ROLE OF VISCOUS DISSIPATIVE COUETTE FLOW WITH WALL CONCENTRATION EFFECT IN A CONVECTIVELY HEATED CHANNEL

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Abstract

The heat and mass transfer problem of a viscous dissipative fluid with symmetric wall concentration conditions is solved analytically using the homotopy perturbation method. Physical components of engineering importance in terms of Sherwood number, Nusselt number, and skin friction are also calculated, along with the governing equations controlling the flow configuration. The study's main findings suggest that reverse flow can be produced and controlled when mass flow with a wall concentration effect is present. Furthermore, a graphic comparison of the present work's findings with previous research for the limiting scenario establishes the validity and accuracy of the current study. A high degree of consistency was verified.

Keywords: *Couette Flow, Mass flow, Wall concentration, Viscous dissipation, Convective boundary conditions, Homotopy perturbation method.*

INTRODUCTION

Free convection in a boundary layer cavity is the flow caused by the interplay of gravity and density changes inside a fluid. These changes are visible as a result of energy or mass diffusion components, or their interaction. Studies on free convection flow in vertical parallel plate channels for a single-phase incompressible viscous fluid have received a lot of attention recently, both theoretically (exact or approximate solutions) and experimentally, because natural convection heat transfer is used in many practical applications (Al-Subaie and Chamkha 2004, Zullkifree *et al*. 2019). Couette flow is the laminar flow of a viscous fluid over two parallel walls with one moving tangentially to the other. A pressure gradient in the flow direction may serve as the driving force behind this flow (Mng'ang'a, 2023). On the aforementioned topic, several scholars have made important contributions. Omokhuale and Ojemeri (2024) employed the homotopy perturbation approach to analyze the effect of viscous dissipation on a buoyancy-induced Couette flow in a vertical channel impacted by Newtonian heating in order to illustrate the phenomena of free convection Couette flow. Their findings suggest that Newtonian heating settings and viscous dissipation are intended to increase fluid velocity. To ascertain how Navier slip heating affects the

Couette flow of a viscoelastic dusty fluid and the transmission of heat in a rotating frame, Khan *et al.* (2022) examined the second law. Heat transfer within two infinitely parallel porous plates in an inclined magnetic field with unstable MHD Couette flow was examined by Joseph *et al.* (2014). Mosayebidorcheh *et al.* (2015) studied the temperature-dependent unsteady magnetized Couette flow and heat transfer of dusty fluid. Thermal radiation with unsteady MHD free convective Couette flow was studied by Jha *et al.* (2015). Raju *et al.* (2016) used the finite element approach to examine the effects of thermal diffusion and diffusion thermodynamics on a natural convection Couette flow. Job and Gunakala (2016) used Galerkin's finite element technique to explain the effect of the radiation factor across an upright porous wall. Additionally, Ali *et al*. (2017) examined the three-dimensional Couette flow of a Maxwell fluid with recurring injection and suction. The instability of an electrically conducting fluid's MHD Couette flow was examined by Hussain *et al.* (2018). The impact of constant and radioactive pressure gradients on an unsteady MHD Couette flow was examined by Anyanwu *et al.* (2020). Taking into account the influence of a heat source factor, Ajibade *et al.* (2021) examined the effect of a viscous dissipative fluid on steady free convection Couette flow across a vertical channel. Furthermore, in the works of Omokhuale and Jabaka (2022a, 2022b) and Omokhuale and Dange (2023), the impacts of heat sink, radiation, heat source, magnetic field, and thermal properties on unstable convective Couette flow were thoroughly examined.

The assumptions of constant surface temperature, ramping wall temperature, or constant surface heat flux are typically used for modeling free convection flows on parallel plates (Rajput 2011a, 2011b, Narahari 2012, Singh and Sarveshanand 2015). The initial assumptions are incorrect, even though in the majority of real-world situations, the heat transfer from the surface is thought to be proportional to the local surface temperature. The proportionality condition of the heat transfer to the local surface temperature is known as Newtonian heating, and these kinds of flows are called conjugate convective flows (Lesnic *et al.* 2004). Merkin (1994) was the first to study the flow of the natural convection boundary layer over a vertical surface with Newtonian heating. Given the foregoing, Hamza *et al.* (2025a, 2025b) recently used the renowned special collocation technique to examine the effects of nonlinear radiation on unsteady free and mixed bioconvection with nonlinear density fluctuation over certain nanoparticles impacted by Newtonian heating. The magnetized flow of a Newtonian fluid influenced by Newtonian heating across a stretched sheet in the presence of a porous material was studied analytically and numerically by Swain *et al*. (2020). The three-dimensional flow of Jeffrey fluid with Newtonian heating was examined by Shehzad *et al*. (2014). The impact of Newtonian heating and Navier slip on the transient flow of an exothermic fluid in a vertical channel was highlighted by Hamza (2016). The unsteady heat and mass transfer of a hydromagnetic Casson fluid over a vertical plate in the presence of thermal radiation and chemical reaction was numerically analyzed by Das *et al.* (2015). The heat and mass transmission of a water-B nanofluid over a stretched sheet while thermal radiation and chemical reaction coexisted was documented by Qayyum *et al.* (2017). The effects of MHD flow of Powell-Eyring fluid by a stretched cylinder with thermal radiation through an angled magnetic field under the Newtonian heating effect were examined by Hayat *et al*. (2018). The unsteady MHD heat and mass transfer flow of Jeffrey fluid over an oscillating vertical plate induced by thermal radiation and Newtonian heating factors was precisely and numerically solved by Zin *et al.* (2018).

Viscous dissipation is the term used to describe the internal heat emission that results from viscous forces in the molecular fluid-particle interaction. The mechanical heat lost in this manner has a significant impact on the thermal buoyancy and thermal properties of fluid motion. Thus, in the majority of lubricating sectors, viscous dissipation is essential. The applications of these fluid flow types in science, engineering, and industrial technologies such as gas drainage, electrical appliance cooling, geothermal energy, plasma physics, gas turbines, the burning of fossil fuels, the food processing industry, and so forth—have drawn a lot of attention (Ajibade *et al.* 2021). Gebhart's (1962) study, which examined the impact of viscous dissipation on natural convection using a perturbation approach, was among the first to consider viscous dissipation. Numerous writers have provided examples of their work in this area over the years. To do this, Isa *et al.* (2024) used the Implicit Finite Difference Method (IFDM) to computationally examine the effects of time-dependent magnetized free convection heat transfer flow of viscous dissipative fluid in an upstanding channel with convective heating conditions. They came to the conclusion that increasing the viscous dissipation parameter over time is seen to increase fluid velocity. Usman *et al.* (2023) studied the combined impacts of Soret and Dufour on a magnetized natural convection flow in the presence of viscous dissipation via an upstanding channel. The effects of MHD and superhydrophobicity on a viscous dissipative fluid in a slit microchannel were examined by Yale *et al.* (2023) employing a semi-analytical method. Considering the effects of MHD, chemical reaction, porous medium, and heat sink, Omokhuale *et al.* (2024a, 2024b) emphasized the importance of viscous dissipation on both free and mixed convective flows of a nanofluid flow over a semi-infinite flat plate in an oscillating system. Uka *et al.* (2023) used a well-known regular perturbation and the Mathematica V.10 method to analytically and numerically address the fluid flow problem in the presence of viscous dissipation and porosity effects caused by a moving wedge. Amar *et al.* (2023) investigated how heat energy and viscous dissipation affect the MHD heat transfer flow of Casson fluid across a moving wedge with a convective boundary condition, taking internal heat production and absorption into account. Under the action of constant suction, Uwanta and Usman (2014) analyzed heat and mass transfer flow affected by thermo-diffusion and diffusion-thermo effects through a viscous dissipative fluid. Other studies that addressed the influence of viscous dissipation on fluid flow include those of Ajibade and Umar (2020a, 2020b), Ojemeri and Onwubuya (2023), and Prakash and Sivakumar (2018), and more.

