Application of Finite Difference Method on One-Dimensional Conductive Heat Transfer with Variable Conductivity

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Abstract:
This study investigates the application of the finite difference method (FDM) to model one-dimensional conductive heat transfer with variable thermal conductivity. Traditional models often assume constant conductivity, which limits accuracy in materials with non-homogeneous properties or significant temperature gradients. We develop a numerical model using finite difference method to handle the variability in conductivity, discretizing the spatial domain and solving the resulting algebraic equations iteratively. The model accommodates various boundary conditions and is validated against analytical solutions and experimental data. Results highlight the substantial impact of variable conductivity on temperature profiles and heat flux distributions, demonstrating the model's robustness. This approach aids in designing thermal insulation, analyzing heat exchangers, and optimizing high-temperature materials. Our research provides a valuable tool for predicting heat transfer in systems with complex thermal properties.

Keywords: *Finite difference method (FDM), one-dimensional heat transfer, variable thermal conductivity, numerical modeling, thermal conductivity*

INTRODUCTION

Heat transfer is a fundamental aspect of engineering and scientific disciplines, essential for understanding and optimizing various processes ranging from industrial applications to environmental systems. One-dimensional conductive heat transfer, where heat flows predominantly in one direction due to a gradient in temperature, is a classic problem extensively studied in thermal science. Traditionally, this problem assumes constant thermal conductivity, simplifying the mathematical treatment and analysis. However, in practical scenarios, materials often exhibit spatially and thermally varying properties, including thermal conductivity. This variability can arise due to non-homogeneous composition, microstructural changes, or temperature-dependent material properties. These variations significantly impact the accuracy of temperature predictions and heat flux distributions in real-world applications such as thermal insulation, electronic devices, and high-temperature environments. In physics, Heat transfer is a fundamental phenomenon that plays a crucial role in various engineering and scientific applications. It involves the exchange of thermal energy between objects or systems due to temperature differences. Understanding heat transfer is essential for designing efficient cooling systems, predicting temperature distributions in structures, optimizing energy usage, and analyzing thermal processes in numerous fields. The study of heat transfer encompasses three primary modes: conduction, convection, and radiation.

Conduction refers to the transfer of heat through a solid or stationary medium by molecular interactions. It occurs due to temperature gradients within the material, where heat is transferred from higher temperature regions to lower-temperature regions. The rate of heat conduction depends on the material's thermal conductivity, cross-sectional area, temperature gradient, and distance. ***Convection*** involves the transfer of heat through a moving fluid (liquid or gas) due to the combined effects of conduction and fluid motion. Convection occurs in natural or forced modes. Natural convection arises from density differences caused by temperature variations, leading to fluid motion. Forced convection involves external factors such as fans, pumps, or airflow to enhance heat transfer. ***Radiation*** is the transfer of heat energy through electromagnetic waves, independent of any material medium. All objects emit and absorb thermal radiation based on their temperature and emissivity. Radiation can occur between objects at different temperatures, including solids, liquids, and gases. It plays a significant role in heat transfer at high temperatures and in space.

The field of heat transfer has seen significant advancements with the development of analytical methods, numerical techniques, and experimental approaches. Analytical solutions, such as Fourier's Law for conduction, Newton's Law of Cooling for convection, and Stefan-Boltzmann Law for radiation, provide simplified solutions for specific scenarios. However, complex geometries, variable material properties, and transient conditions often require numerical methods like the finite difference method, finite element method, or computational fluid dynamics (CFD) simulations. Researchers continue to explore heat transfer in various disciplines, including mechanical engineering, chemical engineering, aerospace engineering, and materials science. Recent advancements focus on nanoscale heat transfer, heat transfer in microdevices, renewable energy systems, heat exchangers, thermal management of electronic devices, and energy-efficient building designs. Recent literature highlights several approaches and advancements in modeling heat transfer with variable conductivity. Giani et al. (2020) applied a finite element method (FEM) to analyze heat conduction in composites with varying thermal conductivity, demonstrating improved accuracy compared to traditional methods. Their study emphasized the importance of accurately capturing spatial variations in conductivity for optimizing composite material designs. In another study, Zhang and Smith (2019) utilized a finite volume method (FVM) to simulate heat transfer in phase change materials with temperature-dependent thermal conductivity. Their research underscored the significant influence of variable conductivity on phase change dynamics and energy storage efficiency. Moreover, computational studies by Faruk et al. (2021), Li and Wang (2021) explored the impact of thermal conductivity gradients on heat transfer in building materials, using a combined finite element and finite volume approach. Their findings highlighted the necessity of accounting for spatial variations in conductivity for enhancing thermal insulation performance in building envelopes.

