IMPACT OF SUCTION/INJECTION ON CONVECTION-RADIATION FLOW IN A SUPERHYDROPHOBIC MICROCHANNEL WITH VISCOUS DISSIPATION

[1]Muhammed Murtala Hamza, [2*]Agnes Marian Ochiba, [3]Ibrahim Muhammad and [4]Davidson Odafe Akpootu

[1,2,3]Department of Mathematics, Faculty of Physical and Computing Sciences, Usmanu Danfodiyo University, P.M.B. 2346, Sokoto.

[4]Department of Physics, Faculty of Physical and Computing Sciences, Usmanu Danfodiyo University, P. M. B. 2346, Sokoto.

**Corresponding author Email: agnesochiba27@gmail.com*

Abstract

Understanding fluid flow in microchannels, especially when hydrodynamic factors are present, has advanced significantly in the past few decades. The role of heat radiation and constant suction/injection in these unique interactions has, however, received little research. Thus, in the presence of a constant suction/injection action, the influence of optically thick thermal radiation on buoyancy-induced magnetized flow with viscous dissipation via a superhydrophobic microchannel is investigated in this work. While the other parallel plate has a no-slip surface (NSS), one of the plates is purposefully altered to create a superhydrophobic surface (SHS). The nonlinear coupled ordinary differential equations that are modeled are semi-analytically solved. A visual representation shows how the main parameters govern the flow behavior in terms of momentum and energy distributions. Because the current study is supported by a comparison with previous research, it is valid for the limiting case. The study's main findings are: A larger fluid injection parameter results in a stronger temperature and velocity, respectively, and increasing values of the thermal radiation R function as an additional assisting force. Medical sciences and technical applications like the cooling and drying of paper and textiles, and the molding of metals and polymers, to mention a few, depend heavily on the research of fluid suction or injection flow.

Keywords: *Suction/injection, Viscous dissipation, Thermal radiation, Magnetohydrodynamics (MHD), Superhydrophobic microchannel.*

INTRODUCTION

Suction or injection will occur on a flat layer or channel surface that has porosity or passages that allow liquids to flow. The suction/injection impact is highly important in frontier layer management, especially in engineering, aerodynamics, space exploration, and medicine. Our investigation into the impact of suction/injection in industrial working fluids was spurred by these kinds of scenarios. The illustrations of this concept were explained by references like Chaudhary and Jain (2007), Khalid *et al.* (2015), and Mishra *et al.* (2022). As a result, the effects of suction and injection on a buoyancy-induced hydromagnetic flow along a porous vertical channel with temperature-dependent features were recently examined by Hamza *et al.* (2025). The impact of wall porosity on unsteady magnetized slip flow influenced by temperature-dependent thermal properties and asymmetric/symmetric wall heating in an up-facing channel was previously investigated by Hamza *et al.* (2024). Jha *et al.* (2022) investigated the effects of injection or suction on steady free convection in a vertical channel driven by absorption effects or point or line heat production using the Laplace transform technique. Uwanta *et al.* (2014)

demonstrated the impact of suction/injection on hydromagnetic convective flow on flat plates in a different study. It has been shown that fluid flow in the channel may be regulated by injection or suction. Jha *et al.* (2018) studied the impact of injection and suction on natural convection flow in transient magneto-hydrodynamics. Additionally, Jha *et al.* (2019) explored how suction and injection affected the flow of hydromagnetic free convection. They also found that injection parameters are lowered by increasing velocity. As shear stress increases, so do the suction characteristics. A heat transfer study in blood fluid-based copper and alumina nanoelements over an unstable stretched sheet impacted by suction/injection was examined by Kamis *et al.* (2021). Furthermore, Kamis *et al.* (2023), Kamis *et al.* (2022), Bachok *et al.* (2023), and Karouei *et al.* (2024) have offered some thorough insights into the effects of unsteady magnetized hybrid nanofluid flow across a stretching or shrinking thin film with suction/injection effects.

Given its importance in building various sophisticated energy-converting devices that can function at extreme temperatures, the investigation of the thermal radiation effect—the electromagnetic wave radiation that a surface produces due to its heat—is receiving increasing amounts of interest, particularly when a magnetic field exists (Jamaludin *et al.* 2020). Other potential benefits include, for example, solar technology, nuclear power plants, and spaceship aerodynamics. In light of this, several researchers have examined the effects of heat radiation in various physical forms. Ojemeri *et al.* (2025) examined the impact of thermal radiation on a viscous Casson fluid in an oscillating plate with varying thermal conductivity while taking into account the many benefits of thermal radiation. Ahmad *et al.* (2023) considered unsteady fluid convection on a magnetic thermal radiation effect across an oscillating plate with uneven heating variables. Onwubuya and Ojemeri (2023) presented a comparative research on mixed convection-radiation flow over a channel with cracks impacted by magnetic influence. Hamza *et al.* (2023) highlighted the effects of superhydrophobicity and thermal radiation on a magnetized natural convection fluid via a heated porous superhydrophobic microchannel. Goud *et al.* (2023) investigated the problem of time-dependent magnetized flow within a porous medium in an erect plate, in the coexistence of viscous dissipation and thermal radiation implications, while Gireesha *et al.* (2020) explained the impacts of thermal radiation and buoyancy-induced flow of a water-based hybrid nanofluid featuring nanoparticles with a porous vertical channel using Darcy's model. Hasan *et al.* (2020) investigated the effects of thermal radiation, heat production, and an imposed magnetic field on the free convection of a double stress fluid in a flux-isothermal upright plate using the indeterminate coefficient approach. In the context of thermal radiation, Bejawada and Nandeppanavar (2022) investigated the impact of the MHD heat transfer issue on the micropolar fluid that flows through a vertically porous shifting plate. A theoretical study of radiative-convection consequences for MHD fluid flow in a heated square container with a non-Darcy square space in the presence of the Hall effect and the heat source or sink was described by Parthiban and Pasad (2023).