It is particularly challenging to find closed-form solutions for flow problems with coupled, nonlinear governing equations. Therefore, such issues can be tackled through numerical systems or approximate solution processes. The perturbation method is among the most effective strategies. In contrast, the solutions of the perturbation approach are limited to modest perturbation parameters. To get around this restriction, a different technique called the homotopy perturbation approach was created. He (1999) applied the approach to linear, nonlinear, or coupled equations in partial or conventional forms. He used a homotopy perturbation procedure to study a nonlinear coupling method (He 2000). A novel nonlinear analytical approach based on homotopy perturbation was later introduced by He (2003). Adamu (2017) also came to the conclusion that HPM is a reliable and efficient method for resolving coupled differential equations. With outstanding outcomes, Ajibade and Tafida (2020) proved the efficacy of their HPM-based approach. According to Nasution (2023), simulations showed that HPM findings largely replicated numerical calculations utilizing the 4th-order Runge-Kutta technique, and demonstrated that HPM is suitable for dealing with nonlinear differential equations.

When heat and mass transfer do not happen concurrently, several chemical engineering operations, including the synthesis of polymers, the creation of ceramics or glassware, and the processing of food, may not be accomplished. Heat and mass transfer take place simultaneously to support processes including drying, evaporation at a water body's surface, energy transfer in a wet cooling tower, and movement in a desert cooler. Moreover, reverse flow is promoted by symmetric wall concentration. These benefits are all the drawbacks of the earlier research. In order to close this gap, this research builds on the work of Kabir and Abdulganiyu (2024) by examining the heat and mass transfer flow of a viscous dissipative Couette flow impacted by the Soret factor and wall concentration conditions in a convectively heated channel. Overall, the results of this study would be important for the chemical, power, and petroleum industries for the manufacturing of glassware and ceramics, catalytic chemical reactors, polymer production, starting exothermic or endothermic chemical reactions, and other uses.

PROBLEM FORMULATION AND SCHEMATIC DIAGRAM

We consider a steady buoyancy-induced Couette flow of an incompressible viscous fluid flowing along two vertical parallel plates affected by symmetric wall concentration conditions. As shown in Figure 1, both walls, $y_0 = 0$ and $y_0 = h$, which are the upper and lower walls, are kept at constant temperatures T_1 and T_0, respectively. It is presumed that the flow occurs in x'-axis, vertically upward through the plates, but the y'-direction is perpendicular to the plates. All the fluid properties are assumed to be constants. The flow variables are functions of space y only. We make further assumptions for the present problem:

i. The flow is steady, laminar, and fully developed.
ii. The fluid is assumed to be affected by Viscous dissipation.
iii. The fluid is assumed to be influenced by the mass transfer effect.
iv. The plates are assumed to be heated asymmetrically, with one plate maintained at a temperature T_0 while the other plate is at a temperature T_h.
v. Due to this temperature gradient between the plates, it is assumed that natural convection flow occurs in the microchannel.
vi. The effect of symmetric wall concentration is considered.

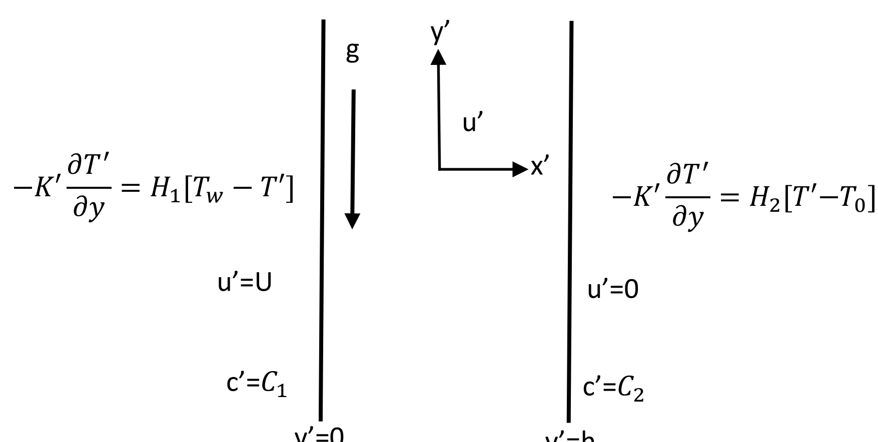

Figure 1: Geometry of the flow

Following Kabir and Abdulganiyu (2024) and Hamza *et al.* (2024), and employing Boussinesq's approximations with convective boundary conditions, the dimensional governing equations for the current formulation can be modelled as:

$$v\frac{d^2u'}{dy'^2} + g\beta_1(T' - T_O) + g\beta_2(C' - C_O) = 0 \tag{1}$$

$$\alpha\frac{d^2T'}{dy'^2} + \frac{\mu}{\rho c_\rho}\left(\frac{du'}{dy'}\right)^2 = 0 \tag{2}$$

$$D_m\frac{d^2C'}{dy'^2} = 0 \tag{3}$$

Subject to the boundary conditions:

$$u' = U, -K' = H_1(T_w - T'), C' = \Gamma_1 \quad \text{at} \quad y' = 0 \tag{4}$$

$$u' = 0, -K' = H_2(T' - T_0), C' = \Gamma_2 \quad \text{at} \quad y' = h \tag{5}$$

Given that the dimensionless quantities and parameters employed are as follows:

$$u = \frac{u'}{U}, y = \frac{y'}{h}, T = \frac{T' - T_0}{T_w - T_o}, C = \frac{C' - C_0}{C_w - C_o}, Pr = \frac{v}{\alpha}, \alpha = \frac{K}{\rho c_\rho}, Ec = \frac{u^2}{c_p(T_w - T_o)}, B_r = EcPr$$

$$Gr = \frac{g\beta h^2(T_w - T_o)}{v}, \mathrm{B}_j = \frac{hd}{,k}, Sc = \frac{v}{D} \tag{6}$$

Substituting eqn (6) into eqns (1-5), we have the following equations in their basic form as

$$\frac{d^2U}{dy^2} + GrT + GcC = 0 \tag{7}$$

$$\frac{d^2T}{dy^2} + EcPr\left[\frac{dU}{dy}\right]^2 = 0 \tag{8}$$

$$\frac{d^2C}{dy^2} = 0 \tag{9}$$

Restricted to the boundary conditions:

$$U = 1, \quad \frac{dT}{dy} = Bj(T - 1), \quad C = \Gamma_c \quad at \quad y = 0 \tag{10}$$

$$U = 0, \quad \frac{dT}{dy} = -BjT, \quad C = -\Gamma_c \quad at \quad y = 1 \tag{11}$$