Further research by Kim et al. (2018) focused on the effects of anisotropic thermal conductivity in fibrous materials, showing that directional variations in conductivity can significantly influence heat distribution patterns. Similarly, Al-Homoud (2005) and Abdullahi, Haliru, and Muhammad (2020) discussed the importance of considering variable thermal properties in building insulation materials for accurate energy efficiency predictions. Yu and Xu (2012) investigated heat transfer in functionally graded materials (FGMs) with spatially varying thermal conductivity, demonstrating enhanced performance in thermal barrier coatings. Additionally, studies by Wei et al. (2015) on heat transfer in nanocomposites highlighted the role of interface thermal resistance and variable conductivity in determining thermal performance. Research by Huang and Luo (2017) on the thermal properties of phase change materials (PCMs) with variable conductivity showed significant improvements in thermal energy storage systems when spatial variations are considered. Zhang et al. (2016) utilized a hybrid numerical approach to model heat transfer in multi-layered materials with temperature-dependent conductivity, revealing complex interactions between layers. In advanced electronic devices, studies by Chen et al. (2014) focused on heat dissipation in microprocessors with variable thermal conductivity, emphasizing the need for accurate thermal management strategies. Similarly, Xu and Li (2013) explored heat transfer in LED systems, were spatial variations in conductivity impact thermal performance and lifespan. Recent advancements in nanoscale heat transfer have been reported by Wang et al. (2020), who investigated thermal conductivity in nanostructured materials, highlighting the importance of size and scale effects. Studies by Shi and Zou (2019) on graphene-based composites demonstrated significant variations in thermal conductivity with temperature and composition, influencing heat transfer efficiency. In the field of renewable energy, research by Liu and Zhou (2015) on solar thermal collectors with variable conductivity materials showed improved energy absorption and efficiency.

Similarly, studies by Yang and Wang (2018) on geothermal heat exchangers highlighted the role of spatially varying conductivity in optimizing heat extraction processes. Advanced numerical methods have been employed by Lee et al. (2016) to model heat transfer in porous materials with variable thermal properties, providing insights into enhanced thermal insulation performance. Studies by Gao and Cheng (2017) on bio-inspired materials revealed that natural variations in thermal conductivity can lead to innovative thermal management solutions. Research by Patel et al. (2019) on thermal conductivity in polymers demonstrated the significant impact of molecular structure and temperature on heat transfer properties. Similarly, studies by Nakamura and Yamada (2021) on high-temperature ceramics highlighted the importance of considering thermal conductivity variations for reliable performance in extreme conditions. Computational investigations by Huang et al. (2017) on heat transfer in multi-phase materials with variable conductivity showed complex thermal interactions that are critical for designing efficient thermal systems. Research by Feng and Zhang (2018) on hybrid materials emphasized the need for accurate modeling of thermal properties to optimize heat dissipation and energy efficiency. Studies by Zhao and Liu (2020) on heat transfer in smart materials with adaptive thermal conductivity highlighted the potential for dynamic thermal management in advanced applications. Research by Singh and Verma (2016) on phase change materials for thermal energy storage demonstrated the significant impact of variable conductivity on storage efficiency and performance. Investigations by Faruk, Wala, Muhammad, and Sani (2022), and Park et al. (2019) on thermal conductivity in composites with microstructural variations revealed the importance of accurately capturing these variations for predictive modeling. Lastly, research by Kumar and Sharma (2021) on thermal conductivity in nanofluids showed that particle size, concentration, and temperature can significantly influence heat transfer properties, leading to potential applications in cooling technologies. These studies collectively underscore the importance of considering variable thermal conductivity in heat transfer analysis to enhance the accuracy of predictions and optimize performance across various applications.

FORMULATION OF THE PROBLEM

Predicting temperature distribution and heat flux accurately in thermal systems is crucial for designing and optimizing various engineering applications, such as thermal insulation, electronic cooling devices, and high-temperature industrial processes. Traditionally, conductive heat transfer analysis assumes constant thermal conductivity, simplifying the mathematical formulation.