The internal energy generation caused by viscous forces in molecular fluid-particle interaction is known as viscous dissipation, and the mechanical energy lost in this manner has a significant impact on the thermal buoyancy and thermal properties of a fluid. As a result, most lubricant operations rely heavily on viscous dissipation. These types of fluid movement have been receiving a lot of attention due to their significance in science, engineering, and industrial technologies, including gas drainage, electrical appliance cooling, geological energy, plasma physics, gas turbines, fossil fuel combustion, food processing sectors, and others (Ajibade *et al.* 2021). Gebhart's (1962) study, which examined the impact of viscous dissipation on natural convection using a perturbation approach, was among the first to take viscous dissipation into account. Numerous writers have provided examples of their work in this area over the years. Omokhuale and Ojemeri (2024) examined the impact of viscous dissipation on a buoyancy-induced Couette flow in a vertical channel impacted by Newtonian heating using the homotopy perturbation technique. Their findings suggest that Newtonian heating settings and viscous dissipation are intended to increase fluid velocity. Isa *et al.* (2024) outlined the effects of time-dependent magnetized free convection heat transfer flow of viscous dissipation in a vertical channel with Newtonian heating numerically. They came to the conclusion that the rising level of the viscous dissipation parameter tends to enhance the fluid velocity with increasing time. The nonlinear thermal impact of a viscous dissipative fluid in a superhydrophobic microchannel with pores and magnetic effects was solved semi-analytically by Ojemeri and Onwubuya (2023). Adopting a regular perturbation technique, Yale *et al.* (2023) studied the effects of MHD and superhydrophobicity on a viscous dissipative fluid in a gaping microchannel. Onwubuya *et al.* (2024) addressed the effects of superhydrophobicity, porous media, and viscous dissipation on the free convection of an electrically conducting fluid across an upstanding microchannel subjected to an applied magnetic field. Incorporating a heat source/sink effect, the plates were heated alternately. Evaluating the effects of MHD, chemical reaction, porous medium, and heat sink, Omokhuale *et al.* (2024a, 2024b) emphasized the importance of viscous dissipation on both free and mixed convective flows of a nanofluid flow over a semi-infinite flat plate in an oscillating plate. The viscous dissipation and thermal effects on magnetized mixed convection stagnation point flow of Maxwell nanofluid across a stretched surface were investigated by Narender *et al*. (2021). Using a well-known regular perturbation, Uka *et al.* (2023) analytically and numerically resolved the fluid flow issue in the presence of viscous dissipation and permeability effect due to a wedge in motion.

Notwithstanding all of this progress, there is still a substantial need to comprehend the dynamics of heat radiation with fluid suction or injection in viscous dissipative fluids, particularly through superhydrophobic microchannels. None of the aforementioned literature studied the combined impact of suction/injection in a viscous dissipative fluid along a superhydrophobic microchannel. The existence of fluid suction or injection is required not only for regulating fluid flow in microchannels but also for adding and removing reactants during chemical reactions. This study attempts to close this gap by extending Yale *et al*.'s (2023) work to include and investigate new interactions between heat radiation, suction/injection, and SHO microchannel designs. This study's investigation of these effects will bring useful insights into fluid flow control in microchannels and chemical reaction optimization.

FORMULATION OF THE PROBLEM

Examine the steady buoyancy-induced flow of an incompressible viscous fluid passing between two vertical parallel plates with heat radiation effects. As seen in Figure 1, the wall at $y_0 = 0$ is caused by the exceedingly difficult-to-wet SHO effect, whereas the NSS remains at $y_0 = L$. A transversely applied magnetic field of strength B_0 is stated to be introduced between two upright microchannel plates. All fluid parameters are considered to be constant. The flow variables are only functions of space y. Due to the current formulation, the following assumptions are made:

i. The fluid is laminar, viscous, and incompressible.
ii. The flow is fully developed and follows the Boussinesq approximation.
iii. The fluid is expected to undergo viscous dissipation.
iv. The fluid is thought to be impacted by thermal radiation.
v. The fluid is believed to be injected and sucked at a constant pace in the channel.
vi. The assumption is that the magnetic Reynolds number is minimal, resulting in a negligible difference between the induced and external magnetic fields.

vii. It is assumed that there is asymmetric heating of the plates, with one at T_0 and the other at T_h.
viii. The temperature differential between plates leads to spontaneous convection flow in the microchannel.

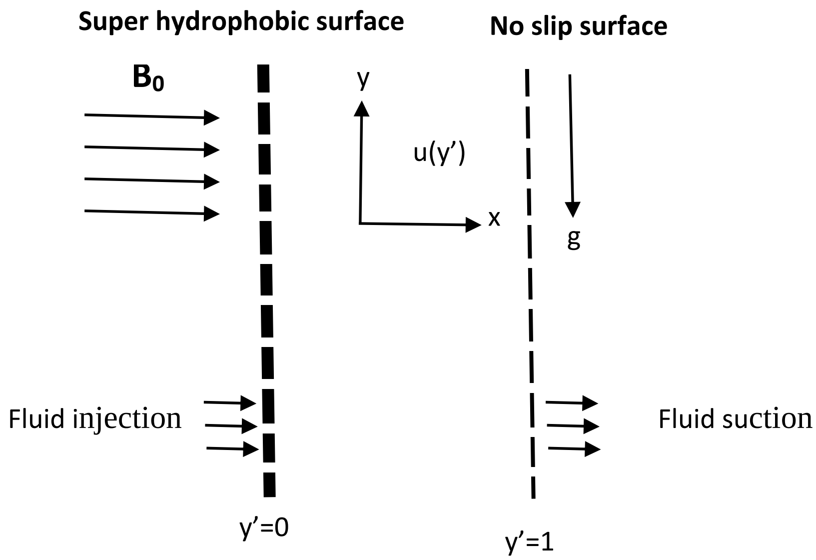

Figure 1: Schematic Diagram of the Flow System

Following Yale *et al.* (2023) and employing the Boussinesq buoyancy and Rosseland approximations with the appropriate boundary conditions, the dimensional governing equations for the present problem can be modeled as:

$$v\frac{d^2u'}{dy'^2} - V_0\frac{du'}{dy'} + g\beta(T' - T_O) - \frac{\sigma B_O^2 u'}{\rho} = 0 \tag{1}$$

$$\frac{k}{\rho c_\rho}\frac{d^2T'}{dy'^2} - V_0\frac{du'}{dy'} - \frac{1}{\rho c_\rho}\frac{dq_r}{dy'} + \frac{v}{c_\rho}\left(\frac{du'}{dy'}\right)^2 = 0 \tag{2}$$

Subject to the boundary conditions

$$\left.\begin{aligned} \left.\begin{aligned} u(y) &= \lambda'\frac{du'}{dy'} \\ T(y') &= T_h + \gamma'\frac{dT'}{dy'} \end{aligned}\right\} & at\ y' = 0 \\ \left.\begin{aligned} u(y') &= 0 \\ T(y') &= T_o \end{aligned}\right\} \qquad & at\ y' = L \end{aligned}\right\} \tag{3}$$

Introducing the dimensionless quantities and variables

$$u = \frac{u'}{U}, y = \frac{y'}{h}, \theta = \frac{T' - T_0}{T_w - T_o}, x = \frac{x'v}{Uh^2}, M^2 = \frac{\sigma\beta_o^2 h^2}{\rho v}, (Y, \gamma, \lambda) = (Y', \gamma', \lambda')/h$$

$$S = \frac{V_0 h}{v}, Pr = \frac{v\rho c_p}{K}, Ec = \frac{u^2}{c_p(T_w - T_o)}, B_r = EcPr \quad (4)$$