METHOD OF SOLUTION

The nonlinear coupled equations are solved using the homotopy perturbation method (HPM), such that the solutions of the velocity, temperature, and concentration are assumed to be in infinite series form as;

$$U = u_0 + pu_1 + p^2u_2 + p^3u_3 + \dots \tag{12}$$

$$T = t_0 + pt_1 + p^2t_2 + p^3t_3 + \dots \tag{13}$$

$$C = c_0 + pc_1 + p^2c_2 + p^3c_3 + \dots \tag{14}$$

Constructing a convex homotopy on eqns (7 – 11) and substituting eqns (12 – 14) into it, so that we have the following set of equations without requiring an initial approximation

$$(1-p)\frac{d^2U}{dy^2} + p[GrT + GcC] = 0 \tag{15}$$

$$(1-p)\frac{d^2T}{dy^2} + p\left[EcPr\left[\frac{dU}{dy}\right]^2\right] = 0 \tag{16}$$

$$(1-p)\frac{d^2C}{dy^2} = p[0] \tag{17}$$

Comparing like powers of p, we obtain the velocity, temperature, and concentration equations with their transformed boundary conditions as follows:

Velocity Gradient

$$p^0: \frac{d^2u_0}{dy^2} = 0 \tag{18}$$

$$p^1: \frac{d^2u_1}{dy^2} = -Grt_0 - Gcc_0 \tag{19}$$

$$p^2: \frac{d^2u_2}{dy^2} = -Grt_1 - Gcc_1 \tag{20}$$

$$p^3: \frac{d^2u_3}{dy^2} = -Grt_2 - Gcc_2 \tag{21}$$

Where the corresponding boundary conditions become:

$$\left.\begin{array}{c} u_0(0) = 1, u_1(0) = u_2(0) = u_3(0) = 0 \\ u_0(1) = u_1(1) = u_2(1) = u_3(1) = 0 \end{array}\right\} \tag{22}$$

Temperature Gradient

$$p^0: \frac{d^2t_0}{dy^2} = 0 \tag{23}$$

$$p^1: \frac{d^2t_1}{dy^2} = -EcPr\left[\frac{du_0}{dy}\right]^2 \tag{24}$$

$$p^2: \frac{d^2 t_2}{dy^2} = -2EcPr \frac{du_0}{dy} . \frac{du_1}{dy} \tag{25}$$

$$p^3: \frac{d^2 t_3}{dy^2} = -2EcPr \frac{du_0}{dy} . \frac{du_2}{dy} - EcPr \left[\frac{du_1}{dy}\right]^2 \tag{26}$$

Where the corresponding boundary conditions becomes:

$$\left.\begin{array}{l} t_0'(0) = Bj(t_0 - 1), t_1'(0) = Bjt_1, t_2'(0) = Bjt_2, t_3'(0) = Bjt_3 \\ t_0'(1) = -Bj, t_1'(1) = -Bjt_1, t_2'(1) = -Bjt_2, t_3'(1) = -Bjt_3 \end{array}\right\} \tag{27}$$

Mass diffusion Gradient

$$p^0: \frac{d^2 c_0}{dy^2} = 0 \tag{28}$$

$$p^1: \frac{d^2 c_1}{dy^2} = 0 \tag{30}$$

$$p^2: \frac{d^2 c_2}{dy^2} = 0 \tag{31}$$

$$p^3: \frac{d^2 c_3}{dy^2} = 0 \tag{32}$$

Where the corresponding boundary conditions becomes:

$$\left.\begin{array}{ll} c_0(0) = \Gamma_c, & c_1(0) = c_2(0) = c_3(0) = 0 \\ c_0(1) = -\Gamma_c, & c_1(1) = c_2(1) = c_3(1) = 0 \end{array}\right\} \tag{33}$$

The solutions of velocity, temperature and concentration have been obtained as follows:

$$u_0 = B_1 y + B_2 \tag{34}$$

$$u_1 = -GrA_1 \frac{y^3}{6} - GrA_2 \frac{y^3}{6} - GcJ_1 \frac{y^3}{6} - GcJ_2 \frac{y^2}{2} + B_3 y + B_4 \tag{35}$$

$$u_2 = GrEcPr \frac{y^3}{6} - GrA_3 \frac{y^3}{6} - GrA_4 \frac{y^2}{2} + B_5 y + B_6 \tag{36}$$

$$u_3 = 2Gr^2 EcPr \left(A_1 \frac{y^6}{720} + A_2 \frac{y^5}{120}\right) + 2EcPrGrGc \left(J_1 \frac{y^6}{720} + J_2 \frac{y^5}{120}\right) - 2EcPrGr \frac{y^4}{24} - GrA_5 \frac{y^3}{6} - GrA_6 \frac{y^2}{2} + B_7 y + B_8 \tag{37}$$

$$\theta_0 = A_1 y + A_2 \tag{38}$$

$$\theta_1 = -B_1 \frac{y^2}{2} + A_3 y + A_4 \tag{39}$$

$$\theta_2 = -2EcPrGr\left(A_1\frac{y^4}{24} + A_2\frac{y^3}{6}\right) - 2EcPrGc\left(J_1\frac{y^4}{24} + J_2\frac{y^3}{6}\right) + 2EcPrB_3\frac{y^2}{2} + A_5y + A_6 \quad (40)$$

$$\theta_3 = 2EcPr\left(GrEcPr\frac{y^5}{120} - GrA_3\frac{y^4}{24} - GrA_4\frac{y^3}{6} + B_5\frac{y^2}{2}\right) - EcPrk_5\frac{y^6}{30} - EcPrk_4\frac{y^5}{20} - EcPrk_5\frac{y^4}{12} - EcPrk_6\frac{y^3}{6} - EcPrB_3^2\frac{y^2}{2} + A_7y + A_8 \quad (41)$$

$$c_0 = J_1y + J_2 \quad (42)$$

$$c_1 = 0 \quad (43)$$

$$c_2 = 0 \quad (44)$$

$$c_3 = 0 \quad (45)$$

Substituting eqns (34 to 37) into eqn (12), we have:

$$U = B_1y + B_2 - GrA_1\frac{y^3}{6} - GrA_2\frac{y^3}{6} - GcJ_1\frac{y^3}{6} - GcJ_2\frac{y^2}{2} + B_3y + B_4 + GrEcPr\frac{y^3}{6} - GrA_3\frac{y^3}{6} - GrA_4\frac{y^2}{2} + B_5y + B_6 + 2Gr^2EcPr\left(A_1\frac{y^6}{720} + A_2\frac{y^5}{120}\right) + 2EcPrGrGc\left(J_1\frac{y^6}{720} + J_2\frac{y^5}{120}\right) - 2EcPrGr\frac{y^4}{24} - GrA_5\frac{y^3}{6} - GrA_6\frac{y^2}{2} + B_7y + B_8 \quad (49)$$