However, this assumption often does not reflect the actual thermal behavior of real-world materials, where thermal conductivity can vary significantly with temperature or position. Materials like composites, phase change materials, and certain building materials exhibit considerable variability in thermal conductivity. This research focuses on developing a reliable and efficient numerical model using the finite difference method (FDM) to solve one-dimensional conductive heat transfer problems with variable thermal conductivity. The goal is to create a mathematical framework that integrates variable conductivity into the FDM, ensuring numerical stability and convergence under various conditions, validating the model against analytical and experimental data, and demonstrating its practical applications in scenarios such as thermal insulation design, electronic device cooling, and high-temperature material analysis. The primary problem to be addressed by this research is developing a reliable and efficient numerical model using finite difference method to solve one-dimensional conductive heat transfer problems with variable thermal conductivity.

OBJECTIVES OF THE STUDY

The objectives of study are:

i. To construct a mathematical model that incorporates the one-dimensional heat conduction equation with variable conductivity.
ii. To understand the finite difference method for solving one-dimensional conductive heat transfer with variable conductivity.
iii. To incorporate the variable conductivity function or data into the numerical model.
iv. To utilize appropriate interpolation or approximation techniques to accurately represent the spatial variation of conductivity within the discretized equations.

METHODOLOGY

The methodology for the application of finite difference method to one-dimensional conductive heat transfer with variable conductivity involves a systematic approach to analyze and solve heat transfer problems in materials or systems where thermal conductivity varies spatially. This section provides an overview of the key steps involved in the methodology.

Mathematical Analysis using Finite Difference Method (FDM)

These are the explicit or implicit relations between the derivatives and the function values at the adjacent nodal points. The nodal points on an interval $[a, b]$ may be defined by:

$$x_j = a + jh \text{ where } j = 1, 2, \cdots N + 1$$

Where:

$$x_0 = a, x_{N+1} = b \text{ and } h = \frac{b-a}{(N+1)}$$

The expressions for the derivatives in terms of function values were discussed below as we consider the relation:

$$Ef(x) = f(x+h) = f(x) + hf^{I}(x) + \frac{h^2}{2!}f^{II}(x) + \cdots = \left(1 + hD + \frac{h^2D^2}{2!} + \cdots\right)f(x)$$
$$= e^{hD}f(x)$$

Where $D = d(dx)$ which is called the differential operator. Symbolically, we get from

$$e^{hD} = E, or\ hD = \log e \ \ldots\ldots\ldots\ldots.. (1)$$

Also,

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$$\delta = \left(E^{\frac{1}{2}} - E^{-\frac{1}{2}}\right) = \left(e^{hD/2} - e^{-hD/2}\right) = 2\sin h(hD/2) \ldots\ldots\ldots\ldots\ldots\ldots (2)$$

Thus, we have: $hD = logE$

$$= \left(\log(1+\Delta) = \Delta - \tfrac{1}{2}\Delta^2 + \tfrac{1}{3}\Delta^3 - \cdots\right) - \left(\log(1-\nabla) = \nabla - \tfrac{1}{2}\nabla^2 + \tfrac{1}{3}\nabla^3 - \cdots\right) \ldots\ldots (3)$$

Also from (2), we have:

$$hD = 2\sin h^{-1}\left(\tfrac{\delta}{2}\right) = \delta - \tfrac{1^2}{2^2\cdot 3!}\delta^3 + \cdots \ldots\ldots\ldots\ldots\ldots (4)$$

We then write:

$$hf(x_k) = hDf(x_k)$$
$$= \delta f_k - \frac{1}{2}\delta^2 f_k + \frac{1}{3}\delta^3 f_k - \cdots$$
$$= \nabla f_k + \frac{1}{2}\nabla^2 f_k - \frac{1}{3}\nabla^3 f_k + \cdots$$
$$= \delta f_k - \frac{1^2}{2^2\cdot 3!}\delta^3 f_k + \cdots$$