Substituting eqn (4) into eqns (1 to 3), we have the basic coupled equations dictating the flow as:

$$\frac{d^2U}{dy^2} - S\frac{dU}{dy} - M^2U = -\theta \quad (5)$$

$$\frac{d^2\theta}{dy^2} - S\frac{d\theta}{dy} + Br\left(\frac{du}{dy}\right)^2 = 0 \quad (6)$$

Subjected to the boundary conditions:

$$\theta(y) = 1 + \gamma\frac{d\theta}{dy}; \qquad u(y) = \lambda\frac{du}{dy} \qquad \text{at } y = 0 \quad (7)$$

$$\theta(y) = 0; \qquad u(y) = 0 \qquad \text{at } y = 1$$

where qr is the radiative heat flux in the optically thick medium flow, from the Roseland approximation (Raptis 1998), which is given by:

$$qr = -\frac{4\sigma^*}{3k^*}\frac{\partial T^4}{\partial y} \quad (8)$$

Now, according to (Raptis 1999), it is assumed that temperature changes inside the flow are significantly meager so that T^4 may be taken as a linear function of temperature in a Taylor series about T_0 and neglecting higher terms, we have:

$$T^4 \cong 4T_0^3T - 3T_0^4 \quad (9)$$

Using equations (8) and (9) equation (6) now becomes

$$\left(1 + \frac{16\sigma^*T^3}{3kk^*}\right)\frac{d^2T}{dy^2} - S\frac{d\theta}{dy} + Br\left(\frac{du}{dy}\right)^2 = 0 \quad (10)$$

Where $R = 16\sigma^*T^3/3kk^*$ is viewed as the radiation parameter, so that equation (10) becomes

$$(1 + R)\frac{d^2T}{dy^2} - S\frac{d\theta}{dy} + Br\left(\frac{du}{dy}\right)^2 = 0 \quad (11)$$

Where M is the magnetic parameter, S is the suction/injection parameter, R is the thermal radiation, Br = EcPr is the Brinkman number, representing the viscous dissipation, λ is the velocity slip, and γ is the temperature jump parameter.

METHOD OF SOLUTION

The coupled nonlinear governing equations are solved using the regular perturbation method, such that the solutions of the temperature and velocity are assumed as;

$$U = u_0 + \mathrm{Br}(u_1) \quad (11)$$

$$T = t_0 + \mathrm{Br}(t_1) \quad (12)$$

Inserting eqns (11) and (12) into eqns (5), (7) and (11), and taking the coefficients of Br^0 and Br, we derive the following system of ordinary differential equations with their corresponding boundary conditions as:

$$\mathrm{Br}^0: \frac{d^2u_0}{dy^2} - S\frac{du_0}{dy} - \mathrm{M}^2 u_0 = -\theta_0 \tag{13}$$

$$\mathrm{Br}: \frac{d^2u_1}{dy^2} - S\frac{du_1}{dy} - \mathrm{M}^2 u_1 = -\theta_1 \tag{14}$$

$$\mathrm{Br}^0: (1+R)\frac{d^2\theta_0}{dy^2} - S\frac{d\theta_0}{dy} = 0 \tag{15}$$

$$\mathrm{Br}: (1+R)\frac{d^2\theta_1}{dy^2} - S\frac{d\theta_1}{dy} + \left[\frac{du_0}{dy}\right]^2 = 0 \tag{16}$$

$$\left.\begin{aligned} \mathrm{Br}^0: \theta_o &= 1 + \gamma\frac{d\theta_o}{dy} \\ \mathrm{Br}: \theta_1 &= \gamma\frac{d\theta_1}{dy} \\ \mathrm{Br}^0: u_0 &= \lambda\frac{dU_o}{dy} \\ \mathrm{Br}: u_1 &= \lambda\frac{dU_1}{dy} \end{aligned}\right\} \text{at } y = 0 \tag{17}$$

$$\text{and} \left.\begin{aligned} \mathrm{Br}^0: \theta_0 &= 0 \\ \mathrm{Br}: \theta_1 &= 0 \\ \mathrm{Br}^0: u_0 &= 0 \\ \mathrm{Br}: u_1 &= 0 \end{aligned}\right\} \text{at } y = 1 \tag{18}$$

The solutions of velocity and temperature are derived as:

$$u_0 = L_{19}\exp(m_3 y) + L_{20}\exp(-m_4 y) \tag{19}$$

$$u_1 = L_{21} + L_{22}\exp(-m_2 y) + L_{23}\exp(2m_3 y) + L_2\exp(-m_2 y) + L_{24}\exp\big((m_3 - m_4)y\big) + L_{25}\exp\big((m_3 + m_4)y\big) + L_{26}\exp\big((m_3 - m_2)y\big) + L_{27}\exp(-2m_4 y) + L_{28}\exp\big((m_1 - m_4)y\big) + L_{29}\exp(-(m_2 + m_4)y) + L_{30}\exp(2m_1 y) + L_{31}\exp\big((m_1 - m_3)y\big) + L_{32}\exp(-2m_2 y) \tag{20}$$

$$\theta_0 = L_1\exp(m_1 y) + L_2\exp(-m_2 y) \tag{21}$$

$$\theta_1 = L_7 + L_8\exp(-m_2 y) + L_9\exp(2m_2 y) + L_{10}\exp\big((m_3 - m_4)y\big) + L_{11}\exp\big((m_3 + m_1)y\big) + L_{12}\exp\big((m_3 - m_2)y\big) + L_{13}\exp(-2m_4 y) + L_{14}\exp\big((m_1 - m_4)y\big) + L_{15}\exp(-(m_2 + m_4)y) + L_{16}\exp(2m_1 y) + L_{17}\exp\big((m_1 - m_2)y\big) + L_{18}\exp(-2m_2 y) \tag{22}$$

Substituting eqns (19) and (20) into eqn (11), we have

$$U = L_{19}\exp(m_3 y) + L_{20}\exp(-m_4 y) + Br\big[L_{21} + L_{22}\exp(-m_2 y) + L_{23}\exp(2m_3 y) + L_{24}\exp\big((m_3 - m_4)y\big) + L_{25}\exp\big((m_3 + m_4)y\big) + L_{26}\exp\big((m_3 - m_2)y\big) + L_{27}\exp(-2m_4 y) + L_{28}\exp\big((m_1 - m_4)y\big) + L_{29}\exp(-(m_2 + m_4)y) + L_{30}\exp(2m_1 y) + L_{31}\exp\big((m_1 - m_3)y\big) + L_{32}\exp(-2m_2 y)\big] \quad (23)$$

Also, substituting eqns (21) and (22) into eqn (12), we have

$$\theta = L_1\exp(m_1 y) + L_2\exp(-m_2 y) + Br\big[L_7 + L_8\exp(-m_2 y) + L_9\exp(2m_2 y) + L_{10}\exp\big((m_3 - m_4)y\big) + L_{11}\exp\big((m_3 + m_1)y\big) + L_{12}\exp\big((m_3 - m_2)y\big) + L_{13}\exp(-2m_4 y) + L_{14}\exp\big((m_1 - m_4)y\big) + L_{15}\exp(-(m_2 + m_4)y) + L_{16}\exp(2m_1 y) + L_{17}\exp\big((m_1 - m_2)y\big) + L_{18}\exp(-2m_2 y)\big] \quad (24)$$