Also, substituting eqns (38 to 41) into eqn (13), we have:

$$\theta = A_1y + A_2 - B_1\frac{y^2}{2} + A_3y + A_4 - 2EcPrGr\left(A_1\frac{y^4}{24} + A_2\frac{y^3}{6}\right) - 2EcPrGc\left(J_1\frac{y^4}{24} + J_2\frac{y^3}{6}\right) + 2EcPrB_3\frac{y^2}{2} + A_5y + A_6 + 2EcPr\left(GrEcPr\frac{y^5}{120} - GrA_3\frac{y^4}{24} - GrA_4\frac{y^3}{6} + B_5\frac{y^2}{2}\right) - EcPrk_5\frac{y^6}{30} - EcPrk_4\frac{y^5}{20} - EcPrk_5\frac{y^4}{12} - EcPrk_6\frac{y^3}{6} - EcPrB_3^2\frac{y^2}{2} + A_7y + A_8 \quad (50)$$

Similarly, substituting eqns (42 to 45) into eqn (14), we have:

$$C = J_1y + J_2 \quad (51)$$

To obtain the mass flux, $Q_m = \int_0^1 Udy$, we have

$$Q_m = \frac{B_1}{2} + B_2 - \frac{GrA_1}{24} - \frac{GrA_2}{6} - \frac{GcJ_1}{24} - \frac{GcJ_2}{6} + \frac{B_3}{2} + B_4 + \frac{GrEcPr}{120} - \frac{GrA_3}{24} - \frac{GrA_4}{6} + B_6 + \frac{Gr^2EcPrA_1}{2520} + \frac{Gr^2EcPrA_2}{360} + \frac{GrEcPrJ_1}{2520} + \frac{GrEcPrJ_2}{360} - \frac{GrEcPrB_3}{60} - \frac{GrA_5}{24} - \frac{GrA_6}{6} + \frac{B_7}{2} + B_8 \quad (52)$$

Where the skin friction, Nusselt number, and Sherwood number are computed as:

$$\frac{dU}{dy}\ |_{y=0} = B_1 + B_2 + B_3 \quad (53)$$

$$\frac{dU}{dy}\ |_{y=1} = B_1 - \frac{GrA_1}{2} - GrA_2 - \frac{GcJ_1}{2} - GcJ_2 + B_3 + \frac{GrEcPr}{6} - \frac{GrA_3}{2} - GrA_4 + B_5 + 2EcPrB_3 + \frac{EcPrGr^2A_1}{60} - \frac{EcPrGr^2A_2}{12} + \frac{EcPrGrJ_1}{60} + \frac{EcPrGcGrJ_2}{12} - \frac{EcPrGrB_3}{3} - \frac{GrA_5}{2} - GrA_6 + B_7 \quad (54)$$

$$\frac{d\theta}{dy}\ |_{y=0} = A_1 + A_2 + A_3 \quad (55)$$

$$\frac{d\theta}{dy}\ |_{y=1} = A_1 - Bj + A_3 - \frac{BjGrA_1}{3} - BjGrA_2 - \frac{BjGcJ_1}{3} - EcPrGcJ_2 + 2EcPrB_3 + A_5 + \frac{Ec^2Pr^2}{12} - \frac{EcPrGrA_3}{3} - EcPrGrA_4 + 2EcPrB_5 - \frac{EcPrk_5}{5} - \frac{EcPrk_4}{4} - \frac{EcPrk_5}{3} - \frac{EcPrk_6}{2} - EcPrB_3^2 + A_7 \quad (56)$$

$$\frac{dC}{dy}\ |_{y=0} = J_1 \quad (57)$$

$$\frac{dC}{dy}\ |_{y=1} = J_1 \quad (58)$$

All the constants used are indicated in the appendix section

RESULTS AND DISCUSSION

A description of the findings is given in this section. Graphs showing the temperature and velocity analytical solutions are examined for a range of monitoring parameter values. Additionally, tables with the computed numerical values for the mean temperature, mass flow, skin friction, and rate of heat transfer are supplied for additional analysis. The default range of values taken for this present study are $1 \leq Bj \leq 100, 1 \leq Ec \leq 0.1, 1 \leq Pr \leq 0.1, 2 \leq Gr \leq 10, 30 \leq Gc \leq 10, 0.1 \leq Ec \leq 1.0,$ as they relate to physical situations in real life.

The impact of the wall concentration (Γ_c) condition on concentration and velocity gradients is described in Figures 2 and 3, respectively. It is fascinating to report that the presence of the wall concentration parameter causes it to rise at the lower plates while it decreases at the upper plate. There is, however, a point of intersection at exactly the middle of the vertical channel where the flow is independent of the parameter under consideration. In other words, the symmetric heating of the channel has created a reverse flow both in the concentration and velocity distributions. The phenomenon of reverse flow or backflow, including its understanding and management, has a significant impact on fluid dynamics applications such as heat transfer improvement, sediment resuspension, mixing and dispersion, and dynamic stall control. Additionally, the results obtained in this work corroborate those reported by Hamza *et al.* (2024). The behavioral pattern of thermal Grashof number (Gr) and solutal Grashof number (Gc) is explained in Figures 4 and 5, respectively, on the velocity gradient. Because the buoyant forces outweigh the viscous forces, it is evident that the action of Gr increases the fluid velocity. As a result, fluid particles in places with higher energy tend to migrate toward areas with lower energy. A point of junction is shown at the center of the channel, indicating that Gc, on the other hand, produces a reverse flow. This is because of the flow configuration's symmetric wall concentration.

Figures 6 and 7 show the temperature and velocity curves for various Eckert number (Ec) values. Higher Eckert numbers cause both distributions to rise because the fluid's kinetic energy exceeds the enthalpy difference in the boundary layer. Convection currents become stronger as a result, decreasing fluid density and raising the temperature and velocity profiles inside the channel. Additionally, it is seen that the temperature distribution is more significantly impacted by the Eckert number in the vicinity of the cold plate, with a point of intersection observed close to the channel's center. Figures 8 and 9 depict the influence of Prandtl number (Pr) on the gradients of fluid temperature and velocity. Temperature and velocity improve as the Prandtl number grows. This is caused by momentum diffusivity outperforming heat diffusivity. Naturally speaking, when the momentum diffusivity rate exceeds the heat diffusivity, the fluid's boundary layer thickness expands, enhancing convection currents through the channel and resulting in larger temperature and velocity distributions. With the help of Figures 10 and 11, the function of the Biot number (Bj) on the velocity and temperature components is elucidated. respectively. The temperature distribution near the heated plate swells as the Biot number goes up, whereas it falls near the cold plate as the Biot number increases, as seen in Figure 10. Figure 11 shows that when the Biot number rises, the velocity profile falls. Because internal conduction resistance is greater than exterior convection resistance, heat transmission to the fluid is impacted, lowering its temperature and impairing fluid velocity.