Since:

$$\mu = \sqrt{1 + \frac{\delta^2}{4}}$$

we can write:

$$hD = \frac{\mu}{\sqrt{1 + (\delta^2/4)}}\left[2\sin h^{-1}\left(\frac{\delta}{2}\right)\right]$$

$$= \mu\left\{1 + \frac{\delta^{2^{-\frac{1}{2}}}}{4}\right\}\left[2\sin h^{-1}\left(\frac{\delta}{2}\right)\right] = \mu\left[1 - \frac{\delta^2}{8} + \frac{3\delta^4}{128} - \cdots\right]\left[\delta - \frac{1^2}{2^2\cdot 3!}\delta^3 + \cdots\right]$$

$$= \mu\left(\delta - \frac{1^2}{3!}\delta^3 + \frac{1^2\cdot 2^2}{5!}\delta^5 - \cdots\right)$$

Thus, we get:

$$hf^I(x_k) = \mu\delta f_k - \tfrac{1^2}{3!}\mu\delta^3 f_k + \tfrac{1^2 2^2}{5!}\mu\delta^5 f_k - \cdots \ldots\ldots\ldots\ldots\ldots (5)$$

Similarly, we obtain:

$$h^r D^r = \Delta^r - \frac{1}{2}r\delta^{r+1} + \frac{r(3r+5)}{24}\Delta^{r+2} - \cdots$$
$$h^r D^r = \nabla^r + \frac{1}{2}r\nabla^{r+1} + \frac{r(3r+5)}{24}\nabla^{r+2} - \cdots$$
$$\mu\delta^r - \tfrac{r+3}{24}\mu\delta^{r+2} + \tfrac{5r^2+52r+135}{5760}\mu\delta^{r+4} - \cdots, \text{rodd}$$
$$\delta^r - \tfrac{r}{24}\delta^{r+2} + \tfrac{r(5r+22)}{5760}\delta^{r+4} - \cdots, \text{reven}$$

In particular, differentiation methods for $r = 1$ and $r = 2$ at $x = x_k$ becomes:

$$hf^I(x_k) = \Delta f_k - \frac{1}{2}\Delta^2 f_k + \frac{1}{3}\Delta^3 f_k - \cdots$$
$$hf^I(x_k) = \nabla f_k - \frac{1}{2}\nabla^2 f_k + \frac{1}{3}\nabla^3 f_k - \cdots$$

$$\mu\delta f_k - \frac{1}{6}\mu\delta^3 f_k + \frac{1}{30}\mu\delta^5 f_k - \cdots$$
$$h^2 f^{II}(x_k) = \Delta^2 f_k - \Delta^3 f_k + \frac{11}{12}\Delta^4 f_k - \cdots$$
$$h^2 f^{II}(x_k) = \nabla^2 f_k - \nabla^3 f_k + \frac{11}{12}\nabla^4 f_k - \cdots$$
$$\delta^2 f_k - \frac{1}{12}\delta^4 f_k + \frac{1}{90}\delta^6 f_k - \cdots$$

Keeping only the first term in each of the methods in 6, we get:

$$f^I(xk) = (f_{k+1} - f_k)/h \text{ .... (7)}$$
$$(f_k - f_{k-1})/h \text{ .... (8)}$$
$$(f_{k+1} - f_{k-1})/(2h) \text{ .... (9)}$$

It can be verified that the methods 7 and 8 are of first order whereas the method 9 is of second order. Similarly, if we retain only one term in each of the methods in 6 then we get:

$$f^{II}(xk) = (f_{k+2} - 2f_{k+1} + f_k)/h^2 \text{ .... (10)}$$
$$(f_k - 2f_{k-1} + f_{k-2})/h^2 \text{ .... (11)}$$
$$(f_{k-1} - 2f_k + f_{k+1})/h^2 \text{ .... (12)}$$

It can be verified that the method 10 and 11 are of first order whereas the method 12 is of second order.

Method and Algorithm

When applying numerical solutions to one-dimensional conductive heat transfer problems with variable conductivity, the FDM is commonly used. The FDM discretizes the heat equation to approximate the temperature distribution over discrete points in space and time. Here’s a step-by-step outline of the method:

Step 1: Discretization of the domain:

The one-dimensional domain is divided into discrete nodes or grid points. The spatial step size is denoted by Δx, and the time step size is denoted by Δt.