The skin friction and Nusselt number are obtained as:

$$\frac{dU}{dy}\Big|_{y=0} = m_3 L_{19} - m_4 L_{20} + Br[-m_2 L_{22} + 2m_3 L_{23} + (m_3 - m_4)L_{24} + (m_3 + m_4)L_{25} + (m_3 - m_2)L_{26} - 2m_4 L_{27} + (m_1 - m_4)L_{28} - (m_2 + m_4)L_{29} + 2m_1 L_{30} + (m_1 - m_3)L_{31} - 2m_2 L_{32}] \quad (25)$$

$$\frac{dU}{dy}\Big|_{y=1} = m_3 L_{19}\exp(m_3) - m_4 L_{20}\exp(-m_4) + Br\big[-m_2 L_{22}\exp(-m_2) + 2m_3 L_{23}\exp(2m_3) + (m_3 - m_4)L_{24}\exp\big((m_3 - m_4)\big) + (m_3 + m_4)L_{25}\exp\big((m_3 + m_4)\big) + (m_3 - m_2)L_{26}\exp\big((m_3 - m_2)\big) - 2m_4 L_{27}\exp(-2m_4) + (m_1 - m_4)L_{28}\exp\big((m_1 - m_4)\big) - (m_2 + m_4)L_{29}\exp\big(-(m_2 + m_4)\big) + 2m_1 L_{30}\exp(2m_1) + L_{31}\exp\big((m_1 - m_3)\big) + L_{32}\exp(-2m_2)\big] \quad (26)$$

$$\frac{d\theta}{dy}\Big|_{y=0} = m_1 L_1 - m_2 L_2 + Br[-m_2 L_8 + 2m_2 L_9 + (m_3 - m_4)L_{10} + (m_3 + m_1)L_{11} + (m_3 - m_2)L_{12} - 2m_4 L_{13} + (m_1 - m_4)L_{14} - (m_2 + m_4)L_{15} + 2m_1 L_{16} + (m_1 - m_2)L_{17} - 2m_2 L_{18}] \quad (27)$$

$$\frac{d\theta}{dy}\Big|_{y=1} = m_1 L_1\exp(m_1) - m_2 L_2\exp(-m_2) + Br\big[-m_2 L_8\exp(-m_2) + 2m_2 L_9\exp(2m_2) + (m_3 - m_4)L_{10}\exp\big((m_3 - m_4)\big) + (m_3 + m_1)L_{11}\exp\big((m_3 + m_1)\big) + (m_3 - m_2)L_{12}\exp\big((m_3 - m_2)\big) - 2m_4 L_{13}\exp(-2m_4) + (m_1 - m_4)L_{14}\exp\big((m_1 - m_4)\big) - (m_2 + m_4)L_{15}\exp\big(-(m_2 + m_4)\big) + 2m_1 L_{16}\exp(2m_1) + (m_1 - m_2)L_{17}\exp\big((m_1 - m_2)\big) - 2m_2 L_{18}\exp(-2m_2)\big] \quad (28)$$

RESULTS AND DISCUSSION

The subject of viscous dissipative flow of an incompressible fluid traveling vertically within a slit microchannel surface is explored under the influence of heat radiation and constant suction/injection impacts. The modeled equations are solved using a standard perturbation technique. Graphs have been plotted to demonstrate the effects of important parameters that influence the structure of the flow. The reference values chosen for this investigation are R=1, S=1, λ=γ=M=1, and Br=0.001. It's worth noting that S>0 indicates fluid injection, while S<0 indicates fluid suction. However, in this study, we only focus on fluid injection.

Figures 2 and 3 explain how suction/injection affects temperature and velocity profiles, respectively. The results show that raising the injection parameter promotes thermal and hydromagnetic fluxes. Suction/injection variables have a substantial impact on boundary-layer management in fields such as engineering, aerodynamics, space exploration, and medicine. As a result, the discoveries can be particularly beneficial for regulating fluid flow in channels and assisting in the addition of reactants during chemical processes. Figure 4 depicts the variance of the radiation parameter R from the temperature gradient while all other parameter values are held constant. This figure clearly shows that increasing R causes the fluid's temperature to rise significantly. This situation happens when the radiation parameter thickens the thermal barrier, allowing the fluid to generate heat energy from the flow domain and stabilizing the system. Figure 5 shows an increase in fluid velocity due to a larger thermal radiation effect generated by heat creation, which increases flow speed. Furthermore, the heat released by the heated wall reinforces the fluid particles. A similar trend is observed in the study of Hamza et al. (2023), confirming the validity of the current experiment. Figures 6 and 7 show the effect of viscous dissipation Br, on steady temperature and velocity gradients. Since the Eckert number emanates from the kinetic energy of flow and heat enthalpy changes, an increase in the Eckert number promotes the kinetic energy. Consequently, higher viscous dissipative factors boost the fluid's temperature and velocity, respectively. Again, it is true that rising levels of the Brickman number represent better convective heating at the channel surface, leading to a stronger temperature and velocity at the plate surfaces. Makinde and Aziz (2011) assert that raising the local Brinkman value causes convective heating at the lower channel wall. As a result of this, higher surface temperatures are produced that penetrate deeper into the still fluid. Figure 8 shows the pattern of fluid motion for varying values of the magnetic parameter M. The flow pattern clearly shows a reduction for all investigated M values. The primary reason for this behavior is the presence of a magnetic influence, which creates a resistive force (the Lorentz force) that limits velocity. Figures 9a and b show the distinctive behavior of the heat transfer rate, commonly known as the Nusselt number, in the application of the injection parameter. It is discovered that increasing the fluid injection parameter at the plates causes an increase in the heat transfer rate. Figures 10a and b show how R and γ affect the heat transfer rate. In the presence of R, the rate of heat transmission slows at the lower plate, whereas the higher plate shows a contrasted pattern (y=1). Figures 11a and b show the impact of varying Br versus γ on Nusselt numbers. The deliberate use of Br reduces heat transmission between both plates in the channel. Figures 12a and b show the considerable effect of suction/injection on shear stress. When SHS and NSS are heated, skin friction increases with larger injection parameters at the bottom plate, whereas the reverse effect occurs at the upper plate. Figures 13a and b show how R affects the shear stress coefficient. It was discovered that increasing R values boosts frictional force at the lower plate (y=0), whereas opposite behavior is observed at the upper plate (y=1). Figures 14a and b show how Br vs λ affects skin friction on both upstanding walls. Figure 14a shows that near the wall (y=0), the shearing stress is significantly strengthened; however, when y=1, the opposite effect occurs, as shown in Figure 14b.