The calculated numerical values for the heat transfer amount at both surfaces are displayed in Table 1. The table makes it clear that when the Eckert number rises, the rate of heat transfer at the lower plate decreases, but at the top plate, the opposite effect is seen. Furthermore, the rate of heat transmission increases as the Prandtl number rises. The computed numerical values of skin friction at both plates are displayed in Table 2. For larger values of Eckert numbers and Prandtl numbers, respectively, it is shown that increasing the wall concentration reduces skin friction at the lower plates, whereas the top plate exhibits the opposite effect. The mass flux values are shown in Table 3. When the solutal Grashof number and wall concentration rise in the context of rising Eckert and Prandtl numbers, mass flow rises as well.

VALIDATION OF RESULTS

We compared our results for temperature and velocity for the limiting situation with those of Kabir and Abdulganiyu (2024) and Muawiya and Tafida (2019) in order to demonstrate the correctness and precision of the current study. A very good consistency is confirmed by the comparison, as shown in Table 4. This further demonstrates the effectiveness of the homotopy perturbation approach in resolving coupled and nonlinear differential equations.

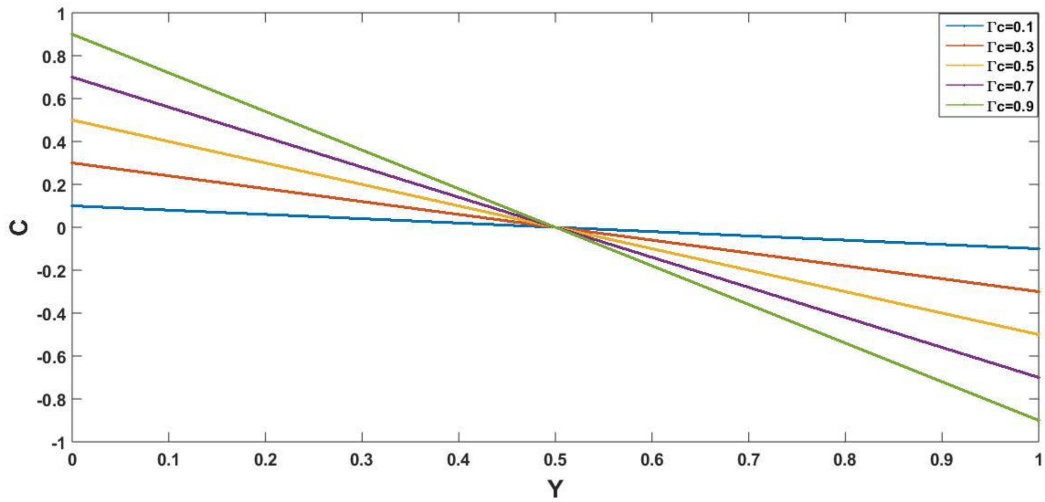

Figure 2: Impact of Γ_c on Concentration Profile

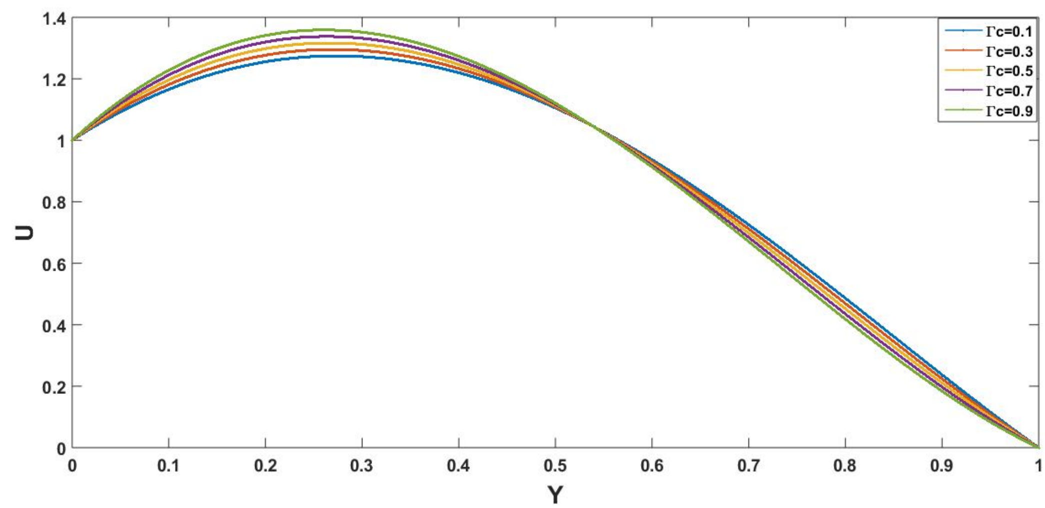

Figure 3: Impact of Γ_c on Velocity Profile

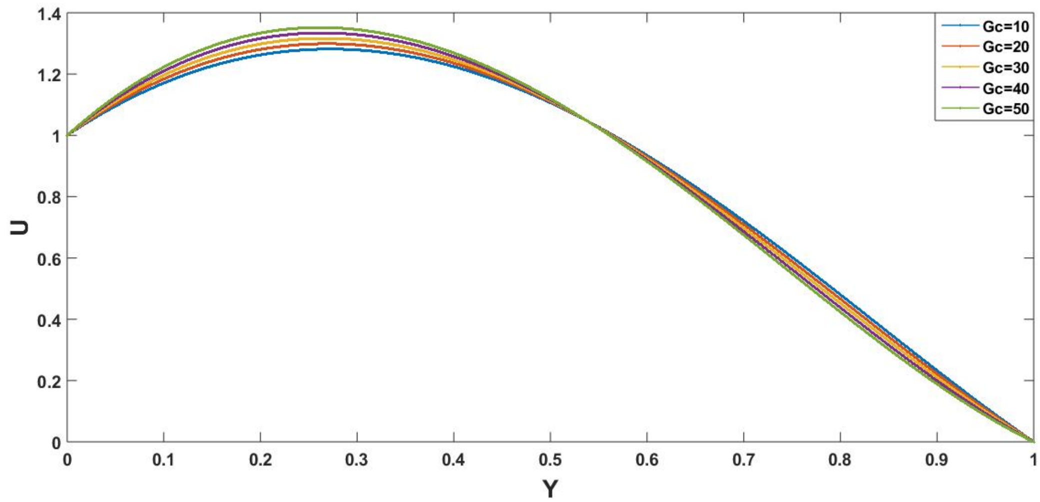

Figure 4: Impact of Gc on Velocity Profile

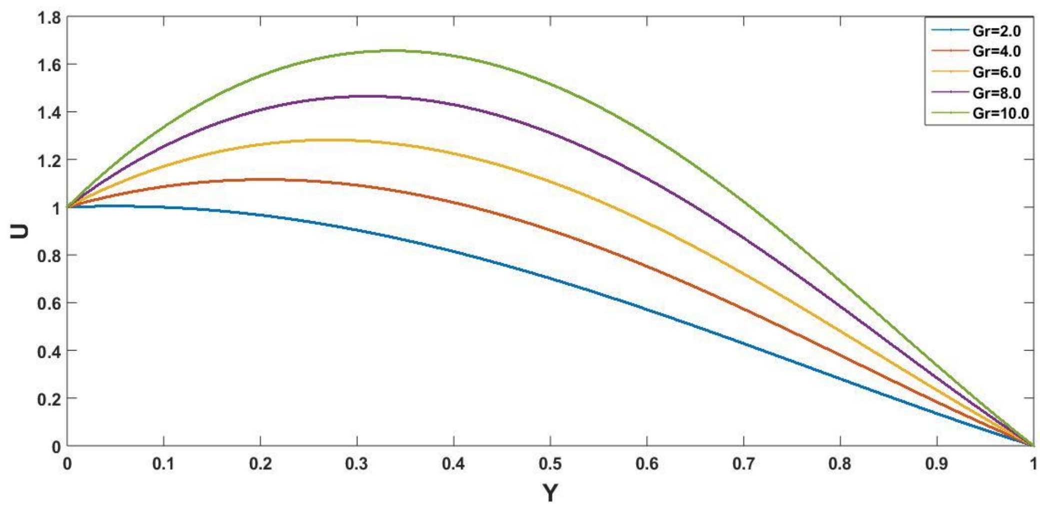

Figure 5: Impact of Gr on Velocity Profile

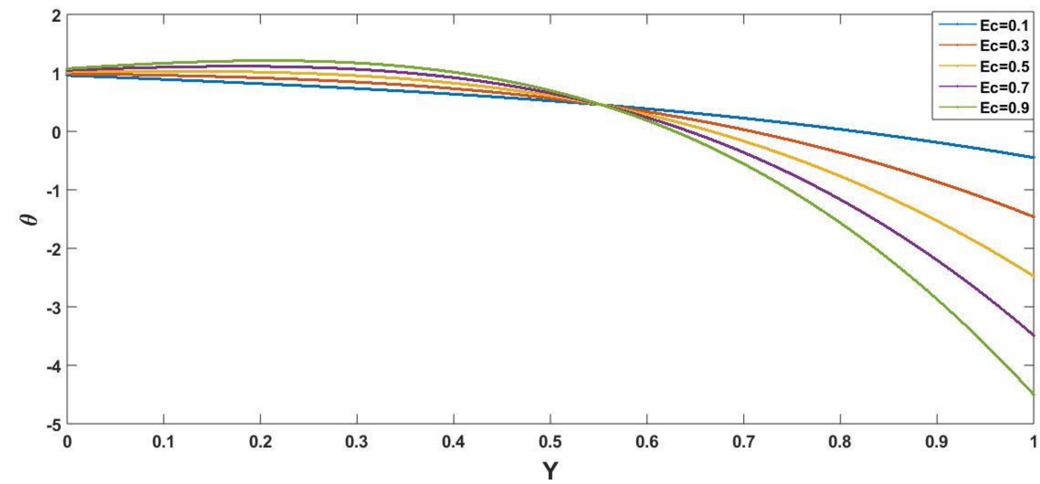

Figure 6: Impact of Ec on Temperature Profile

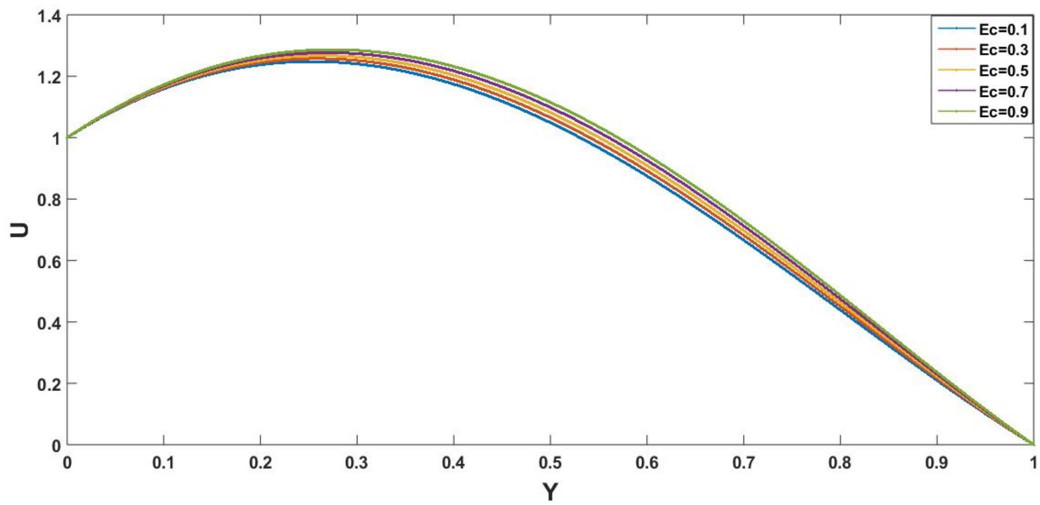

Figure 7: Impact of Ec on Velocity Profile

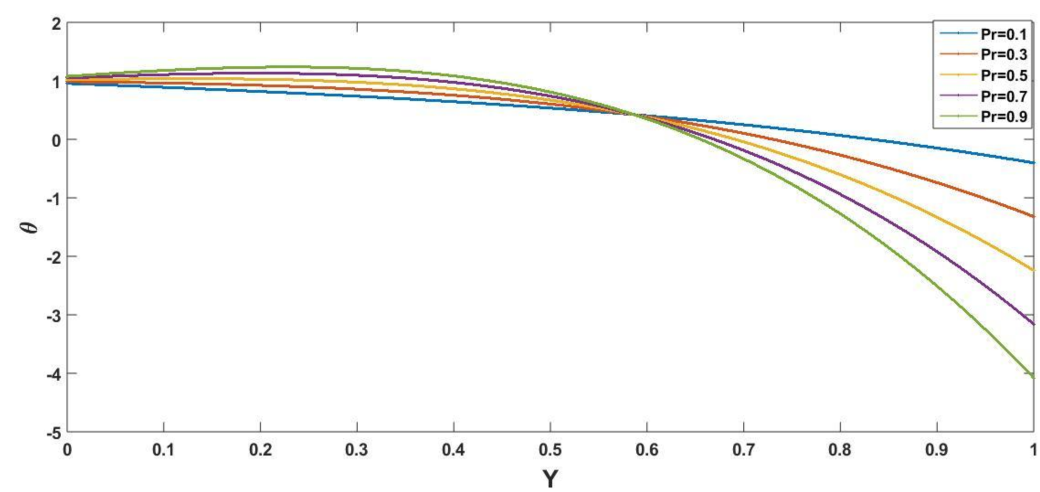

Figure 8: Impact of Pr on Temperature Profile

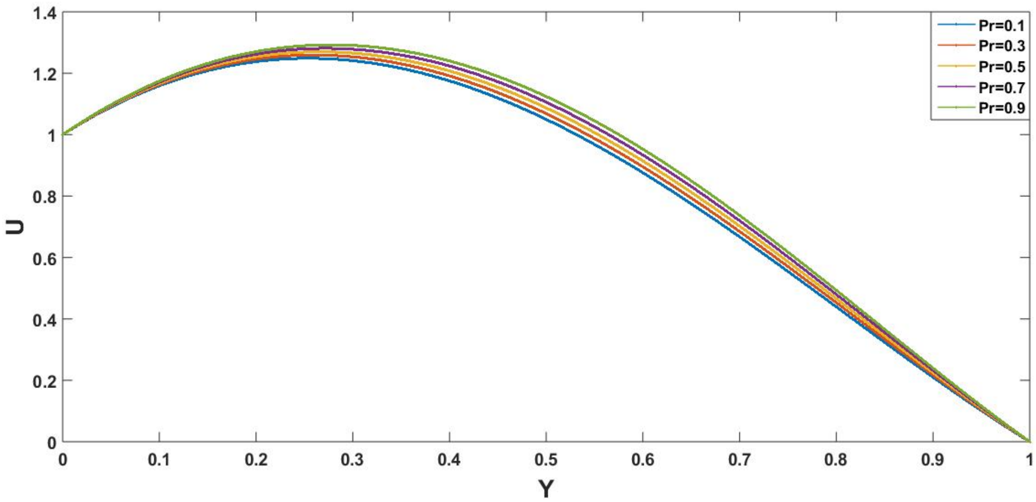

Figure 9: Impact of Pr on Velocity Profile

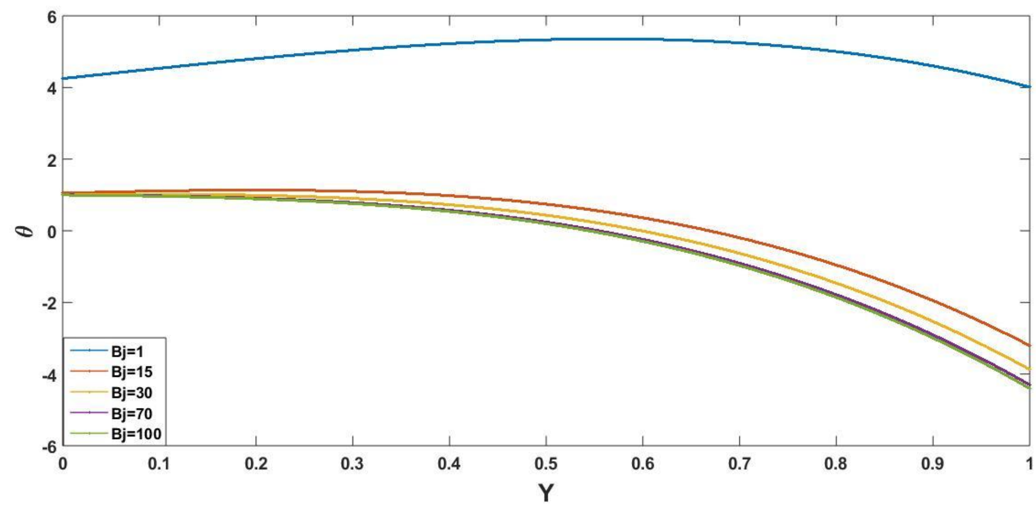

Figure 10: Impact of Bj on Temperature Profile

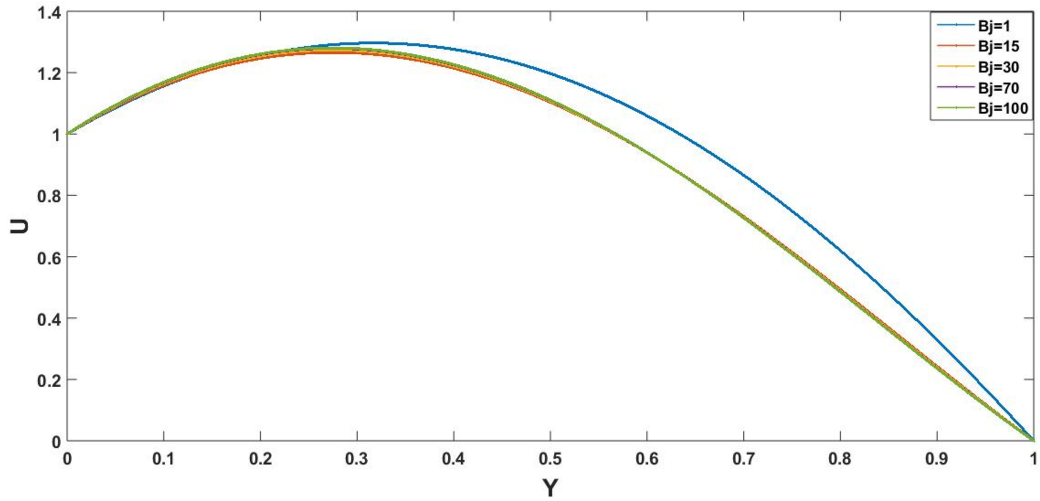

Figure 11: Impact of Bj on Velocity Profile

Table 1: Computed numerical values for the heat transfer amount at both surfaces.

		$\Gamma_c = 0.1, Gr = 6,$ $Gc = 10,$ $Bj = 7.0$		$\Gamma_c = 0.5, Gr = 8,$ $Gc = 20,$ $Bj = 7.0$		$\Gamma_c = 1.0, Gr = 10,$ $Gc = 30,$ $Bj = 17$	
