ANALYSIS OF LENGTH OF STAY OF PATIENTS IN EMERGENCY CARE UNIT OF FEDERAL MEDICAL CENTRE, UMUAHIA ABIA STATE.

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ABSTRACT

The study entitled "Analysis of length of stay of patients in the emergency care unit, Federal Medical Centre, Umuahia (a case study of Crowther ward)" was conducted to access the probability distribution that best fits the length of stay of patients in the ward. The data used in this work was secondary data collected from the Crowther ward of the Federal Medical Centre, Umuahia. The data comprises 100 patients with their age, date of admission, diagnosis, date of discharge, disposal and number of days in the ward. The number of days (length of stay) a patient stayed in the ward followed best the Exponential 2 parameter distribution. We estimated the expected length of stay of patients who entered the ward to be 18 days on average. We found the age and type of disease to be some factors that influenced the length of stay of patients in the Crowther ward. Aged patients (patients of the dependent population) and patients diagnosed with fractures stayed much longer than other patients in the ward. The study therefore recommends that the management of the Crowther ward, Federal Medical Centre, Umuahia, should provide more bed space and quality and timely services to patients. Government, churches and other social organizations should educate and sensitize their members on the implications of road accidents, which will help reduce the rate at which we admit patients into the Crowther ward and also the length of stay of patients in the ward.

Keywords: *Length of stay, Prediction, Probability Distribution, Easyfit, Fracture*

INTRODUCTION

Length of stay (LOS) is the time between hospital admission and discharge. It is an essential factor for hospitals. LOS impacts hospital management and hospital care functions. Because each hospital has a few beds, staff, and care services in each ward, predicting LOS is critical to provide the best hospital services to inpatients. LOS influences the number of hospital employees, beds, treatment outcomes, hospitalization fees, and hospital resource management (Baek *et al*., 2018). Hospitals and health care providers prefer shorter LOS to longer LOS. Decreasing LOS can decrease medication side effects, risk of hospital infection, and hospitalization costs for both patients and hospitals (Marfil-Garza *et al*., 2018). LOS can indicate difficult-to-measure research outcomes such as mortality and disease severity (LaFaro *et al*., 2015). In terms of patient flow, LOS is critical. Patient flow represents how a patient moves through a sequence of procedures (Awad *et al*., 2017; Chaou *et al*., 2017). LOS reduction and the conviction that each patient receives the appropriate care at the appropriate time have an impact on service quality, patient outcomes, and care costs. The payment pressure on governments and health institutions is directly impacted by LOS reduction (Awad *et al*., 2017).

The lengths at which patients who are accident victims stay in the hospital calls for immediate intervention. Patients of varying age groups suffering from fractures or related ailments are currently being treated at the Crowther ward of the Federal Medical Centre in Umuahia. Length of stay (LOS) is a readily available metric that shows the level of hospital activity. It is used for multiple purposes such as managing hospital care, controlling quality, determining the appropriateness of hospital usage, and planning for the hospital's future (Siamisang *et al.*, 2022). According to Ahmed *et al.* (2020), the length of stay of patients in a hospital is the difference between the time of admission and discharge of a patient into a hospital. The time patients spend in a hospital has a large impact on overall health system costs. Shorter stays are more efficient from a hospital's perspective because beds are provided more quickly to provide care for other patients. The most common causes of hospitalization in the Crowther ward are car accidents trauma, etc. Opportunities exist to increase efficiency by reducing the length of hospital stays, provided patients are not put at risk by being sent home too soon.

A road traffic accident is one factor that influences the admission of patients to the emergency care unit. A road traffic accident occurs when a vehicle collides with another vehicle, pedestrian, animal, road debris, or other stationary obstruction, such as a tree or utility pole. People are injured in road accidents every day, more so in developing countries like Nigeria. Worldwide, road traffic accidents lead to death and disability and financial cost to both society and the individual involved (Baek *et al.*, 2018). There is generally increasing incidence, morbidity, and mortality rates of road traffic accidents. The problem is that the enormity of the problem is not appreciated, and enough preventive measures are not taken. Most patients in emergency care are accident victims and most stay longer than patients that are not accident victims, after which some may even die.

A bone may be completely fractured or partially fractured in many ways; crosswise, lengthwise, in multiple pieces, due to motor accidents (road traffic accidents), falls from height, etc. and it takes a lot of days even months for the bone to heal (Puozaa, 2016). A fracture occurs when the physical force exerted on the bone is stronger than the bone itself. The length of stay of patients with fractures is higher in aged people because, as one age, the bone becomes more brittle, and it takes longer time for such bones to heal (Bashkin *et al*., 2016). The probability distribution of the length of stay of patients is a function that assigns a probability to the number of days a patient spends in the hospital. It tells how the length of stay is distributed.

Mohammed *et al*. (2023) in their study found that older patients, middle triage, and hospitalization were all associated with longer LOS. Service-related factors, such as the complexity of care provided, initial ward designation, and ward transfer, had a significant impact as well. Finally, prolonged LOS was associated with a higher ratio of patients per medical doctor and nurse. In contrast, a higher number of residents in the ED were associated with longer patient

LOS. Fekadu *et al*. (2022) found that a significant number of patients stay longer in the Emergency Department. According to the findings of Puozaa (2016), LOS increases with the following factors: age, household size, availability of formal medical care facilities, and the severity of illness. However, there is an inverse relationship between LOS and the cost of care, being a female, resource endowment in the area, and utilization of preventive care. People in lower and higher socioeconomic brackets have higher LOS than people in the middle socioeconomic bracket.

In the Crowther ward of the Federal Medical Centre, Umuahia, demand for hospital beds has increased because of an increase in the population of patients, an increased number of elderly patients, increased provision of medical services, and inadequate medical officers for rescheduling of patients. As a result, the hospital faces pressures to match this demand with available hospital beds to get patients into hospital beds promptly. The lack of spare bed capacity has led to overcrowding in hospitals and congestion in emergency care units with resultant inefficiencies in service delivery, length of stays that may be longer than necessary and greater risk to patients of medical error, avoidable death and complications. But this problem has been so challenging that the increase in demand for bed spaces has taken place in finite capacity health systems, where bed stocks and staff resources have remained almost fairly constant. Also, the increasing rate at which aged patients and patients with different illnesses stay in the Crowther ward has become alarming.