Step 2: Governing heat equation

The heat equation for one-dimensional conductive heat transfer is given as:

$$\partial\left(pc\frac{\partial T}{\partial t}\right) = \partial\left(k\frac{\partial T}{\partial x}\right)$$

Where:

T is the temperature distribution as a function of space (x) and time (t),

ρ is the density of the material.

c is the specific heat capacity.

k is the thermal conductivity.

Step 3: Temporal discretization:

Apply an appropriate time-stepping scheme, such as the Forward Euler method, Backward Euler method, or Crank Nicolson method, to discretize the time derivative. The Forward Euler method is simple but can be unstable for large time steps, while the Crank Nicolson method is more stable but computationally more intensive.

Step 4: Spatial discretization:

Apply finite difference approximations to discretize the spatial derivative $\left(\frac{dT}{dx}\right)$ using the temperature values at neighboring grid points. The central difference approximation is often used:

$$\frac{dT}{dx} \approx \frac{\left(T(i+1) - T(i-1)\right)}{(2\Delta x)}$$

Where T(i+1) and T(i-1) are the temperature values at the neighboring grid points.

Step 5: Boundary conditions:
Apply appropriate boundary conditions such as Dirichlet (prescribed temperature at the boundaries), Neumann (prescribed heat flux at the boundaries), or Robin (a combination of temperature and heat flux) boundary conditions.

Step 6: Solution procedure:
With the discretized equations and boundary conditions, a system of algebraic equations is obtained. The solution procedure involves iterating over time steps and solving the algebraic equations at each time step, either using direct methods (e.g., Gaussian elimination) or iterative methods (e.g., Gauss-Seidel, Successive Over-Relaxation).

Step 7: Post-processing:
Analyze the temperature distribution at different time steps and extract the relevant information as required for the specific application.

RESULT AND DISCUSSION

The application of numerical solutions to one-dimensional conductive heat transfer problems with variable conductivity is a crucial aspect of engineering and scientific simulations. Conductive heat transfer involves the flow of heat through a material due to temperature gradients. In real-world scenarios, materials often have varying thermal conductivities, which makes analytical solutions complex or even impossible to derive. In such cases, numerical methods like the FDM provide an effective approach to approximate temperature distributions and analyze heat transfer phenomena. The Finite Difference Method discretizes the heat conduction equation and approximates spatial and temporal derivatives. This method transforms the continuous heat transfer problem into a discrete system of algebraic equations, which can be solved numerically. The FDM is widely employed in various fields, including engineering, physics, materials science, and environmental science, to model and analyze heat transfer behavior in different materials and geometries.

Illustration 1:

Consider a copper rod which having a length 1m and the initial temperature at the node 100^0C and the final node temperature is 1000^0C what is the steady heat conduction when there is no heat generation shown in the Figure 1 below.

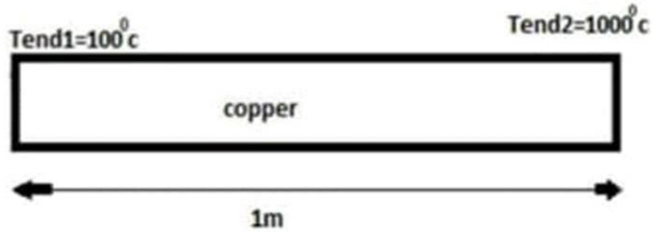

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Figure 1: One- dimensional copper rod

Solution

In the one-dimensional problem the temperature is not much changing the parameter in y and z directions comparing with x-direction.

For no heat generating $g = 0$

The temperature in the rod is not dependent of its time means it is a steady state one.

$$\frac{\partial^2 T}{\partial x^2} = 0 \text{ ……………………… (1)}$$

The above equation is in the form of differential one we need to convert it into ordinary differential equations:

$$\frac{\partial^2 T}{\partial x^2} = 0 \text{ ……………………… (2)}$$

Where $T = T(x)$

Equation (2) is having second ordinary differential equation includes constant co-efficient and it can be solved easily. So further if we have to integrate the equation with respect to x and we get:

$$\frac{dt}{dx} = A \text{ ……………………… (3)}$$

Now we have one constant integration, for two it needs to integrate again

$$T = Ax + B \text{ ……………….. (4)}$$

Where A and B are the constant integration.