Ec	Pr	τ_0	τ_1	τ_0	τ_1	τ_0	τ_1
0.1		0.7733	12.5499	0.7734	14.1729	0.7737	2.0328
0.4		0.7602	12.8664	0.7603	14.6916	0.7601	1.3943
0.7	0.044	0.7470	13.1828	0.7471	15.2104	0.7469	0.7558
1.0		0.7338	13.4994	0.7339	15.7293	0.7337	0.1173
0.1		0.7068	14.1470	0.7069	16.7908	0.7067	7.5042
0.4		0.4938	19.2627	0.4939	25.1739	0.7493	16.6363
0.7	0.71	0.2808	24.3905	0.2809	33.5731	0.2807	40.7566
1.0		0.0678	29.5304	0.0679	41.9884	0.0677	64.8568

Table 2: Computed numerical values for the skin friction at both plates.

		$\Gamma_c = 0.1, Gr = 6,$ $Gc = 10,$ $Bj = 7.0$		$\Gamma_c = 0.5, Gr = 8,$ $Gc = 20,$ $Bj = 7.0$		$\Gamma_c = 1.0, Gr = 10,$ $Gc = 30,$ $Bj = 17$	
Ec	Pr	τ_0	τ_1	τ_0	τ_1	τ_0	τ_1
0.1		1.0553	1.9614	3.1856	0.8885	7.2489	2.0328
0.4		1.0547	2.0122	3.1870	1.1097	7.2587	1.3943
0.7	0.044	1.0540	2.0630	3.1884	1.3308	7.2686	0.7558
1.0		1.0534	2.1137	3.1897	1.5519	7.2784	0.1173
0.1		1.0520	2.2176	3.1925	2.0042	7.2986	1.1887
0.4		1.0414	3.0372	3.2146	5.5725	7.4574	11.4917
0.7	0.71	1.0307	3.8567	3.2367	9.1407	7.6162	21.7946
1.0		1.0201	4.6763	3.2588	12.7090	7.7750	32.0976

Table 3: Computed numerical values of mass flux Qm.