Therefore, given these problems, the researcher seeks to conduct a study on the analysis of the length of stay of patients in the emergency care unit of Federal Medical Centre, Umuahia (a case study of Crowther ward) to evaluate the efficiency of the management system of the ward. The objectives of this study are as follows.

1) To carry out the Chi-Square goodness of fit test and find the suitable probability distribution to be fitted to the length of stay of patients in the Crowther ward.

2) To use the probability density function f(x) of the found out probability distribution to determine the expected number of days on average that patients would stay in the ward

3) To determine whether the length of stay of patients in the Crowther ward depends on such factors as age and type of disease.

LITERATURE REVIEW

Review on Length of Stay of Patients in the Hospital

Length of stay (LOS) is a term commonly used to measure the duration of a single episode of hospitalization. Length of stay at a hospital (the difference between discharge and date of admission into a hospital) has some important features as a measure. It is simple to calculate and easy to understand for everyone (Bashkin *et al*, 2015). This simple measure has implications for different stakeholders in the medical care ecosystem. For a patient, the more they stay in a hospital, the more expensive it could be. The stay in a hospital not only covers the hospitalization charges, medicines and other associated costs like investigations etc., but also room and nursing charges. If an insurance scheme covers the patient, the insurer bears the expenditure (government or private insurance). For a hospital, shorter stays lead to a release of capacity in terms of beds so that more patients will be admitted. Though longer stays will increase revenues, hospitals try to discharge or move the patients quickly to other hospitals to release capacity (Vegting *et al.*, 2015). Hospitals need to balance these diversified objectives while laying down their management policies.

Sisko *et al.* (2010) analyzed the relationship between high hospital occupancy, increased nurse staffing levels, weekend admissions, and seasonal influenza, with a predicted probability of death versus the actual number of deaths. The study found that each of the factors had a statistically significant association with in-hospital mortality. Of the four factors, high hospital occupancy had the third greatest influence on patient mortality, leading to an increase in 0.24% points of actual death versus predicted death (95% CI 0.06-0.43). Because it is difficult to find out when the patient's illness ultimately becomes fatal, (Sisko *et al.* 2010) looked at hospital occupancy on the day of admission. It may have been several days after admission, however, with a different level of hospital occupancy, that the patient became fatally ill. A better understanding of the consequences of high hospital occupancy on the occurrence of all severe adverse events (not just mortality) could help inform capacity management policies.

In many countries, demand for hospital beds has increased through an increased population, increased elderly numbers, increased community expectations, increased provision of hospital services, and a higher availability of therapeutic interventions. There are pressures that all hospitals face to match this demand for beds to get unwell patients into hospital beds on time (Keegan, 2010). This activity has become more challenging because the demand increases have occurred in finite capacity health systems, where bed stocks and staff resources have remained fairly constant on a per capita basis.

Some factors that affect the length of stay of patients in the hospital include:

Inadequate medical personnel to admit treatment to patients: Delays for emergency care begin with the wait to be seen by a physician. Many, if not most, are "emergent" (requiring "immediate" care) or "urgent" (requiring care within a "short" period). Recent studies have revealed that waiting to see a physician is long and getting worse. The overall median wait to see medical staff increased from 22 minutes in 1997 to 30 minutes by 2004. Perhaps even more alarmingly, the median wait for patients diagnosed with accident-related disease increased from 8 minutes in 1997 to 14 minutes in 2004 (Rose *et al.* 2016).

Lack of Bed Space: Although insufficient medical staff levels handle significant and serious delays in emergency care, the single biggest cause of overcrowding in the Crowther ward is a lack of inpatient beds. For hospitals reporting Crowther ward capacity problems, the average time to move a patient to an inpatient bed once the decision to admit is over 4.5 hours and bed delays of over 24 hours are not uncommon (Fekadu *et al.* 2022). These delays lead to large numbers of patients in the accident ward who strain the ability of the physicians and nurses to care for them. When hospitals are on diversion, seriously ill patients must travel further to the nearest hospital, increasing their risk for adverse outcomes. In addition, the increased travel time increases waits for ambulances, further endangering patients. A study based on data from New York City showed that when there are significant levels of ambulance diversion in a borough, the number of deaths because of accidents increases by 44%. Hospitals are often divided into nursing units, and these units typically consist of 20 to 50 beds (LaFaro *et al*., 2015).

Delays for Nursing Care: A more recent report from health grades estimates the number of deaths to be 248,000 per year. Medical errors result in more deaths than accidents. Although the causes of these preventable deaths are many, recent studies have shown that insufficient nurse staffing levels are a major factor in the prevalence of medical errors. This is not surprising, because many nursing tasks are time-sensitive. Some diseases require longer treatment and they stay in the hospital. The distribution of LOS exhibits a long right tail. Observing this, many authors pointed out that the distribution of LOS is right-skewed (Siamisang *et al*., 2022). Empirical work suggested that Gamma and lognormal distributions are appropriate (Chaou *et al.*, 2017).

RESEARCH METHODS

Data Collection

The data used for this study are secondary data which were collected from the Crowther ward of the Federal Medical Centre, Umuahia (Queen Elizabeth Specialist Hospital), Umuahia, Abia State, Nigeria from the year 2010 - 2013. The data comprises the date of admission, age, diagnosis, date of discharge, disposal and number of days of patients with fractures in the Crowther ward.

METHOD OF DATA ANALYSIS

In this study, descriptive statistics were used to determine the distribution of patients by age. The frequency distribution histogram and **Easyfit software** were used to guess the probability distribution(s) that fit the data (length of stay of patients with fractures and related diseases in the Crowther ward) based on the nature of the graph. The chi-square goodness of fit test was also employed to test how well the probability distribution fits the data. Also, the Chi-Square test of independence was used to ascertain whether the length of stay is independent of age and the type of disease.

Histogram

Histogram is one of the several methods of presenting a frequency distribution graphically. A histogram is a graphical representation of the distribution of numerical data. It is an estimate of the probability distribution of a continuous variable (a quantitative variable). To construct a histogram, the first step is to divide the entire range of values into a series of small intervals and then count how many values fall into each interval. A rectangle is drawn with a height proportional to the count and a width equal to the interval size, so that the rectangles are about each other to indicate that the original variable continues. The bins (intervals) must be adjacent and usually of equal size.

