation that the body acceleration parameter, radius of stenosis, the slip velocity and the Casson fluid parameters are the strong parameters influencing the flow with due consideration of axial velocity slip at the constricted wall. So, this study is more useful for the purpose of simulation and validation of different models in different conditions of arteriosclerosis. The presence of body acceleration is to increase the flow rate but reduce the flow resistance. It is interesting to note that the model developed in the paper will throw light on the influence of various parameters on flow characteristics and to ascertain which of the parameters has the most dominating role.

Also, it is hoped that the analytical study will help the physicians in estimating the severity of stenosis and its consequence. Thus, the models developed in this paper will throw light on the clinical treatment of the obstruction of fluid movement due to formulation of multiple stenoses in the arterial system and may reduce some of the major complications for the development of ischemia and coronary thrombosis.

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