CASSON FLUID FLOW IN A MAGNETIZED OSCILLATORY SYSTEM AFFECTED BY VARIABLE THERMAL CONDUCTIVITY WITH ASYMMETRIC HEATING

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Abstract

Thermal conductivity plays a significant role in thermal transmission and engineering processes, particularly when the fluid serves as an energy source. Against the popular notion that thermal conductivity should be kept constant, recent trends in experimental findings reveal that, most often, thermal conductivity varies in relation to temperature, pressure, etc. Therefore, this paper is aimed at exploring the impact of variable thermal conductivity and Casson fluid flow on a magnetized oscillatory system within a permeable plate immersed in a porous material. The flow is affected by the non-uniform wall heating condition. The solutions of the resultant nonlinear dimensionless equations have been determined using regular perturbation. In view of the assumed oscillatory pressure gradient, the nonlinear partial differential equations were reduced to a boundary-valued problem where the unsteady flow is superimposed on the mean steady flow. The physical flow behavior of velocity, temperature, shear stress, and heat transfer rate are demonstrated graphically and discussed thoroughly in view of the embedded parameters. It is concluded from this computational analysis that the involvement of variable thermal conductivity improves the temperature and velocity components more effectively compared to the commonly assumed constant thermal conductivity. The action of the Casson fluid parameter is to suppress the fluid velocity. Interestingly, the results obtained for the limiting case in this research are consistent with previous literature, thereby establishing the accuracy and validity of the present analysis.

Keywords: *variable thermal conductivity, casson fluid parameter, magnetized oscillatory flow, darcy porous number, slip parameter*

INTRODUCTION

The term "Casson fluid" describes a type of non-Newtonian fluid characterized by variable viscosity. The dominance of viscous force in this system can be attributed to the action of variable viscosity in the fluid. Fluids commonly employed in many applications include paints, diverse polymer solutions, blood, honey, etc. Typical instances of Casson fluids encompass synthetic lubricants, mud extraction, clay coatings, and biomedical fluids. The Casson fluid simulations that are readily accessible are classified based on their distinct rheological properties, including Oldroyd-B, Eyring-Powell, Oldroyd-A, Maxwell, Carreau, Jeffrey, and Burger. In the field of modern technology, there are some flow features that defy comprehension when analyzed solely through the lens of the Newtonian flow model. Consequently, the utilization of the non-Newtonian fluid notion is more advantageous. Some noteworthy works like Amaraikannan *et al*. (2021a), Shahzad *et al.* (2021), Amaraikannan *et al.* (2021b), and Ali *et al.* (2021) provide valuable insights for readers on this topic. Additionally, Ojemeri *et al*. (2023) put forth magnetized Casson flow of an incompressible fluid with thermal radiation impact in an up-facing porous plate. Also, Nandeppanavar (2018) described the movement pattern of Casson fluid through a moving plate. Sheikh *et al.* (2020) have presented a study on the implication of the flow of a non-Newtonian fluid across a moving sheet. Ahmad *et al*. (2019) introduced a novel approach to modelling a Casson fluid with fractional derivatives in a more recent study. Further, Sarkar and Endalew (2019, 2020), Hamid *et al.* (2019), Das *et al.* (2021), Amjad *et al.* (2021), and Sarkar *et al.* (2020) have published the flow behavior of the Casson fluid in various physical settings.

The study of electrically conductive liquids, including plasma, electrolytes, salt water, and liquid metals, is called magnetohydrodynamics (MHD). There are numerous technical and commercial applications for this type of liquid, including magnetic drug targeting, crystal formation, power generation, reactor cooling, and MHD sensors (Jamaludin 2020). Hartmann and Lazarus (1937) were the first to conduct an experimental study of modern MHD flow in laboratories. This research laid the groundwork for the creation of numerous MHD devices, including MHD pumps, MHD generators, flow meters, and geothermal-powered energy exploration. Afterwards, numerous studies have been published to analyze the effects of MHD convections along different flow media. Buoyancy-induced flow along a magnetized oscillating system with heat transfer across different geometrical settings has been a topic of global attention and interest among many scientists today owing to its vast benefits in geophysics, solid mechanics, hydrology, oil recovery, and in the field of engineering, to cite a few (2018). In this regard, Onwubuya *et al.* (2024) recently simulated the impact of MHD Casson fluid affected by porosity and thermal radiation factors along a permeable channel in an oscillating system. Again, Omokhuale *et al.* (2024) emphasized the function of viscous dissipation on heat and mass flow driven by nanoparticles through a boundary layer cavity in an oscillating system, while Usman *et al.* (2024) demonstrated the effect of thermophoresis on MHD oscillatory flow equipped with nano elements across a boundary layer regime. In a different work, Usman and Sanusi (2023) investigated the impacts of Casson nanofluid flow within a semi-infinite flat plate entrenched in a porous medium affected by thermal radiation and thermal diffusion. Sharma *et al.* (2022) presented the numerical analysis of magnetized oscillatory flow of viscous dissipative fluid along an up-facing channel with porous medium due to heat source and thermal radiation impacts. Falade *et al.* (2017) deliberated on wall porosity impacts in a magnetized oscillatory slip flow along a vertical plate with non-uniform wall temperature. Hamza *et al*. (2011) outlined the influence of thermally and chemically driven slip flow on a transient MHD-free convection via a vertical plate filled with porous medium in an oscillating system.

Thermal conductivity has been assumed to be constant in some previously published works. However, it is well known that temperature variations can substantially alter the physical features of the fluid. The fluctuation of thermal conductivity with temperature must be put into consideration so as to accurately envisage flow behavior and heat transfer rate. Thermal characteristics, in particular, thermal diffusivity and conductivity, are crucial bedrock material parameters that regulate heat transmission and temperature increases in the repository zone (Hamza et al. 2015, Ajibade and Kabir 2020). Variable thermophysical fluid characteristics are therefore crucial for improving fluid heat transmission. The heat transfer system is drastically impacted by temperature-dependent thermal conductivity and viscosity. Several published works on steady flow with varying thermal conductivity are available. In view of this, Hamza *et al.* (2024) considered the implication of variable thermal conductivity on nonlinear mixed convection in an upstanding channel affected by asymmetric/symmetric heating/cooling conditions using a semi-implicit finite difference and regular perturbation techniques. Salawu *et al.* (2021) considered free convection of a non-fixed upstanding plate with the buoyancy force modelled as quadratic Boussinesq approximation and thermal property using an appropriate shooting numerical technique. They concluded that temperature differences enhance heat flow formation in the channel. Pradip *et al.* (2021) searched out the impact of heat generation, thermal property, and variable viscosity on MHD natural convection across an isothermally heated vertical plate. Ewis (2021) scrutinized the impact of variable thermal conductivity, porosity, and Grashof number on natural convection of viscoelastic fluid flow and heat transfer. Hamza *et al.* (2022) discussed the fully developed mixed convection affected by variable thermal conductivity across a microchannel whose plates contemplated velocity slip and temperature jump effects. Other works that exemplify this phenomenon can be found in references: Nalivale *et al.* (2022), Megahed *et al.* (2019), and Haque *et al.* (2015).