A frequency polygon is plotted (join the midpoints of the upper horizontal side of each rectangle with the adjacent ones by straight lines) to see which probability distribution fits the data by observing the nature of the graph, which may be symmetric, right-skewed or left-skewed. The histogram in this work was used to ascertain the probability distribution that best fits the length of stay of patients in the Crowther ward by plotting the frequency of the length of stay against a given range of intervals, say 1-10, 11-120, etc. Also plotting a frequency polygon on the rectangles of the histogram and observing whether the curve is skewed (not having a normal bell-shaped form when cut along a vertical line through the centre the two halves are not equal) or not.

The Chi-Square Goodness of Fit Test

The chi-square test denoted by χ^2 is used to test if a sample of data came from a population with a specific distribution. It is very popularly known as test of goodness of fit for the reason that it enables us to ascertain how well the theoretical distribution such as Poisson, Gamma, Normal etc., fits empirical distribution i.e. those obtained from sample data. Two values are involved, an observed value which is the frequency of a category from a sample, and the expected frequency which is calculated based upon the claimed distribution.

The Chi-Squared goodness of fit test were used to observe how good a particular probability distribution fits the data (length of stay of patients in the Crowther ward) by first grouping the length of stay into intervals, say 1-10, 11-20, . . , 111-120, and also obtaining the frequency (which is also the observed frequency) of each interval. We also obtain the class mark and class boundaries for each interval from which we obtain the mean and variance of the sample data.

Furthermore, we obtain the probability distributions for each of the intervals with knowledge of the mean, variance and class boundaries. Next, we obtain the expected frequency for the sample data by multiplying the various probabilities by the sample size. Also, we obtain the Chi-Square test statistic following the steps below. Note that the knowledge of the probability distribution that best fits the data will enable us to estimate the expected length of stay of a patient in the ward.

Steps in Constructing the Chi-Square Test of Goodness of Fit

Step 1: Statement of Hypothesis

The chi-square test is defined for the hypothesis: H_0: the data follow a specified distribution HI: the data do not follow a specified distribution

Step 2: Stating the level of significance

Here the significance level will be stated.

Step 3: Stating the Test Statistic

The Test Statistic for the Chi-Square goodness of fit test is given by:

$$\chi^2{}_{cal} = \sum_{i=1}^{k} \frac{(O_i - E_i)^2}{E_i}, \chi^2{}_{k-1-u} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad 3.1$$

With $v = k - 1 - u$ degrees of freedom

Where O_i = observed frequency

E_i = expected frequency

k = number of classes for which comparison between observed and expected frequency is going to be made.

u = number of parameters estimated from the observed data.

Step 4: Stating the critical or table value

Here the tabulated value of the Chi-Square distribution is obtained based on the specified degree of freedom and level of significance.

Step 5: Stating the Decision rule

Reject the null hypotheses H_0 if $\chi^2{}_{cal} \geq x^2_{tab}$

Step 6: Conclusion

Here we draw conclusion or make inference based on step 5 above.

Properties of Chi-Square Test of Goodness of Fit

- ❖ No expected frequency should be less than five (5). If any expected frequency is less than five (5), we combine the expected frequency in the adjacent cells as appropriate until the combined expected frequency is at least five (5).
- ❖ The data are obtained from a random sample.
- ❖ The data are the observed frequencies. This means that there is only one data value for each category.

Chi-Square Test of Independence

The Chi-Square test of independence was used to check if length of stay of patients in the Crowther ward is independent of some factors such as age and type/nature of the disease of the patients. The number of days a patient stays in the hospital (length of stay) were divided into two groups/categories (less than one month and above one month due to the severity/nature of the illness, the age of the patient etc.). The types of disease were also divided into two groups (i.e. those in which the bone is injured (fracture) and those in which the bone is not injured) as directed by the medical staff.

The age of the patients was equally classified into two groups i.e. 15-64 (working population), 65 and above (dependent population). It is well known that people belonging to working population always has more strength than the elderly belonging to the dependent population and their bone is more strong compare to the aged people that is very brittle and can easily break. Also, most patients of the working population due to their business, education etc, will want to leave the hospital as soon as possible to meet up with their activities which is not so with dependent population.

Things to note about Chi-Square Test of Independence

- ❖ No expected frequency should be less than five (5). If any expected frequency is less than five (5), we combine the expected frequency in the adjacent cells as appropriate until the combined expected frequency is at least five (5).
- ❖ The two variables should consist of two or more categorical, independent groups. Example of independent variable that meet this criterion includes; gender (male and females) etc.

Statement of Hypotheses

H_0 = the criteria of classification are independent.

H_1 = the criteria of classification are not independent.

Test statistic

$$\chi^2{}_{cal} = \sum_{i=1}^{r}\sum_{j=1}^{c}\frac{\left(O_{ij} - E_{ij}\right)^2}{E_{ij}} \sim \chi^2{}_{(1-\propto,v)} \quad . \ . \ . \ . \ . \ . \ . \ . \ . \ . \quad 3.2$$

with $v = (r-1)(c-1)$ degrees of freedom

where $r =$ Number of rows

$c =$ Number columns

$\alpha =$ Level of significance

$O_{ij} =$ The observed frequency of the ij^{th} cell (number of days in the hospital)

$E_{ij} =$ The expected frequency of the ij^{th} cell

$$E_{ij} = \frac{R_i - C_j}{N} \quad . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \quad 3.3$$

Where $R_i =$ Row total

$C_j =$ Column total

$N =$ Grand total

Decision Rule: Reject the null hypothesis H_0 if $x^2{}_{cal} \geq \chi^2{}_{(1-\propto,v)}$, otherwise accept H_0.

Determination Of Expected Length Of Stay Of Patients In The Crowther Ward

To determine the expected length of stay of patients in the Crowther ward, we apply

$$E\,(X) = \begin{cases} \sum_{i}^{k} xf(x) \text{ for discrete random variables} \\ \int_{-\infty}^{\infty} xf(x)dx \text{ for discrete random variables} \end{cases} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad 3.4$$

Where $f(x) =$ probability density function fitted to the data.

DATA ANALYSIS

Fitting Probability Distribution to the Length of Stay Data

We construct a histogram below which tells the number of patients that stayed in the ward for a certain interval of days and also a frequency polygon which enables us to suggest the probability distribution that fits the data.