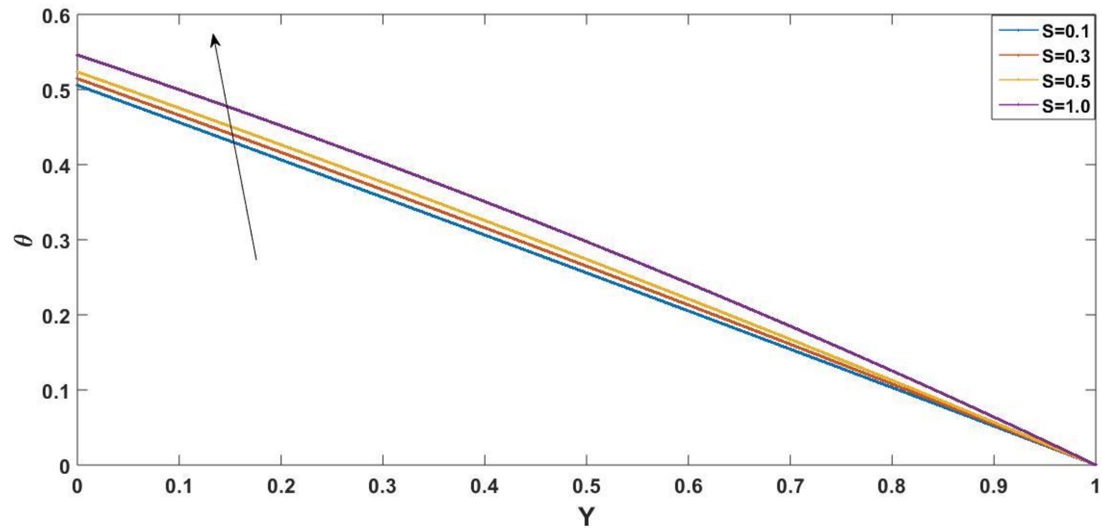

Figure 2: Function of S on Temperature Distribution

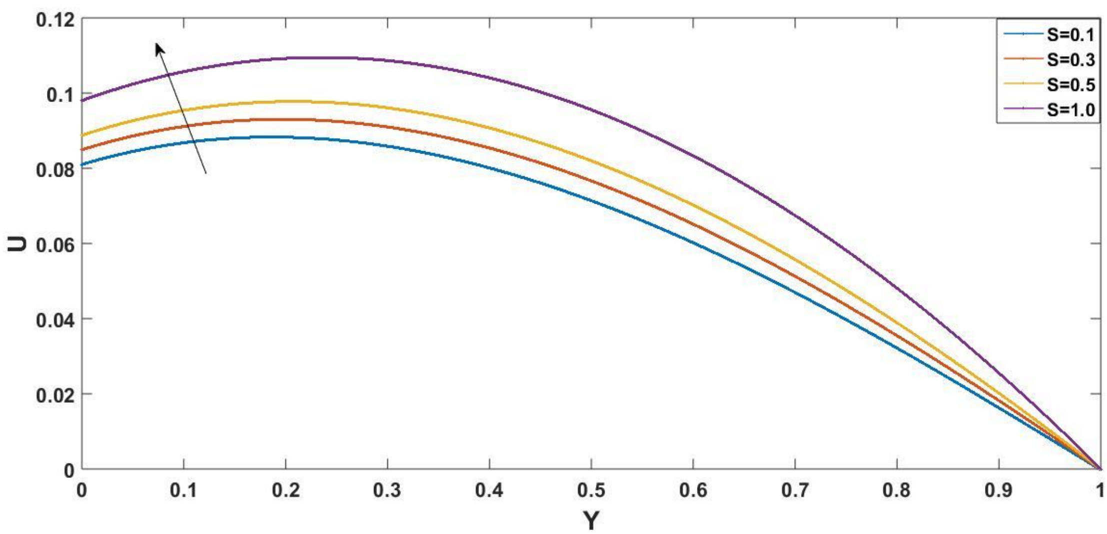

Figure 3: Function of S on Velocity Distribution

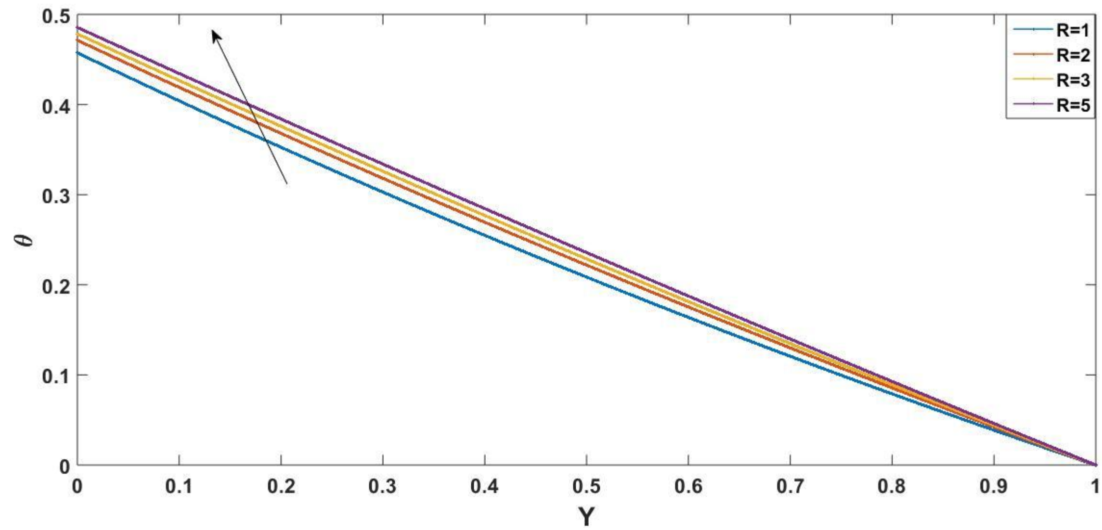

Figure 4: Function of R on Temperature Distribution

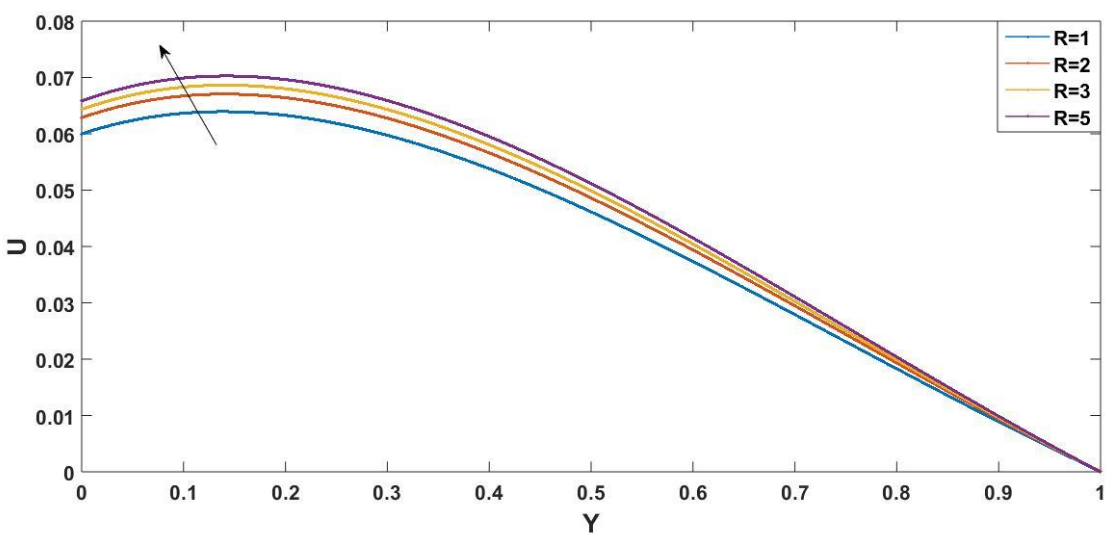

Figure 5: Function of R on Velocity Distribution

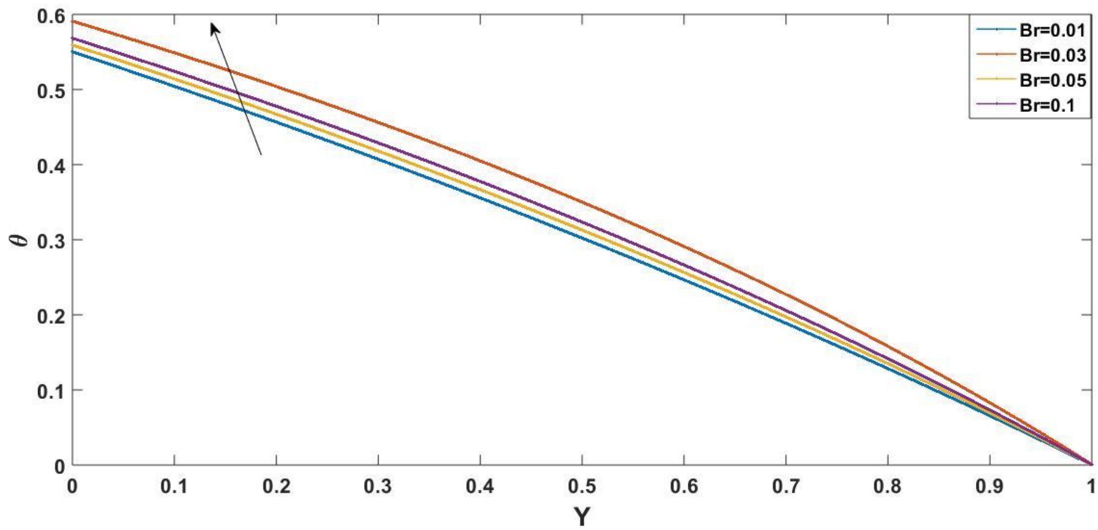

Figure 6: Function of Br on Temperature Distribution

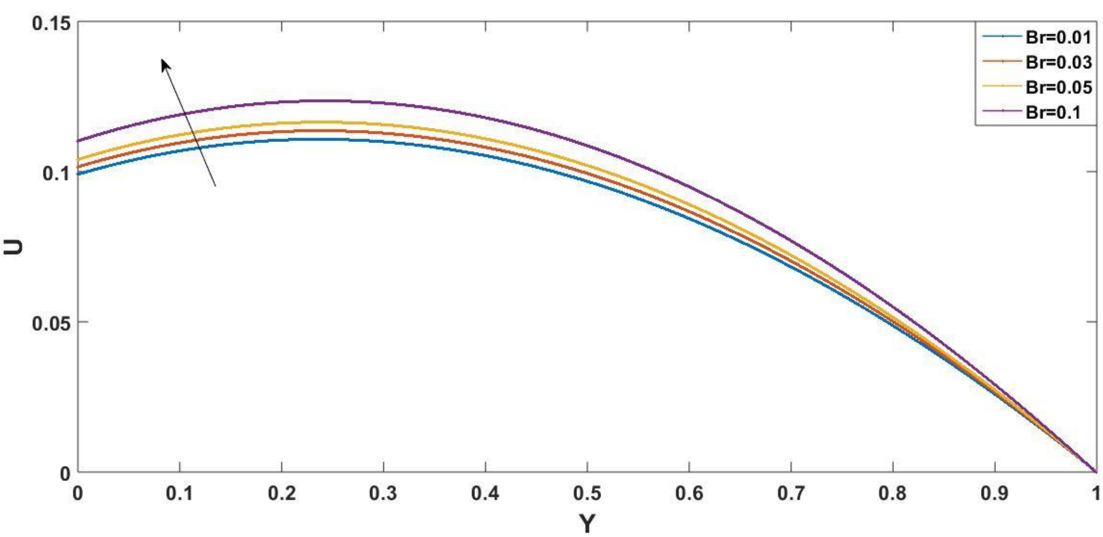