Based on the preceding discourse, the primary objective of the current study is to build on the work of Onwubuya *et al*. (2024) by considering the effect of a temperature-dependent variable thermal conductivity on a magnetized oscillatory Casson fluid flow coated with suction/injection effects across a permeable vertical channel. It is useful to state here that this research is a generalization of the work of Onwubuya *et al.* (2024) by incorporating variable thermal conductivity. The novelty in this present investigation is to derive the solution to the resultant nonlinear partial differential equations using regular perturbation in the variable thermal conductivity term after the unsteady flow is superimposed on the mean steady flow. This model will not only present a unique combination for improving the wave oscillation performance in heat control systems but will also serve as an improvement of previous works. The results of this kind of research would also be useful in hydraulic engineering and industry applications, particularly for exploration of crude oil from petroleum products and in aeronautical technologies to aid proper speed in times of lower pressure.

DESCRIPTION OF THE FLOW SYSTEM

Suppose a natural oscillatory flow of a Casson fluid with thermal property along a permeable channel is investigated under the effect of an imposed magnetic field. It is thought that there is an absence of applied voltage, which signifies the non-involvement of an electric field. The flow moves in the upward direction in the x-direction across the plate, while the y-axis is perpendicular to it and the z-axis across the wideness of the channel, as sketched in Figure 1. Also, it is assumed that the whole system is rotating with a constant vector Ω about the y-axis. Since it is presumed that the plate surface is semi-infinite, the flow variables are functions of y only. The thermal conductivity (kf) of the liquid is taken to be varying linearly with respect to temperature in the form $kf = km\,[1 + \delta\,(T' - T'm)]$ (Elbarbary and Elgazery 2004), where the free-flow heat of the liquid is conductivity and δ is a constant that varies with the properties of the fluid, with $\delta > 0$ for fluids namely water and air and for fluids like lubricating oils (Elbarbary and Elgazery 2004). Following Falade *et al.* (2017) and Onwubuya *et al*. (2024) and taking variable thermal conductivity into account while obeying the Bousinesq approximation, the resultant equations of this present model can be expressed as:

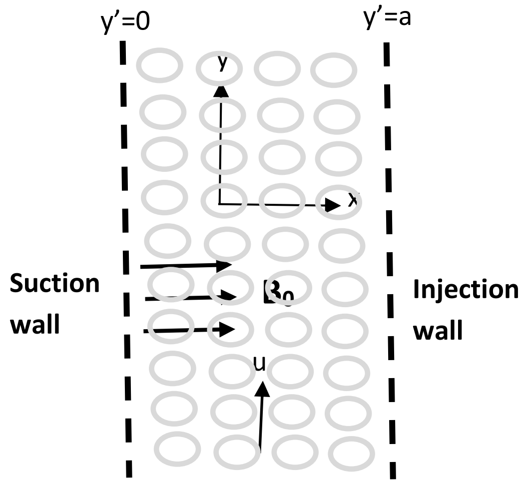

Figure 1. Schematic diagram of the flow

$$\frac{\partial u'}{\partial t'} - v_0\frac{\partial u'}{\partial y'} = -\frac{1}{\rho}\frac{dP}{dx'} + v\left(1+\frac{1}{\xi}\right)\frac{\partial^2 u'}{\partial y'^2} - \frac{v}{K}u' - \frac{\sigma_e B_0^2 u'}{\rho} + g\beta(T' - T_O) \quad (1)$$

$$\frac{\partial T'}{\partial t'} - v_0\frac{\partial T'}{\partial y'} = \frac{4\alpha^2}{\rho c_\rho}(T - T_O) + \frac{1}{\rho c_\rho}k_f\,\left[\frac{\partial T'}{\partial t'}\right] \quad (2)$$

With the boundary condition $T = T_1$

$$u' = \frac{\sqrt{K}}{\alpha_s}\frac{\partial u'}{\partial y'}, T = T_0 \;\; on\, y' = 0 \quad (3)$$

$$u' = 0, T' = T_1 \;\; on\, y' = a \quad (4)$$

While the dimensionless quantities used are provided below:

$$(x\;\;y) = \frac{(x', y\prime)}{h}, u = \frac{hu\prime}{v}, t = \frac{vt\prime}{h^2}, p = \frac{h^2 p\prime}{\rho v^2}, Gr = \frac{g\beta(T_1 - T_0)h^3}{v^2}, Pr = \frac{h^2 p\prime}{\rho v^2}, \theta = \frac{T - T_0}{T_1 - T_0}$$

$$\delta = \frac{4x^2h^2}{\rho C_p v}, \gamma = \frac{\sqrt{K}}{\alpha_s h}, Ha^2 = \frac{\sigma_e B_0^2 h^2}{\rho v}, Da = \frac{K}{h^2}, \alpha = \delta(T_h' - T_m'), s = \frac{v_0 h}{v} \quad (5)$$

Inserting equation (5) into equations (1 – 4), we have the dimensionless governing equations as follows:

$$\frac{\partial u}{\partial t} - s\frac{\partial u}{\partial y} = -\frac{dP}{dx} + \left(1+\frac{1}{\xi}\right)\frac{\partial^2 u}{\partial y^2} - \left(Ha^2 + \frac{1}{Da}\right)u + Gr\theta \quad (6)$$

$$\frac{\partial \theta}{\partial t} - s\frac{\partial \theta}{\partial y} = \frac{1}{Pr}\frac{\partial^2 \theta}{\partial y^2} + D\theta + \alpha\left[\frac{\partial \theta}{\partial y}\right]^2 \quad (7)$$

With the appropriate boundary conditions

$$\left.\begin{array}{l} u = \gamma\frac{du}{dy}, \quad \theta = 0 \;\; on \quad y = 0 \\ u = 0, \;\; \theta = 1 \quad on \quad y = 1 \end{array}\right\} \quad (8)$$

METHOD OF SOLUTION

We employed the theory of simultaneous differential equations to address the resultant nonlinear partial differential equations restricted to relevant boundary conditions after the unsteady flow is superimposed on the mean steady flow, so that in the neighbourhood of the plate, and assuming that an oscillatory pressure gradient is shown in eqn (9), the solutions of velocity and temperature are in the form:

$$-\frac{dP}{dx} = \lambda e^{iwt}, \;\; \mathrm{u(t,y)} = u_0(y)e^{iwt}, \;\; \theta(t,y) = \theta_0(y)e^{iwt} \quad (9)$$