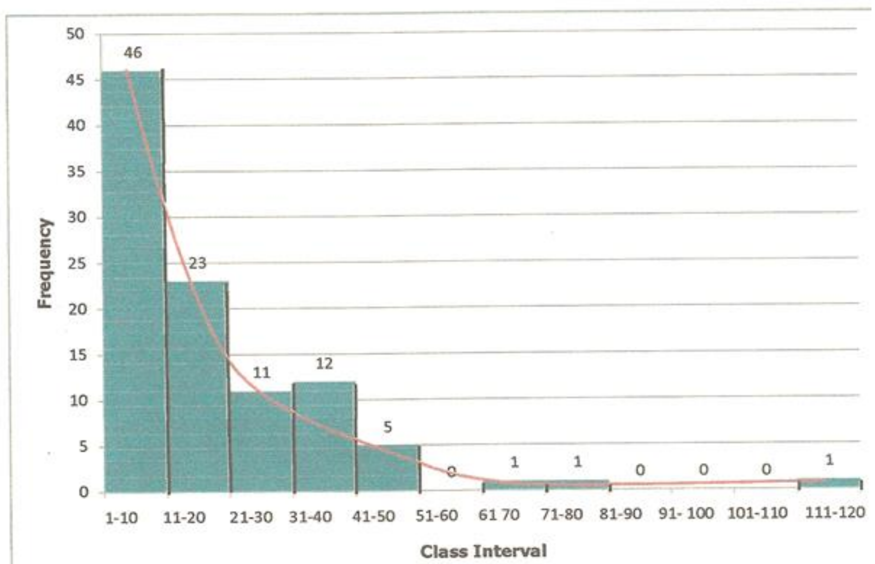

Figure 1: Histogram with a frequency polygon showing the length of stay of patients in the Crowther ward.

Exponential 2 Parameter Distribution

A continuous random variable is said to have the exponential 2 parameter distribution with parameters λ and γ if its probability density function is given by:

$$f(x) = \begin{cases} \lambda e^{-\lambda(x-\gamma),} & x \geq c \;\; \lambda > 0 \\ 0, & \text{otherwise} \end{cases} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad 4.1$$

Where λ is the scale parameter and λ is the location parameter. The mean and variance of the exponential 2 parameter distribution is given as:

Mean $= \mu$ and Variance $\sigma^2 = \frac{1}{\lambda^2}$

From the Easyfit 5.6 software, we obtain the Goodness of fit test for the Chi-Squared for the Exponential 2 parameter as shown in the table below:

Table 1: Table showing the results of Chi-Squared Goodness of fit test from Easyfit 5.6 Standard version of Exponential 2 parameter distribution.

Chi-Squared					
Deg. of freedom	6				
Statistic	3.4897				
P-Value	0.74534				
Rank	7				
$\boldsymbol{\alpha}$	0.2	0.1	0.05	0.02	0.01
Critical Value	8.5581	10.645	12.592	15.033	16.812
Reject?	No	No	No	No	No

Statement of Hypothesis

H_0= The data (length of stay of patients in the Crowther ward) follows the Exponential 2 parameter distribution.

H_1= The data (length of stay of patients in the Crowther ward) does not follows the Exponential 2 parameter distribution.

Test Statistic

From the table above, the Chi-Squared test statistic is given as:

$$\chi^2{}_{cal} = \sum_{i=1}^{k} \frac{(O_i - E_i)^2}{E_i} = 3.4897, \qquad \chi^2{}_{tab} = 12.592 \text{ at } \alpha = 0.05 \quad . \quad . \quad . \quad . \quad . \; 4.2$$

Decision rule: We reject H_0 (null hypothesis) if $\chi^2{}_{cal} \geq \chi^2{}_{tab}$, otherwise we accept H_0.

Conclusion: Since $\chi^2{}_{cal}(3.4897) < \chi^2{}_{tab}$ (12.592), we reject the null hypothesis and thus conclude that the data follows Exponential 2 parameter distribution.

b) Lognormal Distribution

A continuous probability distribution is said to have the lognormal distribution with parameter μ (location parameter) and σ (scale parameter) if its probability density function is given by

$$f(x; \mu, \sigma) = \frac{1}{x\sigma\sqrt{2\pi}} e^{-\frac{(lnx-\mu)^2}{2\sigma^2}}, x > 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad 4.3$$

Mean $= e^{\mu+\frac{\sigma^2}{2}}$

Variance $= (e^{\sigma^2} - 1)\, e^{2\mu+\sigma^2}$

From the Easyfit 5.6 software, we obtain the Goodness of fit test for the Chi-Squared for the lognormal distribution as shown in the table below:

Table 2: Table showing the result of Goodness of fit test from Easyfit 5.6 Standard version for lognormal distribution.

Chi-Squared					
Deg. of freedom	6				
Statistic	4.492				
P-Value	0.61041				
Rank	15				
$\boldsymbol{\alpha}$	0.2	0.1	0.05	0.02	0.01
Critical Value	8.5581	10.645	12.592	15.033	16.812
Reject?	No	No	No	No	No

Statement of Hypothesis

H_0= The data (length of stay of patients in the Crowther ward) fits the Lognormal distribution.

H_1= The data (length of stay of patients in the Crowther ward) does not fits the Lognormal distribution.

Test Statistic

From the table above, the Chi-Squared test statistic is given as:

$$\chi^2{}_{cal} = \sum_{i=1}^{k} \frac{(O_i - E_i)^2}{E_i} = 4.492, \qquad \chi^2{}_{tab} = 12.592 \text{ at } \alpha = 0.05 \quad . \quad . \quad . \quad . \quad . \; 4.4$$

Decision rule: We reject H_0 (null hypothesis) if $\chi^2{}_{cal} \geq \chi^2{}_{tab}$, otherwise we accept H_0.

Conclusion: Since $\chi^2{}_{cal}(4.492) < \chi^2{}_{tab}$ (12.592), we reject the null hypothesis and thus conclude that the data fits the Lognormal distribution.

b) Pareto Distribution

A continuous random variable is said to have a Pareto distribution with parameters α and β if its probability density function (pdf) is given by;

$$f(x;\alpha,\beta) = \begin{cases} \frac{\alpha\beta^{\alpha}}{x^{\alpha+1}}; & \beta \leq x < \infty;\ \alpha,\beta > 0 \\ 0, & \text{otherwise} \end{cases} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad 4.5$$

Where $\alpha$is the scale parameter and β is the location parameter

A few well known properties follows:

Mean $E(X) = \frac{\alpha\beta}{\alpha-1},\ \alpha > 1.$ 4.6

Variance $Var\ (X) = \frac{\alpha\beta^2}{[(\alpha-1)^2(\alpha-2)]},\ \alpha > 2$ 4.7

From the Easyfit 5.6 software, we obtain the Goodness of fit test for the Chi-Squared for the Pareto distribution as shown in the table below:

Table 3: Table showing the result of Goodness of fit test from Easyfit 5.6 Standard version for Pareto distribution.