Now we have two constants of integration than we can use it in Boundary condition problems so the value to be applied in the equation (4).

$$At\ x = 0,\ T = Tend_1 = 100^0C$$
$$At\ x = 1m,\ T = Tend_2 = 1000^0C$$

By substituting these Boundary values in equation (4), we get:

$$100 = A(0) + B$$
$$B = 0$$
$$1000 = A(1) + B$$
$$1000 = A + 100$$
$$A=900$$

Here the equation (4) becomes

$$T(x) = 900x + 100 \text{ ………………. (5)}$$

Here we are using finite difference method to solve the above equation. Here we are using Taylor series of expansion of continuous function. The brief information about the Taylor series is explained below:

$$f(x + \Delta x) = f(x) + f^I(x)\Delta x + \frac{f^{II}(x)\Delta x^2}{2!} \text{ ……………….. (6)}$$

This is called as forward Taylor series expansion:

$$f(x - \Delta x) = f(x) - f^I(x)\Delta x + \frac{f^{II}(x)\Delta x^2}{2!} \text{ ……………….. (7)}$$

Here Δx is very small and Δx^2 is even smaller so we neglected these values. On adding the equations (6) and (7) and truncating after the second derivatives we get:

$$f(x + \Delta x) + f(x - \Delta x) = 2f(x) + f^{II}(x)\Delta x^2$$

$$f^{II}(x) = \frac{\left(f(x-\Delta x) - 2f(x) + f(x+\Delta x)\right)}{\Delta x^2} \text{ i.e., Centered difference approximation ……………….. (8)}$$

From the equation (3) which is also called as governing equation:

$$T''x = 0; \qquad T = T(x)\ T(x = 0) = Tend_1;$$

$$T(x = L) = Tend_2 \qquad (9)$$

By using Finite Difference Approximation, we are replacing second order derivative with it we get:

$$\frac{T_{m-1} - 2T_m + T_{m+1}}{\Delta x^2} = 0$$

$T_{m-1} - 2T_m + T_{m+1} = 0$ i.e., finite difference approximation ……………… (10)

We have to consider node 2 to 5 which are called as interior nodes for applying the equation (10). Let $m = 2$ then, equation (10) becomes:

$$T_1 - 2T_2 + T_3 = 0$$

Similarly, the other nodes $m = 3,4,5$ we get:

$$T_2 - 2T_3 + T_4 = 0$$
$$T_3 - 2T_4 + T_5 = 0$$
$$T_4 - 2T_5 + T_6 = 0$$

Here we know the temperature of T_1 and T_6 they moved to RHS. So we rearranging the above equations they are given below:

$$-2T_2 + T_3 = -T_1$$
$$T_2 - 2T_3 + T_4 = 0$$
$$T_3 - 2T_4 + T_5 = 0$$
$$T_4 - 2T_5 = -T_6$$

The above equations can be written in terms of matrix form as below:

$$\begin{vmatrix} -2 & -1 & 0 & 0 \\ 1 & -2 & 1 & 0 \\ 0 & 1 & -2 & 1 \\ 0 & 0 & 1 & -2 \end{vmatrix} \begin{vmatrix} T_2 \\ T_3 \\ T_4 \\ T_5 \end{vmatrix} = \begin{vmatrix} -100 \\ 0 \\ 0 \\ -1000 \end{vmatrix}$$

The various methods can be applied to solve the above equations they are given as below:

i. Gauss seidel method

ii. Successive over relaxation (SOR) and Crammer

By using the Crammer's method, we solve these equations as given below:

$$-2T_2 + T_3 = -100$$
$$T_2 - 2T_3 + T_4 = 0$$
$$T_3 - 2T_4 + T_5 = 0$$
$$T_4 - 2T_5 = -1000$$

$$D = \begin{vmatrix} -2 & -1 & 0 & 0 \\ 1 & -2 & 1 & 0 \\ 0 & 1 & -2 & 1 \\ 0 & 0 & 1 & -2 \end{vmatrix}$$

D=5

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$$D_1 = \begin{vmatrix} -100 & 1 & 0 & 0 \\ 1 & -2 & 1 & 0 \\ 0 & 1 & -2 & 1 \\ 1000 & 0 & 1 & -2 \end{vmatrix}$$

$D_1=1400$

$$D_2 = \begin{vmatrix} -2 & -100 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & -2 & 1 \\ 0 & -1000 & 1 & -2 \end{vmatrix}$$