		$\Gamma_c = 0.1, Gr = 6,$ $Gc = 10, Bj = 7.0$	$\Gamma_c = 0.5, Gr = 8,$ $Gc = 20, Bj = 7.0$	$\Gamma_c = 1.0, Gr = 8,$ $Gc = 20, Bj = 7.0$
Ec	Pr	Qm	Qm	Qm
0.1		2.4171	3.0563	3.0565
0.4		2.4184	3.0587	3.0595
0.7	0.044	2.4197	3.0610	3.0625
1.0		2.4210	3.0633	3.0654
0.1		2.4236	3.0681	3.0715
0.4		2.4444	3.1057	3.1193
0.7	0.71	2.4652	3.1434	3.1671
1.0		2.4860	3.1810	3.2148

Table 4: Comparison of calculated numerical values between the present work and previous works

	Gr=6, Gc=10, Pr=0.71, y=0.5		Gr=6, Gc=10, Bj=130, y=0.5		Present Work	
	Mauyiwa and Tafida (2019)		Kabr and Abdulganiyu (2024)			
Ec	θ	U	θ	U	θ	U
0.1	0.4824	0.7761	0.4817	0.7703	0.4818	0.7700
0.4	0.4607	0.7538	0.4601	0.7510	0.4600	0.7514
0.7	0.4210	0.7210	0.4178	0.7193	0.4180	0.7190
1.0	0.3986	0.6993	0.3952	0.6907	0.3953	0.6910

CONCLUSION

This study investigated the impact of steady buoyancy-induced Couette flow in a convectively heated vertical channel on mass flow with symmetric wall concentration in a viscous dissipative fluid. The nonlinear ordinary differential equations were solved using the homotopy perturbation method, and the results were described in terms of important governing factors and visually presented. Reverse flow, a complicated phenomenon, can have both beneficial and detrimental effects. In many scientific and engineering domains, its comprehension and appropriate handling are crucial. The following is a summary of the important findings:

(i) The presence of the wall concentration effect on mass and velocity gradients, respectively, creates a reverse flow phenomenon.

(ii) The solutal Grashof parameter decreases the fluid flow at the lower plate before raising it towards the upper plate. This is due to the symmetric heating of the plate.

(iii) Both temperature and velocity profiles grow with the mounting of the viscous dissipation parameter

(iv)The velocity and temperature gradients reduce as the Biot number is increased.

(v) Higher Prandtl number and Eckert number lead to an improved rate of heat transfer, especially at the upper plate.

(vi) A stronger frictional force is achieved for growing levels of viscous dissipation effect.

(vii) The mass flux improves when the wall concentration and viscous dissipation are increased.

(viii). In the future, this present work will be expanded to study the impact of thermal diffusion.

Appendix

$J_1 = -2\Gamma_c, J_2 = \Gamma_c, A_1 = \frac{-Bj}{2+Bj}, A_2 = \frac{1+Bj}{2+Bj}, A_3 = BjA_4, A_4 = \frac{1}{(2Bj+Bj^2)}\left[\frac{EcPrBj}{2} + EcPr\right]$

$k_1 = -2EcPrGr\left(\frac{A_1}{6} + \frac{A_2}{2}\right) - 2EcPrGc\left(\frac{J_1}{6} + \frac{J_2}{2}\right) + 2EcPrB_3, k_2 = -2EcPrGr\left(\frac{A_1}{24} + \frac{A_2}{6}\right)$

$-2EcPrGc\left(\frac{J_1}{24} + \frac{J_2}{6}\right) + EcPrB_3, A_5 = BjA_6, A_6 = \frac{-Bj-k_1}{(2Bj+Bj^2)}, k_3 = \frac{Gr^2A_1^2}{4} + \frac{GrGcJ_1A_1}{2} + \frac{Gc^2J_1}{4},$

$k_4 = Gr^2A_2A_1 + GrGcJ_2A_1 + \frac{GrGcJ_1A_2}{2} + \frac{Gc^2J_1J_2}{2}, \; k_5 = Gr^2A_2^2 - GrA_1B_3 + 2GrGcJ_2A_2 -$
$GcJ_1B_3 + Gc^2J_2^2, k_5 = -2Gr\,A_2B_3 - 2GcJ_2B_3, \; k_7 = 2EcPr(\frac{EcPrGr_1}{24} - \frac{GrA_3}{6} - \frac{GrA_4}{2} + B_5)$

$k_8 = -\frac{EcPrk_3}{5} - \frac{EcPrk_4}{4} - \frac{EcPrk_5}{3} - \frac{EcPrk_6}{2} - EcPrB_3^2, k_9 = 2EcPr(\frac{EcPrGr_1}{120} - \frac{GrA_3}{24} - \frac{GrA_4}{6} + \frac{B_5}{2})$

$k_{10} = -\frac{EcPrk_3}{30} - \frac{EcPrk_4}{20} - \frac{EcPrk_5}{12} - \frac{EcPrk_6}{6} - \frac{EcPrB_3^2}{2}, A_7 = BjA_8, A_8 = \frac{-Bjk_9 - Bjk_{10} - k_7 - k_8}{(2Bj+Bj^2)}$

$B_1 = -1, B_2 = 1, B_3 = Gr\left(\frac{A_1}{6} + \frac{A_2}{2}\right) + Gc\left(\frac{J_1}{6} + \frac{J_2}{2}\right), B_4 = 0, B_5 = \frac{GrA_3}{6} + \frac{GrA_4}{2} - \frac{EcPrGr}{24}, B_6 = 0,$

$k_{11} = 2EcPrGr^2\left(\frac{A_1}{720} + \frac{A_2}{120}\right), k_{12} = 2EcPrGrGc\left(\frac{A_1}{720} + \frac{A_2}{120}\right), k_{13} = \frac{EcPrGrB_3}{12} + \frac{GrA_5}{6} + \frac{GrA_6}{2}$

$B_7 = k_{13} - k_{11} - k_{12}, B_8 = 0.$

Nomenclature

g= Acceleration to Due Gravity [ms^{-2}]

β_1 = Coefficient of Thermal Expansion [K^{-1}]

β_2 = Coefficient of Mass Expansion [K^{-1}]

h= Width of the Channel [m]

μ= Coefficient of Viscosity [$Kgm^{-1}s^{-1}$]

T'= Dimensional Fluid Temperature [K]

C'= Dimensional Fluid Concentration [K]

ν =Kinematic Viscosity [m^2s^{-1}]

T_w= Channel Wall Temperature [K]

C_w =Channel Wall Concentration [K]

Pr =Prandtl Number

T_0= Temperature of the Ambience [K]

C_0= Concentration of the Ambience [K]

G_r =Thermal Grashof Number

G_c =Solutal Grashof Number

T =Dimensionless Fluid Temperature

Ec= Eckert Number

Γ_c =wall concentration parameter

u'= Dimensional Velocity [ms^{-1}]

Bj =Biot Number

u =Dimensionless Velocity

C_ρ = Specific Heat at Constant Pressure [$m^2s^{-2}K^{-1}$]

U = Dimensional Velocity of the Moving Plate [ms^{-1}]

ρ =Density of the Fluid [Kgm^{-3}]

y'= Coordinate Perpendicular to the Plate [m]

α =Thermal Diffusivity of the Fluid [Kgm^{-3}]

y = Dimensionless Co-ordinate Perpendicular to the Plate

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