Chi-Squared					
Deg. of freedom	4				
Statistic	50.416				
P-Value	2.9562E-10				
Rank	47				
$\boldsymbol{\alpha}$	0.2	0.1	0.05	0.02	0.01
Critical Value	5.9886	7.7794	9.4877	11.668	13.277
Reject?	Yes	Yes	Yes	Yes	Yes

Statement of Hypothesis

H_0= The data (length of stay of patients in the Crowther ward) fits the Pareto distribution.

H_1= The data (length of stay of patients in the Crowther ward) does not fits the Pareto distribution.

Test Statistic

From the table above, the Chi-Squared test statistic is given as:

$$\chi^2{}_{cal} = \sum_{i=1}^{k} \frac{(O_i - E_i)^2}{E_i} = 50.416, \qquad \chi^2{}_{tab} = 9.4877 \text{ at } \alpha = 0.05 \quad . \quad . \quad . \quad . \quad . \ 4.8$$

Decision rule: We reject H_0 (null hypothesis) if $\chi^2{}_{cal} \geq \chi^2{}_{tab}$, otherwise we accept H_0.

Conclusion: Since $\chi^2{}_{cal}(50.416) > \chi^2{}_{tab}$ (9.4877), we reject the null hypothesis and thus conclude that the data fits the Pareto distribution.

From the above tests, we observed that the Pareto distribution is not a good fit for the data since the null hypothesis was rejected. We also observed that Exponential 2 parameter and the Lognormal distribution fits the data but the Exponential 2 parameter distribution fits the data most since its Chi-Squared ranking is smaller than that of the Lognormal distribution as given by the Easyfit software and hence the Exponential 2 parameter distribution were used to estimate the expected length of stay on the average of patients in the Crowther ward.

Thus, since the Exponential 2 parameter distribution best fit the data, we give the probability density function as;

$$f(x) = \begin{cases} \lambda e^{-\lambda(x-\gamma)}, & x \geq \gamma \quad \lambda > 0 \\ 0, & \text{otherwise} \end{cases} \qquad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad 4.9$$

But from the Easyfit software, the parameters were given as $\lambda = 0.0598, \gamma = 1$. Hence, we obtain

$$f(x) = \begin{cases} 0.0598 e^{-0.0598(x-1)}, & x \geq \gamma \quad \lambda > 0 \\ 0, & \text{otherwise} \end{cases} \qquad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad 4.10$$

Estimation of the expected Length of stay of Patients in the Crowther Ward.

Here we try to estimate the expected length of stay of patients in the Crowther ward. We will apply the formula

$$E(X) = \int_{-\infty}^{\infty} x f(x) dx \qquad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad 4.11$$

Where $f(x) =$ probability density function fitted to the data.

The Exponential 2 parameter distribution was used to estimate the expected length of stay of patients in the ward since it fits the data most.

From the Easyfit software, the parameters of the distribution λ and γ were given as

$\lambda = 0.0598$ and $\gamma = 1$.

Hence $E(X) = \int_{-\infty}^{\infty} x f(x) dx$ implies

$$E(X) = \int_{-\infty}^{\infty} x \lambda e^{-\lambda(x-\gamma)} \, dx \qquad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad 4.12$$

$$E(X) = \int_{-\infty}^{\infty} x 0.0598 e^{-0.0598(x-1)} \, dx \qquad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad 4.13$$

∴ Using the overall class boundary of (0.5 – 120.5), we have

$$E(X) = \int_{-0.5}^{120.5} x 0.0598 e^{-0.0598(x-1)} \, dx \qquad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad 4.14$$

$$E(X) = 0.0598 \int_{-0.5}^{120.5} x e^{-0.0598(x-1)} \, dx \qquad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad 4.15$$

Applying integration by parts i.e. $\int udv = uv - \int vdu,$ we have

Let $u = x \implies \frac{du}{dx} = 1 \implies du = dx$

Also, let $dv = e^{-0.0598(x-1)} \implies v = \frac{e^{-0.0598(x-1)}}{0.0598}$

Hence

$$E(X) = 0.0598\left[x\left(-\frac{e^{-0.0598(x-1)}}{0.0598}\right)_{0.5}^{120.5} - \int_{-0.5}^{120.5} -\frac{e^{-0.0598(x-1)}}{0.0598}dx\right] \quad . \quad . \quad 4.16$$

$$= 0.0598\left[x\left(-\frac{e^{-0.0598(x-1)}}{0.0598}\right)_{0.5}^{120.5} + \frac{1}{0.0598}\int_{-0.5}^{120.5} e^{-0.0598(x-1)}\ dx\right]$$

$$= 0.0598\left[x\left(-\frac{e^{-0.0598(x-1)}}{0.0598}\right)_{0.5}^{120.5} - \left(\frac{1}{0.0598}\ e^{-0.0598(x-1)}\ \right)_{0.5}^{120.5}\right]$$

$$= 0.0598\left[\left(-\frac{120.5e^{-0.0598(120.5-1)}}{0.0598} + \frac{0.5e^{-0.0598(0.5-1)}}{0.0598}\right) - \left(\frac{e^{-0.0598(120.5-1)}}{0.0598} - \frac{e^{-0.0598(0.5-1)}}{0.0598}\right)\right]$$

$$= 0.0598\left[\frac{1}{0.0598}\left(-120.5e^{-0.0598(120.5-1)} + 0.5e^{-0.0598(0.5-1)} - e^{-0.0598(120.5-1)} + e^{-0.0598(0.5-1)}\right)\right]$$

$= -0.09495 + 0.51518 + 0.01318 + 17.22996$

$= 17.63701 \ \cong 18$

$\therefore\ E(X) = 18$ days. 4.17

Hence the expected length of stay of patients in the Crowther ward is estimated to be 18 days between 2010 – 2013.

But since for a distribution that is skewed, it is more preferable to use the median rather than the mean in planning, hence we obtain the median as follows;

For the Exponential 2 parameter distribution, the median is given by;

$M = \gamma + \frac{1}{\lambda} \times 0.693$ 4.18

Substituting the values of $\lambda = 0.0598$ and $\gamma = 1$ into equation 3.21 above, we obtain

$M = 1 + \frac{1}{0.0598} \times 0.693$

$M = 1 + 16.7224 \times 0.693$

$M = 1 + 11.5886$

$\therefore M = 12.5886\ \cong 13$ days. 4.19

Hence the expected length of stay on the average of patients in the Crowther ward is estimated to be 13 days between 2010 – 2013.