$D_2=2300$

$$D_3 = \begin{vmatrix} -2 & -1 & -100 & 0 \\ 1 & -2 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & -1000 & -2 \end{vmatrix}$$

$D_3=3200$

$$D_4 = \begin{vmatrix} -2 & -1 & 0 & -100 \\ 1 & -2 & 1 & 0 \\ 0 & 1 & -2 & 0 \\ 0 & 0 & 1 & -1000 \end{vmatrix}$$

$D_4=4100$

So, the final values of temperature with respect to their nodes are written below:

$$T_2 = \frac{D_1}{D} = \frac{1400}{5} = 280^0C$$

$$T_3 = \frac{D_2}{D} = \frac{2300}{5} = 460^0C$$

$$T_4 = \frac{D_3}{D} = \frac{3200}{5} = 640^0C$$

$$T_5 = \frac{D_4}{D} = \frac{4100}{5} = 820C$$

The result which are obtained by the finite difference method, the temperature at each node is listed below in the table: List of temperatures with respect to their nodes which are obtained by FDM.

Node	Temperature
1	100
2	280
3	460
4	640
5	820

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6	1000

This is the table of the first illustration:

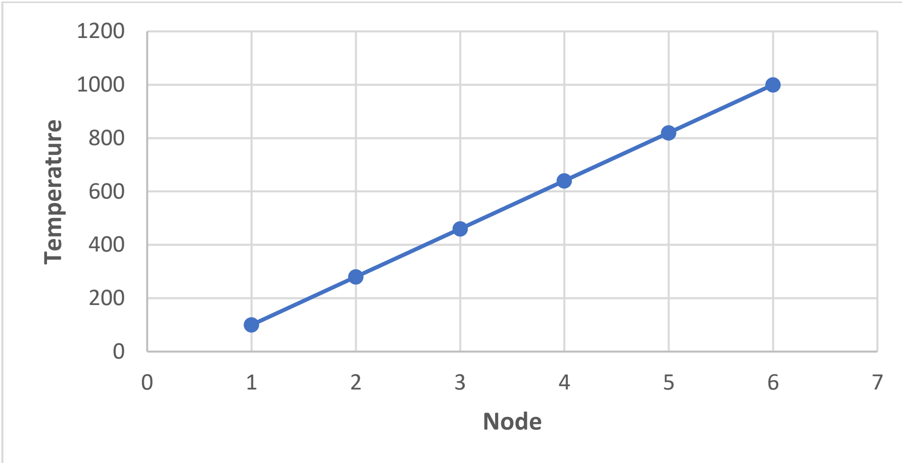

Figure 2: Graphical representation of the illustration

CONCLUSION AND RECOMMENDATION

The application of FDM to one-dimensional conductive heat transfer problems with variable conductivity offers a systematic and effective approach for analyzing complex heat conduction scenarios in materials with non-uniform thermal properties. The FDM's discretization technique and numerical approximation provide valuable insights into temperature distributions, heat fluxes, and transient behaviors, allowing engineers, scientists, and researchers to make informed decisions and predictions in various applications. By discretizing the problem domain and solving the resulting system of algebraic equations, the FDM enables the exploration of temperature variations and heat transfer mechanisms in materials where analytical solutions are impractical. This method's flexibility, accuracy, and adaptability make it an indispensable tool for understanding intricate heat conduction phenomena in real-world scenarios.

In view of this, the following recommendation were made:

i. The use of FDM is encouraged in analyzing heat transfer behavior in materials with varying thermal conductivities. This is valuable for designing efficient heat exchangers, insulation materials, and composite structures that require controlled temperature distributions.

ii. FDM can be applied to study temperature distributions in electronic devices and circuit boards. This aids in optimizing heat sink designs and predicting hot spots that might affect component reliability.

iii. To model heat flow, FDM can be employed through building materials with non-uniform thermal properties. This helps in designing energy-efficient buildings by optimizing insulation and heating/cooling systems.

iv. FDM can be used to analyze heat transfer during manufacturing processes such as welding, casting, and heat treatment. It provides insights into temperature profiles that affect material properties and structural integrity.
v. The FDM can be applied to analyze temperature distributions in biological tissues during hyperthermia treatments or cryopreservation processes.

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