Where λ is any constant, and ω is the number of oscillation (See Falade *et al.* [23]). In view of (9), eqns (6 – 8) transformed to a boundary-valued-problem as follows:

$$u_0'' + \frac{s}{a_1}u_0' - \frac{1}{a_1}\left(Ha^2 + \frac{1}{Da} + i\omega\right)u_0 = -\frac{\lambda}{a_1} - \frac{Gr}{a_1}\theta \quad (10)$$
$$\theta_0'' + sPr\theta_0' - (D - i\omega)\theta_0 + \alpha[\theta_0']^2 = 0 \quad (11)$$

Subject to relevant boundary conditions:

$$\left.\begin{array}{l} u_0 = \gamma u_0', \;\; \theta_0 = 0 \quad on \quad y = 0 \\ u_0 = 0, \quad \theta_0 = 1 \quad on \quad y = 1 \end{array}\right\} \quad (12)$$

As a result of the nonlinearity of the resultant governing equations, we used a regular perturbation technique in the variable thermal conductivity parameter λ to create an exact solution to (10) and (11) restricted to (12) as follows:

$$\theta_0 = \theta_{00} + \alpha\theta_{01} \quad (13)$$
$$u_0 = u_{00} + \alpha u_{01} \quad (14)$$

So that,

$$\theta = \theta_0(y)e^{iwt} \quad (15)$$
$$u = u_0(y)e^{iwt} \quad (16)$$

Inserting eqns (13) and (14) into eqns (10 – 12), and comparing the like powers of $\propto^0$ and $\propto$, we derive the following set of equations:

$$\propto^0 : \theta_{00}'' + sPr\theta_{00}' + (D - i\omega)Pr\theta_{00} = 0 \tag{17}$$
$$\propto : \theta_{01}'' + sPr\theta_{01}' + (D - i\omega)Pr\theta_{01} + \theta'^2_{00} = 0 \tag{18}$$
$$\propto^0 : u_{00}'' + \frac{s}{a_1}u_{00}' - \frac{1}{a_1}\left(Ha^2 + \frac{1}{Da} + i\omega\right)u_{00} = -\frac{\lambda}{a_1} - \frac{Gr}{a_1}\theta_{00} \tag{19}$$
$$\propto \ : u_{01}'' + \frac{s}{a_1}u_{01}' - \frac{1}{a_1}\left(Ha^2 + \frac{1}{Da} + i\omega\right)u_{01} = -\frac{\lambda}{a_1} - \frac{Gr}{a_1}\theta_{01} \tag{20}$$

The transformed boundary conditions now becomes

$$\left.\begin{array}{l} \theta_{00} = 0 \\ \theta_{01} = 0 \quad at \quad y = 0 \\ \quad and \\ \theta_{00} = 1 \\ \theta_{01} = 0 \quad at \quad y = 1 \end{array}\right\} \tag{21}$$

$$\left.\begin{array}{l} u_{00} = \gamma u_{00}' \\ u_{01} = \gamma u_{01}' \quad at \quad y = 0 \\ \quad and \\ u_{00} = 0 \\ u_{01} = 0 \quad at \quad y = 1 \end{array}\right\} \tag{22}$$

The exact solutions of temperature and velocity is obtained as:

$$\theta_{00}(t, y) = (J_1 e^{m_1 y} + J_2 e^{m_2 y})e^{iwt} \tag{23}$$
$$\theta_{01}(t, y) = \left(J_3 e^{m_1 y} + J_4 e^{m_2 y} + J_5 e^{2m_1 y} + J_6 e^{(m_1+m_2)y} + J_7 e^{2m_2 y}\right)e^{iwt} \tag{24}$$
$$u_{00}(t, y) = (E_1 e^{m_3 y} + E_2 e^{m_4 y} + E_3 + E_4 e^{m_1 y} + E_5 e^{m_2 y})e^{iwt} \tag{25}$$
$$u_{01}(t, y) = \begin{pmatrix} E_6 e^{m_3 y} + E_7 e^{m_4 y} + E_8 + E_9 e^{m_1 y} + E_{10} e^{m_2 y} + E_{11} e^{2m_1 y} \\ +E_{12} e^{(m_1+m_2)y} + E_{13} e^{2m_2 y} \end{pmatrix} e^{iwt} \tag{26}$$

The rate of heat transfers and shear stress is also calculated as follows:

$$Nu = \frac{\partial\theta}{\partial y} = \left[m_1 J_1 e^{m_1 y} + m_2 J_2 e^{m_2 y} + \alpha\left\{m_1 J_3 e^{m_1 y} + m_2 J_4 e^{m_2 y} + 2m_1 J_5 e^{2m_1 y} + (m_1 + m_2)J_6 e^{(m_1+m_2)y} + 2m_2 J_7 e^{2m_2 y}\right\}\right]e^{iwt} \tag{27}$$

$$S_f = \frac{\partial u}{\partial y} = \left(m_3 E_1 e^{m_3 y} + m_4 E_2 e^{m_4 y} + \begin{array}{c} m_1 E_4 e^{m_1 y} + m_2 E_5 e^{m_2 y} + \cdots \end{array} \alpha\left\{ \begin{array}{c} m_3 E_6 e^{m_3 y} + m_4 E_7 e^{m_4 y} + m_1 E_9 e^{m_1 y} + m_2 E_{10} e^{m_2 y} + 2m_1 E_{11} e^{2m_1 y} \ldots \\ +(m_1 + m_2)E_{12} e^{(m_1+m_2)y} + 2m_2 E_{13} e^{2m_2 y} \end{array} \right\} \right) e^{iwt} \tag{28}$$

All the constants used are indicated in the appendix section

RESULTS AND DISCUSSION

The impact of Casson fluid on a magnetized oscillating system affected by variable thermal conductivity and porous medium factors on natural convection flow along an infinite vertical porous plate has been studied. The flow is instigated by buoyancy-induced growing pressure gradient along an upward-facing plate. In order to point out the effects of physical parameters such as variable thermal conductivity α, Casson fluid parameter ξ, suction parameter s, magnetic parameter Ha, and thermal Grashof number Gr on the flow behaviours, computational analysis of the flow fields is performed. The influences of the major controlling parameters on the temperature and velocity distributions have been presented and explained in Figures 3 to 15. The main default values selected for this analysis as they relate to real-life applications are s=1, Ha=1, Da=1, and Gr=1. The graphical comparison of the work of Onwubuya *et al.* [18] and the present investigation is portrayed in Figure 2. Interestingly, the comparison displays excellent agreement for the limiting case when variable thermal conductivity and Casson parameter = 1000.