Hence we suggest that the management of the Crowther ward instead of channeling most of their resources in making plans for patients that will come and stay for months and so on, they should rather channel most their resources on patients that stays on the average of 13 days.

Test for Independence of Length of Stay on some factors (Age and Type of Disease)

The Chi-Square test of independence/relatedness classifies the experimental unit in the Crowther ward by two criteria. The first criterion; number of days a patient stayed in the hospital (length of stay) is divided into two groups/categories (less than month and above one month due to the severity of the illness, immune system of the patient, the age of the patient etc.) and the second criterion; type of disease is divided into two groups (i.e. those whose bones were affected i.e. fractured and those whose bones were not affected), and also age of the patients which is equally classified into two groups i.e. 15-64 (working population), 65 and above (dependent population).

The analysis is to determine whether or not the length of stay depends on the age of patients and the type of the disease at a particular level of significance.

SPSS version 20 was used to calculate the values of the expected frequencies.

a) Length of Stay vs Age

Statement of Hypotheses

H_0= The length of stay of patients in the Crowther ward is independent on age of the patients.

H_1= The length of stay of patients in the Crowther ward is dependent on age of the patients.

Table 4: Contingency table for the age group and length of stay of patients in the Crowther ward of Federal Medical Center Umuahia.

Length of stay	**Age group**		**Total**
	Working population	**Dependent population**	
Less than one month	69	7	76
Above one month	10	14	24
Total	79	21	100

From Table 4 above, it is observed that 69 patients who were of the working population age stayed in the ward for less than one month, 7 patients who were of the dependent population stayed in the ward for less than one month, also 10 patients who were of the working population stayed in the ward for above one month, and 14 patients of the dependent population stayed in the ward for above one month.

Table 5: Computation of Chi-Square test of independence for the length of stay of patients and age in the Crowther ward.

O_i	E_i	$O_i - E_i$	$(O_i - E_i)^2$	$(O_i - E_i)^2/E_i$
69	60	9	81	1.3500
7	16	-9	81	5.0625
10	19	-9	81	4.2632

14	5	9	81	5.7857
				16.4614

From the above computation, the Test Statistic is given as;

$$\chi^2{}_{cal} = \sum_{i=1}^{r}\sum_{j=1}^{c}\frac{\left(O_{ij} - E_{ij}\right)^2}{E_{ij}} = 16.4614 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad 4.20$$

But,

Number of rows $r = 2$, Number of columns $c = 2$, level of significance $\alpha = 0.05$

$\therefore \chi^2{}_{1-\alpha,\ \ (r-1)(c-1)} = \chi^2{}_{1-0.05\ \ (2-1)(2-1)} = \chi^2{}_{0.05,1} = 3.841$

Decision Rule: Reject H_0 if $\chi^2{}_{cal} \geq \chi^2{}_{tab}$, otherwise do not reject H_0.

Conclusion: Since $\chi^2{}_{cal} > \chi^2{}_{tab}$, we reject H_0 and conclude that the length of stay of patients in the Crowther ward is dependent on the age of the patients.

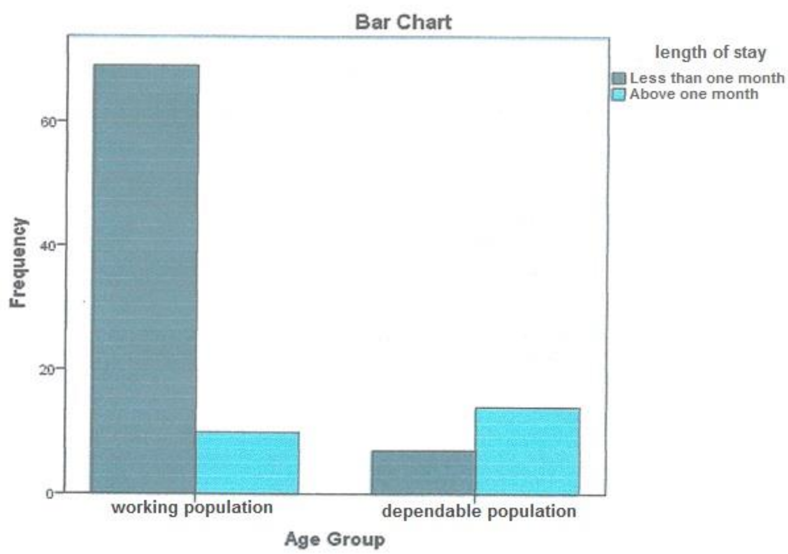

Figure 2: Multiple Bar Chart showing the relationship between length of stay and age of patients in the Crowther ward of the Federal Medical Center Umuahia.

From the figure 2 above, it is observed that for the length of stay of less than one month interval, patients of the working population age stayed longer in the Crowther ward compared to patients of the dependent population age. Also, for length of stay of above one month interval, patients of the dependent population stayed longer in the Crowther ward compared to patients of the working population.

This implies that patients who are aged i.e. patients of the population stayed longer in the ward because as one aged, the bones gets weaker and their rates of recovery becomes very low compared to bones of the working population that is very strong. Hence, increase in the age of a patient leads to increase in the length of stay of the patient in the ward. Thus as one advances in age, he/she will have the tendency of staying much longer in the ward.

b) Length of stay vs Type of disease

Statement of Hypotheses

H_0= The length of stay of patients in the Crowther ward is independent on the type of disease of the patients.

H_1= The length of stay of patients in the Crowther ward is dependent on the type of disease of the patients.

Table 6: Contingency table for the type of disease and length of stay of patients in the Crowther ward of Federal Medical Center Umuahia.

Length of stay	**Type of Disease**		**Total**
	Bone affected	**Bone not affected**	
Less than one month	29	47	76
Above one month	22	2	24
Total	51	49	100

From Table 6 above, it is observed that 29 patients who were suffered from broken bone stayed in the ward for less than one month, 47 patients that were diagnosed of fracture stayed in the ward for less than one month, also 22 patients who were diagnosed of fracture stayed in the ward for above one month, while 2 patients who were not diagnosed of fracture stayed in the ward for above one month. This implies that patients who suffered from fracture has a longer length of stay in the ward since it takes a lot of time for the fractured bone(s) to heal.