Figure 7: Function of Br on Velocity Distribution

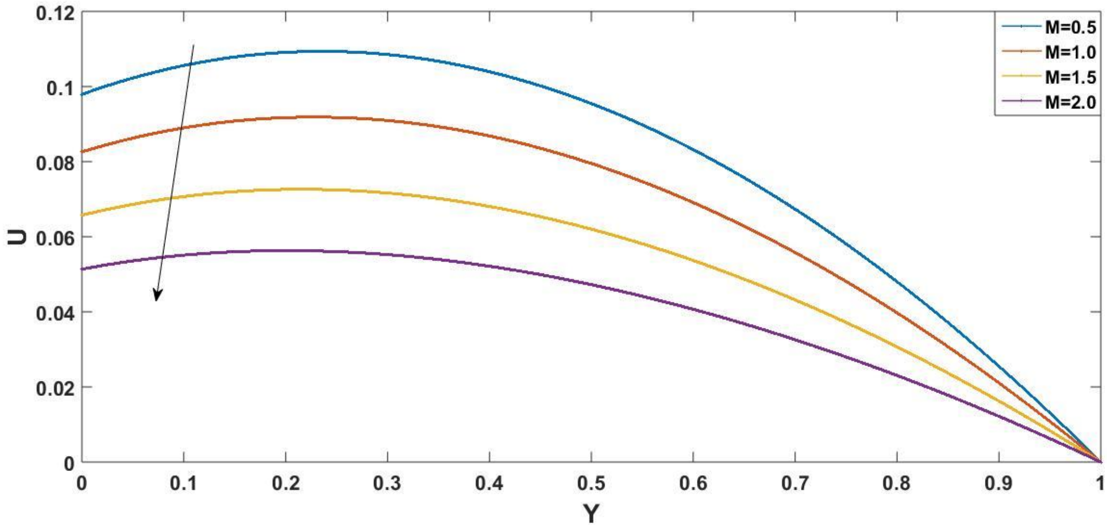

Figure 8: Function of M on Velocity Distribution

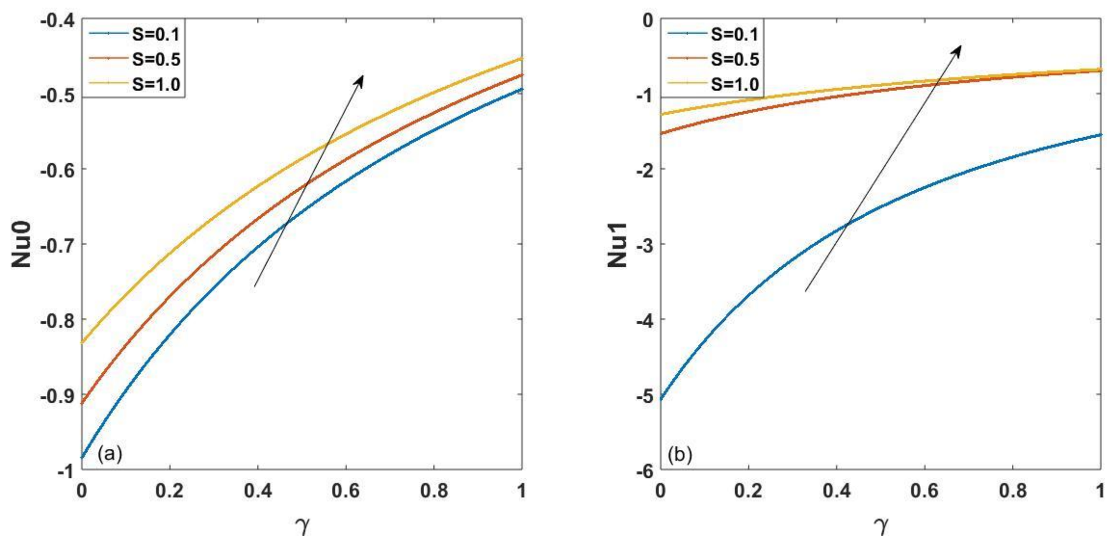

Figure 9: Function of S against γ on Nusselt number

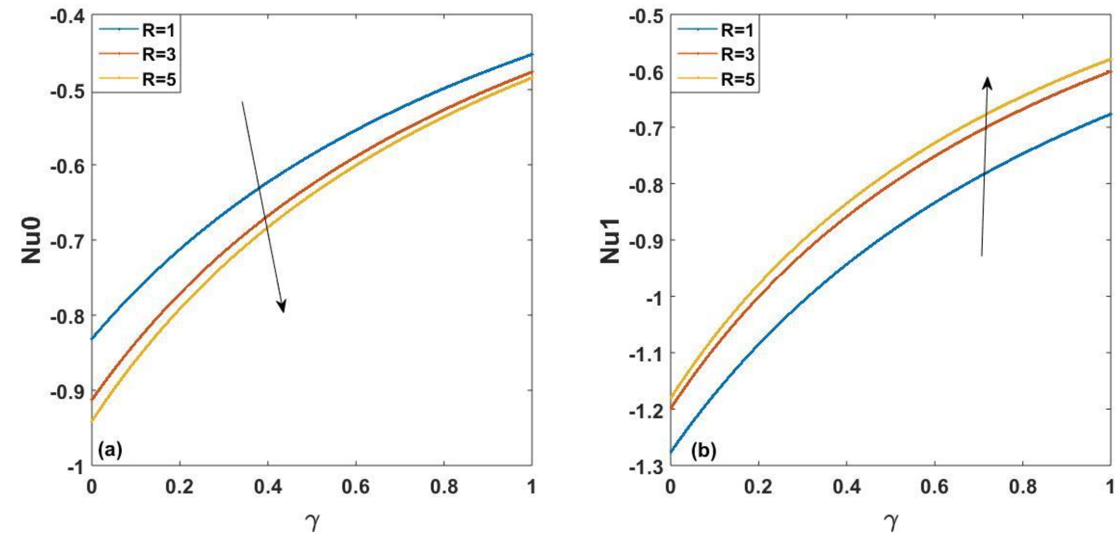

Figure 10: Function of R against γ on Nusselt number

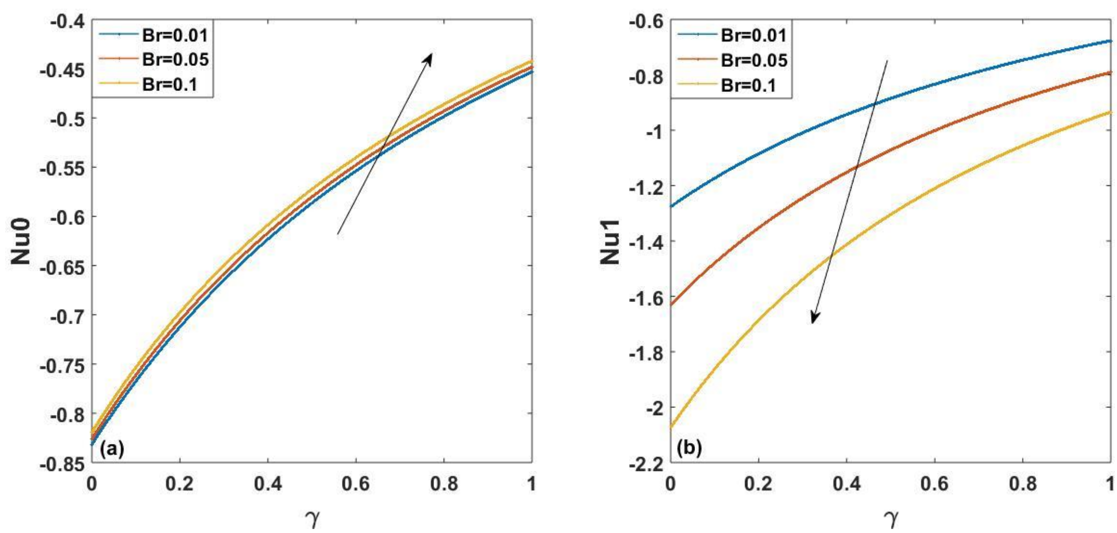

Figure 11: Function of Br against γ on Nusselt number

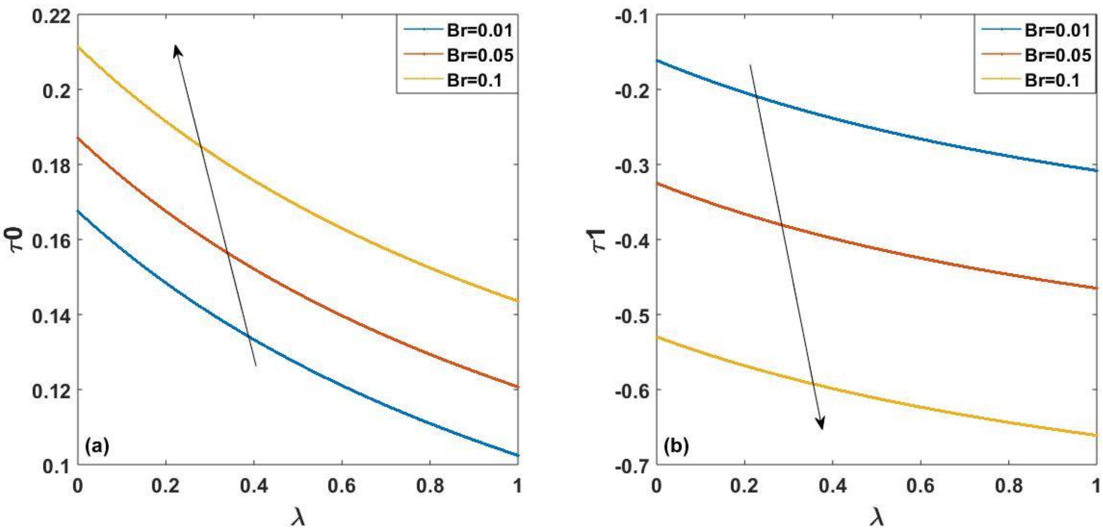

Figure 12: Function of S against γ on Skin Friction