The actions of variable thermal conductivity on the fluid temperature and velocity are illustrated in Figures 3 and 4, respectively. From these graphs, it is evident that an escalation in the temperature and velocity of the fluid happens as the variable thermal conductivity parameter is increased. The action of the Casson fluid parameter on the fluid velocity is depicted in Figure 5. It can clearly be seen that with increasing Casson fluid parameters, the velocity decreases. The velocity gradient for the application of various values of magnetic effect is illustrated in Figure 6. The action of a magnetic field perpendicular to the flow in an electrically conducting fluid produces a Lorentz force that opposes the flow. With the aid of Figure 7, we comprehend the behavior of fluid velocity as the Darcy porous medium is varied. It is apparent from this diagram that the fluid motion grows as Darcy number is raised. This is true since, with stronger permeability of the porous material, the barriers placed on the flow path reduce, thereby encouraging free flow leading to stronger fluid speed. This action makes the fluid boundary wall and thickness rise, which in turn escalates the fluid velocity.

Thermal radiation's impact on the fluid temperature and velocity are seen in Figures 8 and 9, respectively. It is noteworthy to report that when the thermal radiation is increased, the temperature of the fluid noticeably improves. This is attributable to the heat flow from the upper surface to the fluid because the fluid takes in its own radiation. Further, as displayed in Figure 8, the fluid motion is accelerating as a function of the growing thermal radiation effect owing to heat production that elevates the flow movement. This is so because the heat emitted from the heated wall strengthens the fluid particles. The consequences of varying suction/injection parameters on energy and momentum gradients are demonstrated in Figures 10 and 11, respectively. From the sketch in Figure 10, it is seen that the mounting level of the suction/injection parameter encourages the fluid temperature. The concavity with the rise in the suction/injection parameter is as a result of the direction of temperature flow from the hot plate along the cold wall. Similarly, it is noteworthy that increasing the suction/injection raises the fluid velocity towards the cold wall, as shown in Figure 11. Figure 12 explains that increasing the number of oscillations retards the temperature of the fluid inside the channel, which is due to the weakening in the heat transfer amount as the heating strength grows.

The effect of the Casson fluid parameter on the skin friction is demonstrated in Figure 13. It is seen that a rise in the skin friction is established in the cold plate while a reverse attribute happens in the heated wall. However, a point of intersection is viewed near the middle of the vertical channel. The deviation of the variable thermal conductivity parameter on the rate of heat transfer and shear stress is captured in figures 14 and 15, respectively. It was observed that the application of thermal property is identified to strengthen the frictional force and heat transfer amount, respectively. There is, however, a point of significant intersection in the middle of the channel for the rate of heat transfer, which is due to the effect of asymmetric heating at the upper plate.

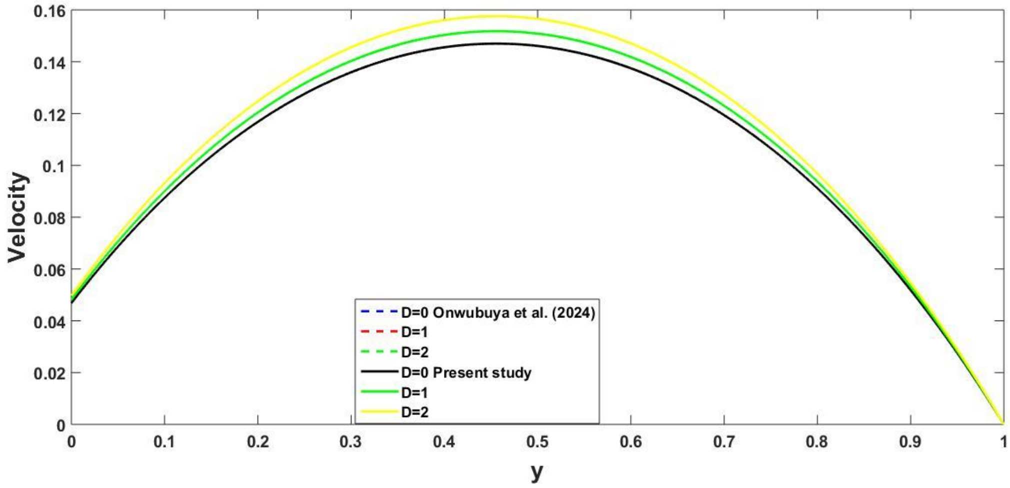

Figure 2: Comparison between the work of Onwubuya *et al.* (2024) and the current analysis

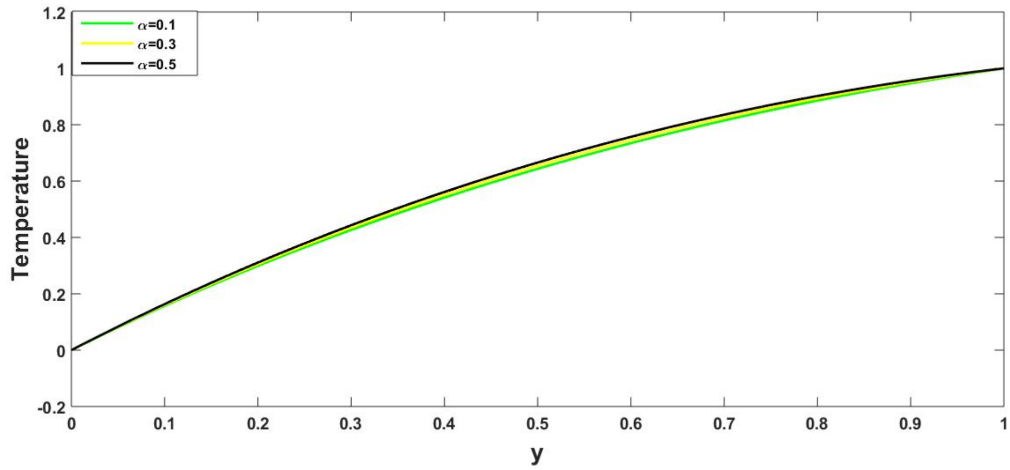

Figure 3 Demonstration of Variable thermal conductivity on Temperature distribution