Computation

Table 7: Computation of Chi-Square test of independence for the length of stay of patients and type of disease in the Crowther ward.

$\boldsymbol{O_i}$	$\boldsymbol{E_i}$	$\boldsymbol{O_i - E_i}$	$(\boldsymbol{O_i - E_i})^2$	$(\boldsymbol{O_i - E_i})^2/\boldsymbol{E_i}$
29	38.8	-9.8	96.04	2.4753
22	12.2	9.8	96.04	7.8721
47	37.2	9.8	96.04	2.5817
2	11.8	-9.8	96.04	8.1390
				21.0681

From the above computation, the Test Statistic is given as;

$$\chi^2{}_{cal} = \sum_{i=1}^{r}\sum_{j=1}^{c}\frac{\left(O_{ij} - E_{ij}\right)^2}{E_{ij}} = 21.0681 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad 3.27$$

But,

Number of rows $r = 2$, Number of columns $c = 2$, level of significance $\alpha = 0.05$

$\therefore \chi^2{}_{1-\alpha,\ \ (r-1)(c-1)} = \chi^2{}_{1-0.05\ \ (2-1)(2-1)} = \chi^2{}_{0.05,1} \ = 3.841$

Decision Rule: Reject H_0 if $\chi^2{}_{cal} \geq \chi^2{}_{tab}$, otherwise do not reject H_0.

Conclusion: Since $\chi^2{}_{cal} > \chi^2{}_{tab}$, we reject H_0 and conclude that the length of stay of patients in the Crowther ward is dependent on the age of the patients.

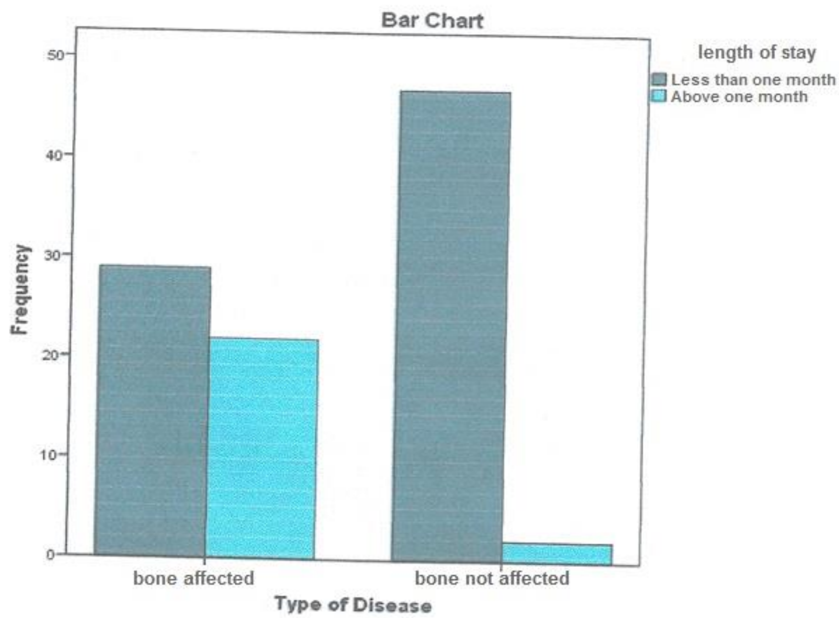

Figure 3: Multiple Bar Chart showing the Relationship between Length of stay and type of disease of patients in the Crowther ward of the Federal Medical Centre Umuahia

From the diagram above, it is observed that for the length of stay of less than one-month interval, patients who were diagnosed of fracture or related diseases stayed longer in the Crowther ward compared to the patients who were not diagnosed of fracture or related disease. Also for length of stay of above one month interval, patients who were diagnosed of fracture or related diseases stayed longer in the ward compared to patients who were diagnosed of fracture or related disease.

This implies that patients who suffered from fracture has a longer length of stay in the ward since it takes a lot of time for the fractured bone(s) to heal.

Diagrammatic View of the Data.

In this survey, the data collected on the length of stay of patients in the Crowther ward of the Federal Medical Centre Umuahia is presented in the following pie chart below.

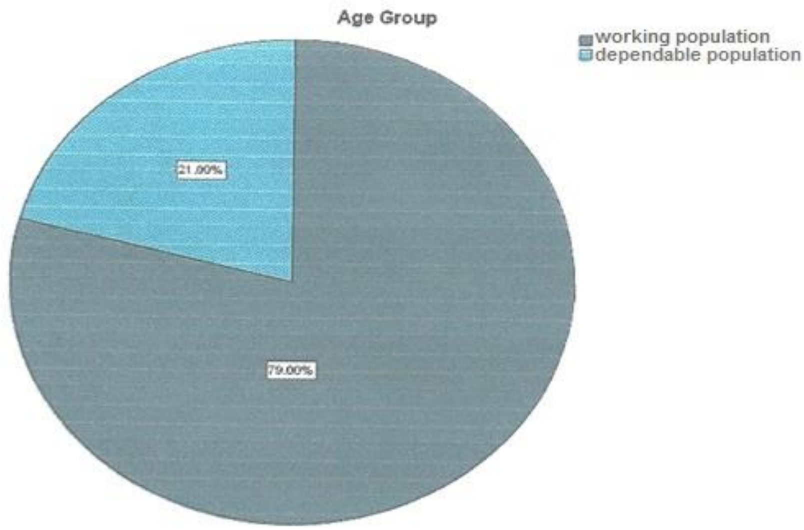

Figure 4: A Pie Chart showing the Age Group of patients in the Crowther ward.

From the diagram above, 21% of the patients in the ward are of the dependent population while 79% are of the working population. This implies that patients who are in the hospital are mostly the working population.