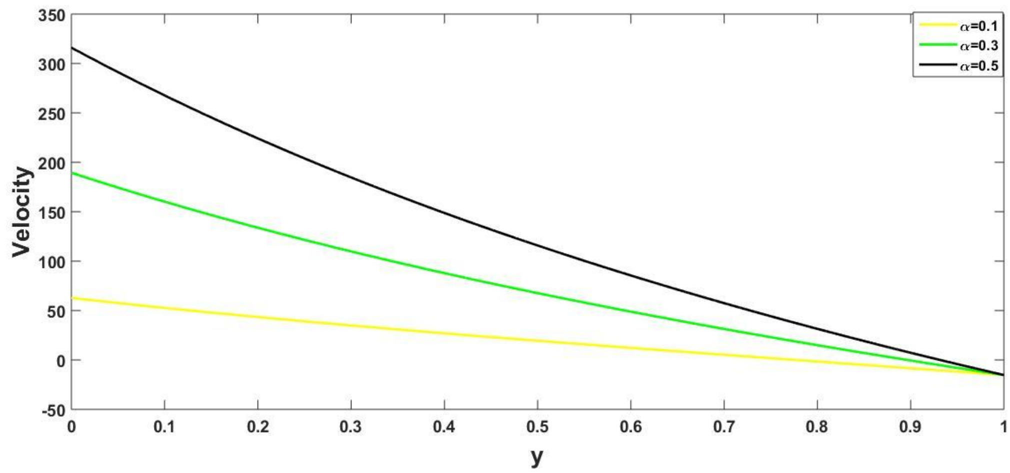

Figure 4. Demonstration of Variable thermal conductivity on Velocity distribution

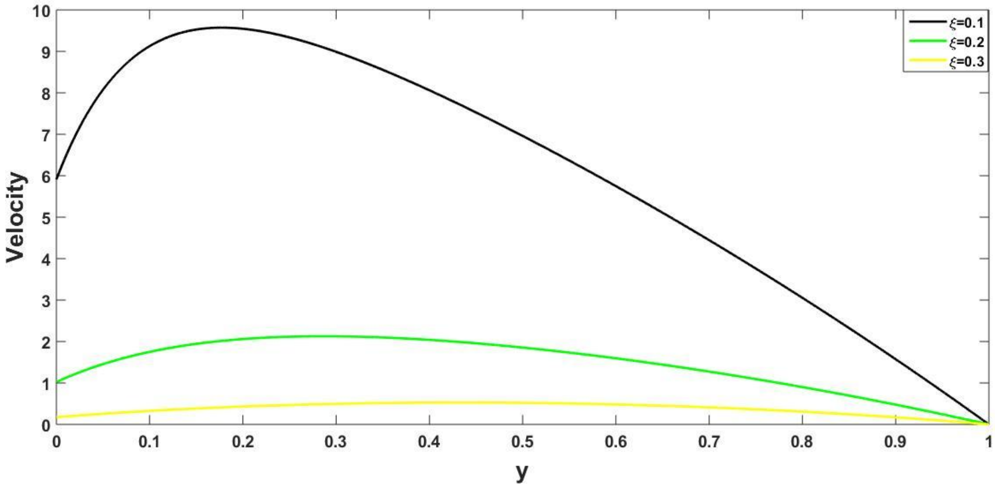

Figure 5: Demonstration of Casson parameter on Velocity distribution

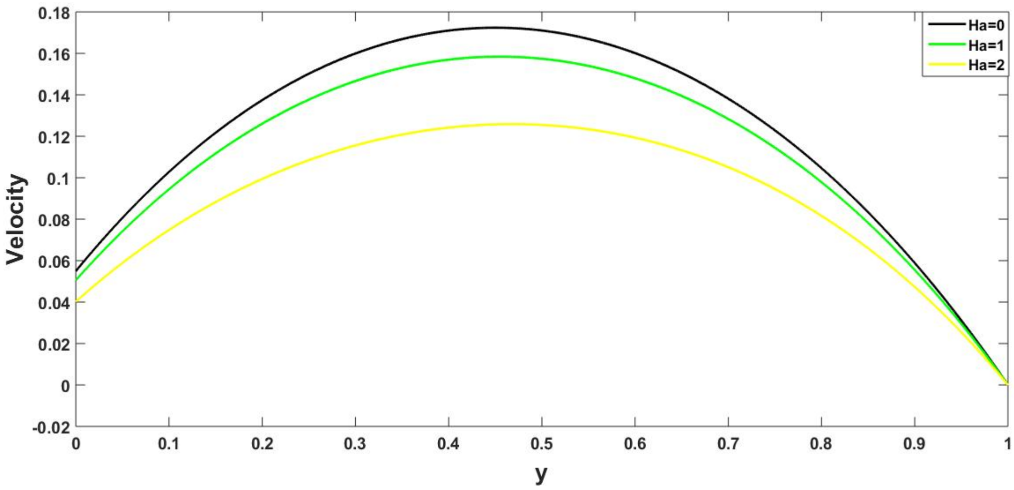

Figure 6: Demonstration of Hartmann number on Velocity distribution

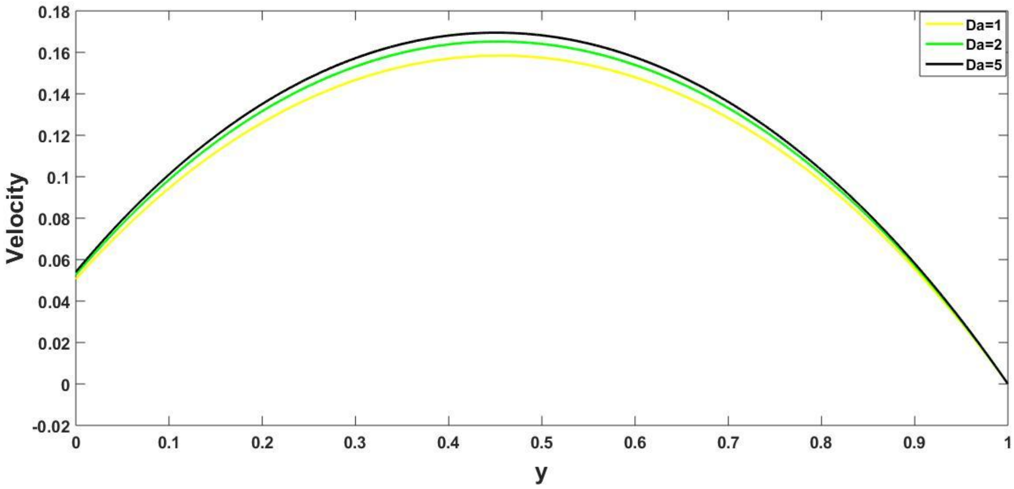

Figure 7: Demonstration of Darcy number on Velocity distribution

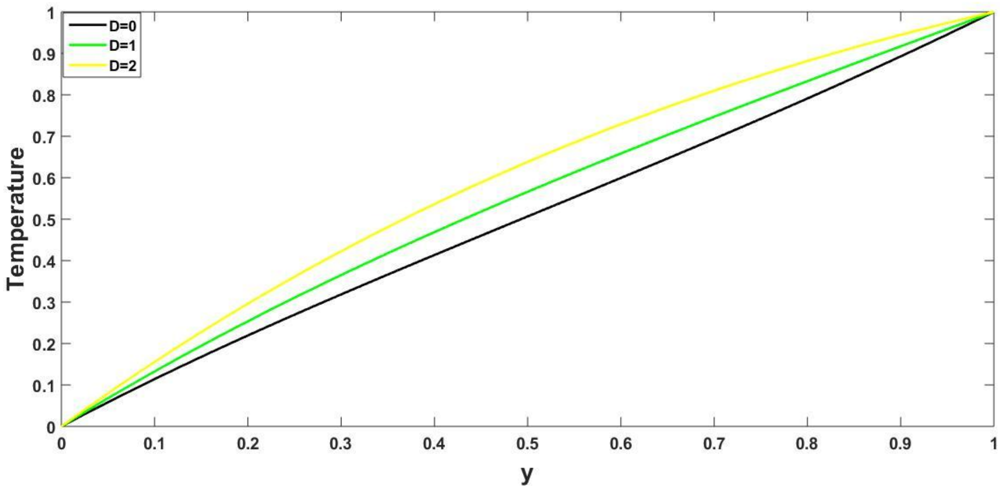

Figure 8: Demonstration of thermal radiation on Temperature distribution

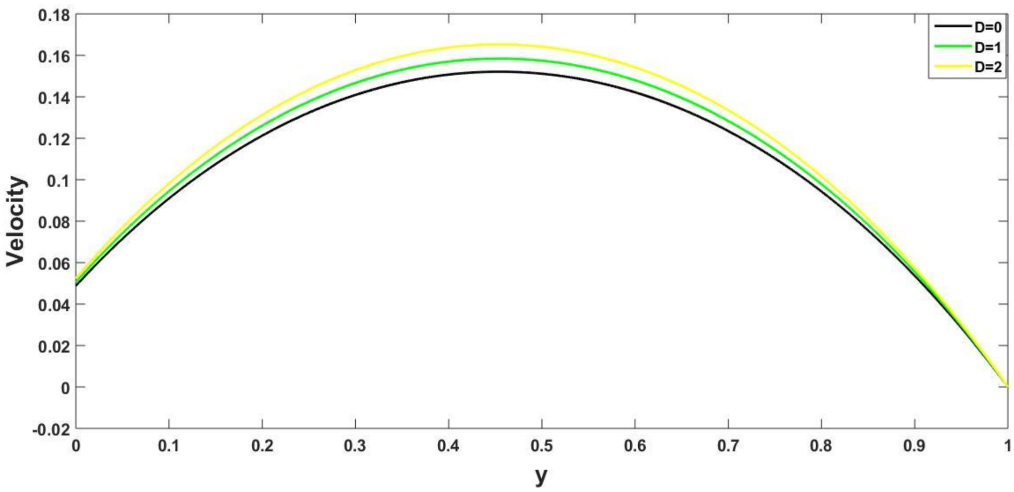

Figure 9: Demonstration of thermal radiation on Velocity distribution

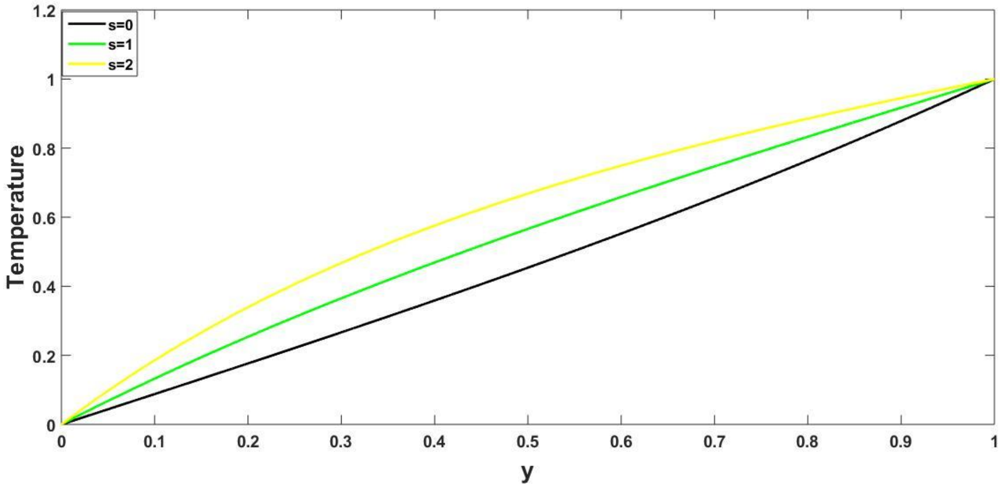

Figure 10: Demonstration of suction/injection on Temperature distribution

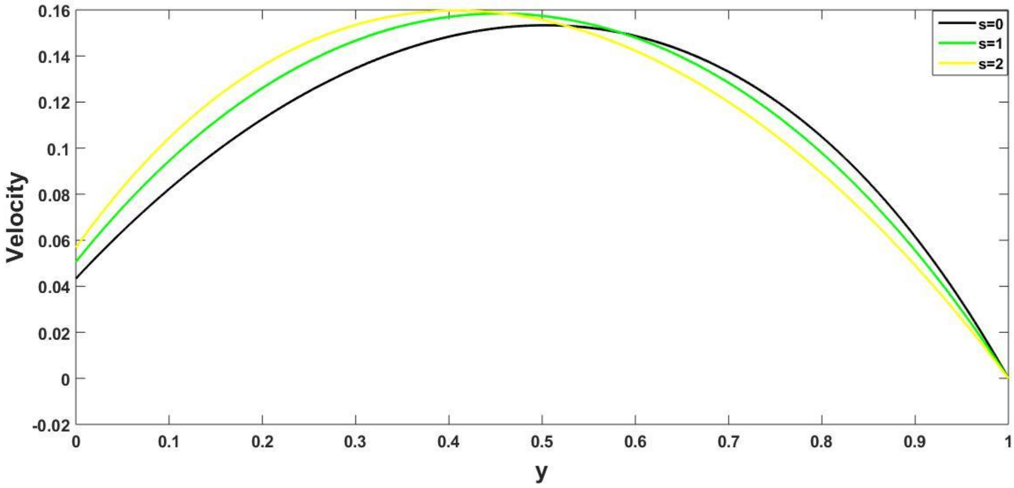

Figure 11: Demonstration of suction/injection on Velocity distribution

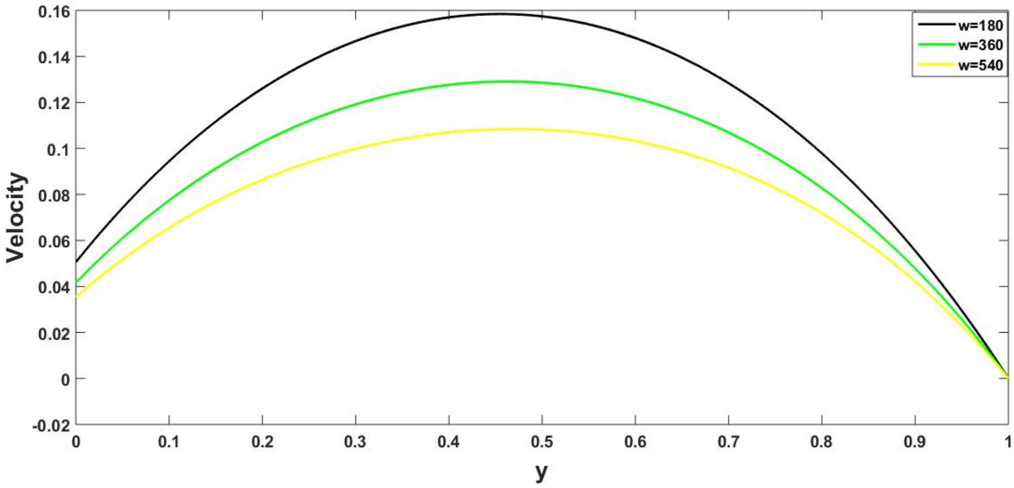

Figure 12: Demonstration of oscillation parameter on Velocity distribution

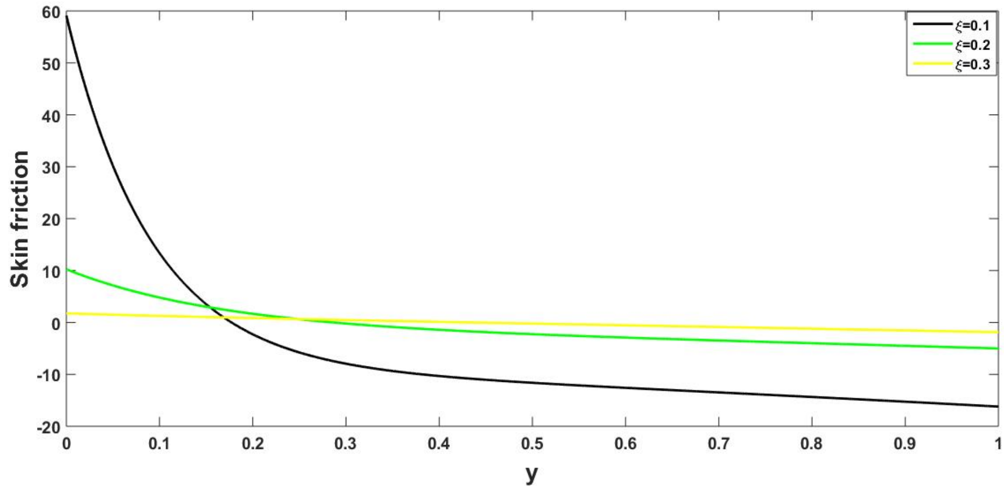

Figure 13: Demonstration of Casson parameter on Skin friction

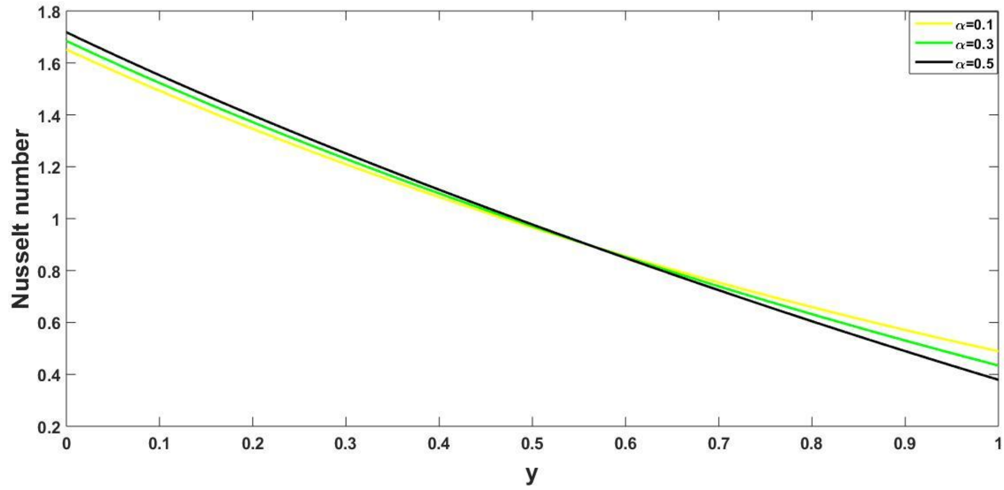