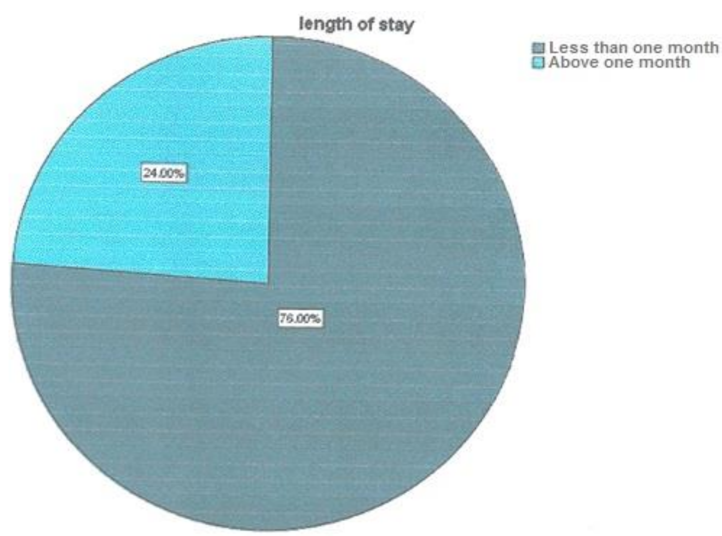

Figure 5: A Pie Chart showing the Length of stay of patients in the Crowther ward.

From the chart above, it is seen that 24% of the patients stayed in the ward for less than one month while 76% of the patients stayed above one month. This implies that majority of the patients admitted to this ward irrespective of the age and type of disease stayed above one month.

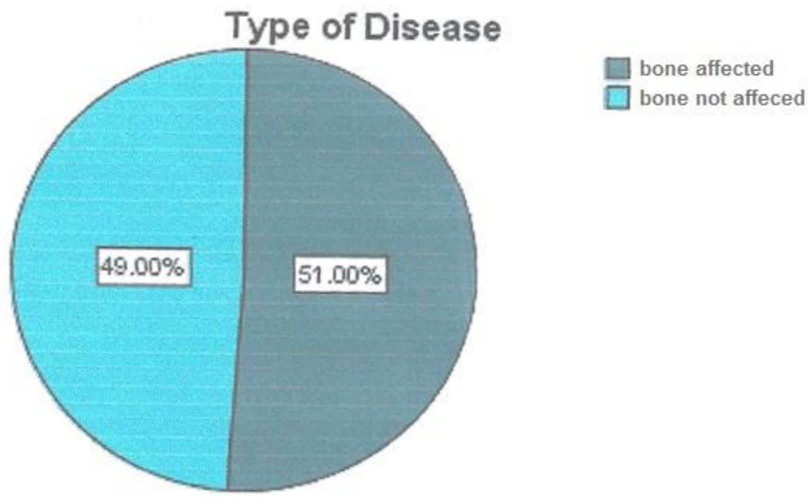

Figure 6: A Pie Chart showing the type of disease of the patients in the Crowther ward.

From the chart, it is seen that 49% of the patients who were admitted in the ward suffered from fracture (broken bone) while 51% of the patients was not diagnosed of fracture.

DISCUSSION OF RESULTS

Atienza (2005) discovered that the distribution of length of stay is right skewed and Marazzi (1998) said the empirical work on length of stay suggested that gamma, Weibull and Lognormal distributions are appropriate for the length of stay data using the Anderson Darling goodness of fit test. From the above results, it has been observed that the length of stay of patients in the Crowther ward of the Federal Medical Centre Umuahia follows the exponential 2 parameter distribution (i.e. the probability distribution that best fits the data is the exponential 2 parameter distribution) which is right skewed using the Chi-Squared goodness for fit test and not Gamma or Weibull distribution suggested by Marazzi (1998). Using the exponential 2 parameter distribution, it was estimated that patients admitted into the ward will stay for expected length of 13 days on the average. This implies that instead of the management of the Crowther ward making plans for patients that will come and stay for months and so on, they should rather channel most of their resources on patients that stay on the average of 13 days.

Martin *et al.* (1996) showed that demographic characteristics of patients and some hospital characteristics are two important determinants of LOS. The important demographic characteristics are age, gender, type of disease etc., but in this work age and type of disease were the demographic characteristics used to carry out Chi-Squared test of independence. It was observed that length of stay of the patients in the ward was influenced by factors as age and the type of disease of the patient. Patients of the dependent population were found to stay longer in the ward because as gets old, their body system becomes weak and will invariably takes time to accept treatment. We observed that 69 patients who are of the working population age stayed in the ward for less than one month, 7 patients who are of the dependent population stayed in the ward for less than one month, also 10 patients who are of the working population stayed in the ward for above one month, and 14 patients of the dependent population stayed in the ward for above one month.

It was also observed that for the length of stay of less than one month interval, patients of the working population age stayed most in the Crowther ward compared to the patients in the dependent population age. Also for length of stay of above one month interval, patients of the dependent population stayed longer in the ward compared to patients of the working population age. Patients who are aged i.e. patients of the dependent population stayed most in the ward because as one aged, the bones gets weaker and their rates of recovery becomes very low compared to bones of the working population that is very strong. Hence increase in the age of a patient leads to increase in the length of stay of the patient in the ward. Thus as one advances in age, he/she will have the tendency of staying much longer in the ward.

It was found that 21% of the patients in the ward are of the dependent population while 79% are of the working population which implies that patients who are in the hospital are mostly the working population and that 24% of the patients stayed in the ward for less than one month while 76% of the patients stayed above one month which implies that majority of the patients admitted to this ward irrespective of the age and type of disease stayed above one month. It was also seen that 49% of the patients who were admitted in the ward suffered from fracture (broken bone) while 51% of the patients was not diagnosed of fracture.

CONCLUSION

This work investigated the length of stay of patients in the emergency care unit of the Federal Medical Centre Umuahia (a case study of the Crowther ward). The number of patients for the survey was 100. The data used for the analysis was from the Crowther ward of the Federal Medical Centre, Umuahia. From the survey and analysis, the Exponential 2 parameter distribution best fits the length of stay of patients in the Crowther ward and the expected length of stay (number of days) for patients admitted to the ward was estimated to be 18 days on average. Aged (dependent population) patients who came to the ward, and the patients diagnosed with broken bone (fracture) stayed much longer in the ward.

We can conclude that age and the type of disease are some of the major causes of the length of stay of patients in the ward. Also, accidents are one of the major reasons patients are admitted to the emergency care unit of the Federal Medical Centre Umuahia because they lead to several injuries, wounds, fractures etc, which leads to an increase in the length of stay.

RECOMMENDATIONS

Based on the findings, we recommend that the management of the Crowther Ward Federal Medical Centre Umuahia should provide more bed space, and quality and timely services to patients.

The management of the Crowther ward should channel most of their resources on patients that will stay for an average of 13 days, rather than channel their resources on patients that will stay for months.

Government, churches, and other social organizations should educate and sensitize their members on the implications of road accidents which will help reduce the rate at which we admit patients into the Crowther ward and also the length of stay of patients in the ward

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