Figure 14: Demonstration of variable thermal conductivity on Nusselt number

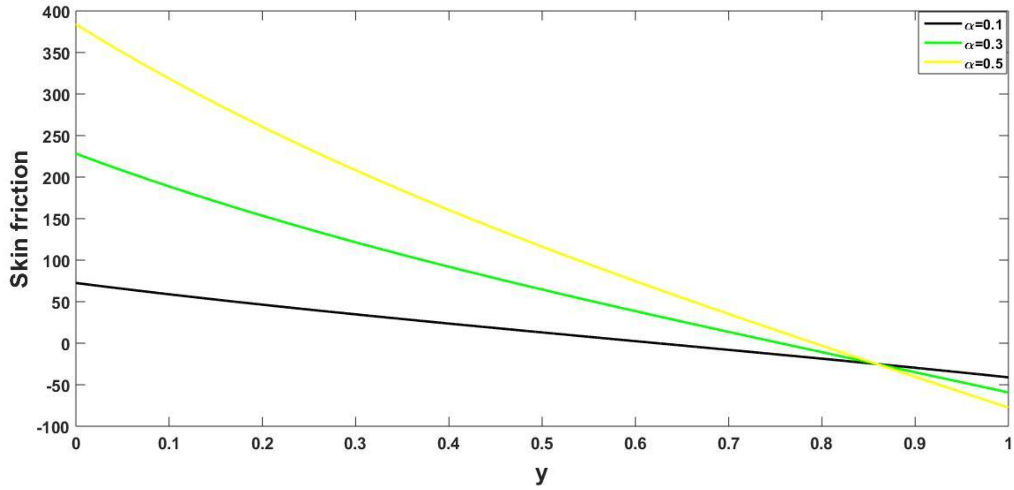

Figure 15: Demonstration of variable thermal conductivity on Skin Friction

CONCLUSION

The unsteady convection flow of Casson fluid and thermal property effects on magnetized oscillatory flow along a porous channel saturated with porous material have been analyzed in the optically thin thermal radiation regime. The solutions of the non-dimensional nonlinear equations have been derived, and the impacts of pertinent incorporated parameters controlling the flow characteristics have been illustrated graphically and discussed. The Casson fluid model is an easier model that adequately explains the physiological and peristaltic flows of non-Newtonian form. Further, variable thermophysical fluid characteristics have been found to be very useful for improving fluid heat transfer. The key findings from this study are summarized below:

i. The application of the Casson fluid parameter is revealed to substantially suppress the fluid velocity.

ii. The fluid temperature and velocity gradient are seen to escalate by uplifting the value of variable thermal conductivity.

iii. Suction/injection increases the fluid temperature and velocity, respectively.

iv. The amount of heat transfer is decreased at the heated wall for growing values of suction/injection, while a counter attribute happens at the cold plate.

v. The effect of the Casson fluid parameter is observed to raise the shear stress at the cold plate, while a reverse attribute happens in the heated wall.

vi. The application of variable thermal conductivity is envisaged to improve the skin friction and the rate of heat transfer, respectively.

vii. In the future, the impact of variable viscosity will be studied on this model.

NOMENCLATURE

B_0 = Constant magnetic flux density [kg/s^2.m^2]

g= Gravitational acceleration [m/s^2]

h= Channel width [m]

$C_p C_v$= Specific heats at constant pressure and constant volume [Jkg^{-1}K^{-1}]

$t' =$ time

$u' =$ axial velocity

$v_0 =$ constant horizontal velocity

P' = fluid pressure

γ = Navier slip parameter

δ=Thermal radiation parameter

Gr= thermal Grashof number

ω = frequency of oscillation

α = Variable thermal conductivity

λ = Positive constant

M= Magnetic field

S=Suction/injection parameter

Nu= Dimensionless heat transfer rate

T= Dimensionless temperature of the fluid [K]

T_0=Reference temperature [K]

T_∞^* = The temperature far away from the plate K

u=Dimensionless velocity of the fluid [ms^{-1}]

y= Dimensionless distance between plates

U_0= Reference velocity [ms^{-1}]

K^* = The mean absorption coefficient [m^{-1}]

β^*=Thermal expansion coefficient [K^{-1}]

μ = Variable fluid viscosity [$kgm^{-1}s^{-1}$]

k= Thermal conductivity [$m.kg/s^3.K$]

q_r^* = The radiative heat flux [Wm^{-2}]

α = Thermal diffusivity [m^2s^{-1}]

σ_e= Electrical conductivity of the fluid [s^3m^2/kg]

ρ = Density of the fluid [kgm^{-3}]

ν = Fluid kinematic viscosity [m^2s^{-1}]

Appendix

$$m1=\frac{sPr+\sqrt{(sPr)^2-4\Pr(D-i\omega)}}{2}\quad m2=\frac{-sPr-\sqrt{(sPr)^2-4\Pr(D-i\omega)}}{2},m3=\frac{\frac{s}{a_1}+\sqrt{\left(\frac{s}{a_1}\right)^2-\frac{4a_2}{a_1}}}{2}$$

$$m4=\frac{-\frac{s}{a_1}+\sqrt{\left(\frac{s}{a_1}\right)^2-\frac{4a_2}{a_1}}}{2},J_1=-\frac{1}{e^{m2}-e^{m1}},J_2=\frac{1}{e^{m2}-e^{m1}},J_5=-\frac{-J_1^2}{4m_1^2+2sPrm1+(\delta-i\omega)},$$

$$J_6=-\frac{-2J_1J_2}{4m_1^2+2sPrm1+(\delta-i\omega)},J_7=-\frac{-J_2^2}{4m_2^2+2sPrm_2+(\delta-i\omega)},\ J_4=\frac{(J_5+J_6+J_7)e^{m1}-b_1}{e^{m2}-e^{m1}},\ J_3=\frac{J_4e^{m_2}+b_1}{e^{m_1}}$$

$$b_1=J_5e^{2m_1}+J_6e^{(m_1+m_2)}+J_7e^{2m_2},\ a_1=1+\frac{1}{\xi},a_2=Ha^2+\frac{1}{Da}+i\omega,\quad a_3=E_3+E_4+E_5,a_4=m_1E_4+m_2E_5,a_5=1-\gamma m_3,a_6=1-\gamma m_4,a_7=\frac{a_4-a_3}{a_5},a_8=\frac{a_6}{a_5},E_1=a_7-E_2a_8,a_8=E_3+E_4e^{m1}+E_5e^{m2},a_{10}=m_1E_4e^{m1}+m_2E_5e^{m2},a_{11}=\gamma m_3a_8e^{m3}-a_8e^{m3}-\gamma m_4a_7e^{m3},a_{12}=\gamma m_3a_7e^{m3}+a_{10}-a_9-a_7e^{m3},E_2=\frac{a_{12}}{a_{11}},E_3=\frac{\lambda}{a_1}$$

$$E_4=\frac{-GrJ_1/a_1}{m_1^2+m_2/a_1-\frac{a_2}{a_1}},E_5=\frac{-GrJ_2/a_1}{m_2^2+sm_2/a_1-a_2/a_1},E_8=\frac{\lambda}{a_2},E_9=\frac{-\frac{GrJ_3}{a_1}}{m_1^2+\frac{sm_1}{a_1}-\frac{a_2}{a_1}},E_{10}=\frac{-\frac{GrJ_4}{a_1}}{m_2^2+\frac{sm_2}{a_1}-\frac{a_2}{a_1}},E_{11}=\frac{-\frac{GrJ_5}{a_1}}{4m_1^2+\frac{2sm_1}{a_1}-\frac{a_2}{a_1}},E_{12}=\frac{-\frac{GrJ_6}{a_1}}{(m_1+m_2)^2+\frac{2s(m_1+m_2)}{a_1}-\frac{a_2}{a_1}},E_{13}=\frac{-\frac{GrJ_7}{a_1}}{4m_2^2+\frac{2sm_2}{a_1}-\frac{a_2}{a_1}},a_{13}=E_3+E_4e^{m1}+E_5e^{m2},a_{14}=\gamma(m_1E_9+m_2E_{10}+2m_1E_{11}+(m_1+m_2)E_{12}+2m_1E_{13}),a_{15}=1-\gamma m_3,a_{16}=1-\gamma m_4,a_{17}=E_8+E_9e^{m1}+E_{10}e^{m2}+E_{11}e^{2m1}+E_{12}e^{(m1+m2)}+E_{13}e^{2m2},a_{18}=\frac{a_{14}-a_{13}}{a_{15}},a_{19}=\frac{a_{16}}{a_{15}},E_6=a_{18}-E_7a_{19},E_7=-\frac{a_{18}e^{m3}-a_{17}}{e^{m4}-a_{19}e^{m3}}$$

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