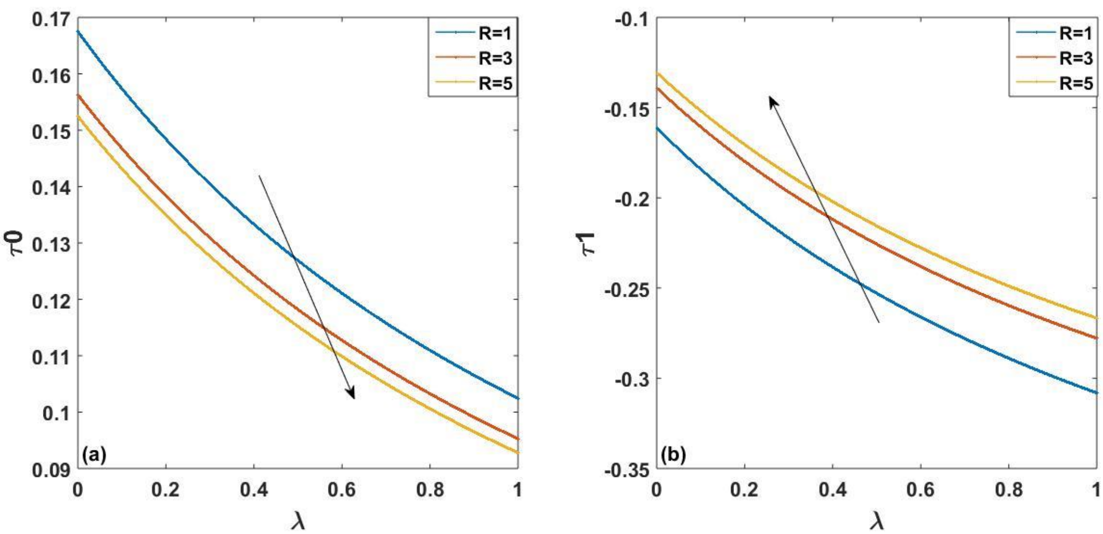

Figure 13: Function of R against γ on Skin Friction

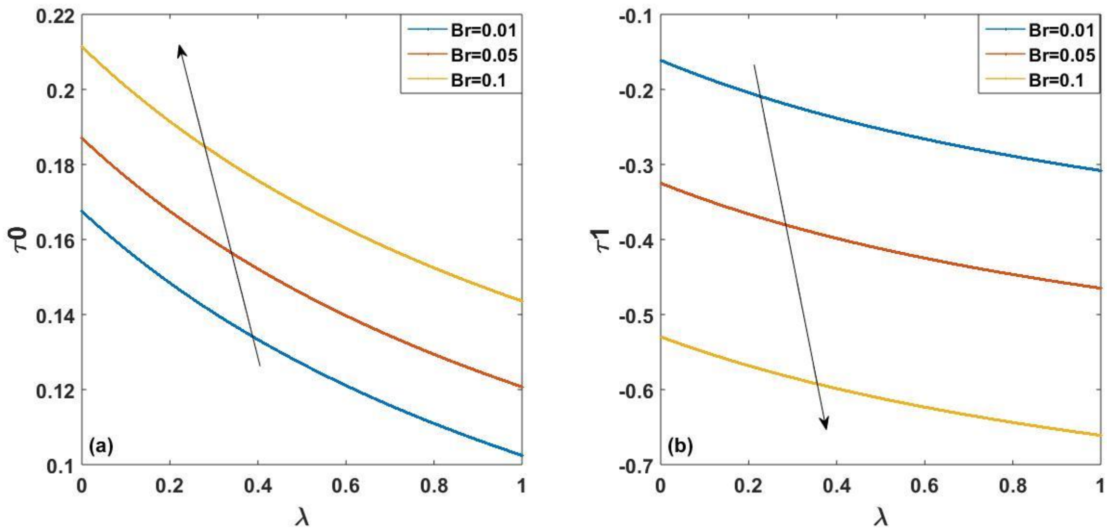

Figure 14: Function of Br against γ on Skin Friction

RESULTS VALIDATION

Table 1: A numerical comparison between the work of Yale et al. (2023) and the current study for λ = γ = 1, M = 0.5, Br = 0.001, and setting R and S to zero, respectively. The comparison shows a decent match.

Y	Yale *et al*. (2023)		Present work $R = 0, S = 0$		Present work $R = 1, S = 0.1$		Present work $R = 5, S = 1$	
	θ (Y)	U(Y)	θ (Y)	U(Y)	θ (Y)	U(Y)	θ (Y)	U(Y)
0.1	0.4500	0.0843	0.4500	0.0843	0.4684	0.0896	0.4691	0.0978
0.2	0.4000	0.0856	0.4000	0.0856	0.4193	0.0910	0.4199	0.1010
0.3	0.3500	0.0831	0.3500	0.0831	0.3696	0.0884	0.3700	0.1003
0.4	0.3000	0.0772	0.3000	0.0772	0.3193	0.0823	0.3195	0.0959
0.5	0.2500	0.0686	0.2500	0.0686	0.2684	0.0731	0.2682	0.0879

CONCLUSION

This study analyzed the roles of thermal radiation and suction/injection on steady, fully developed, magnetized flow in a microchannel with SHO features, in a viscous, dissipative fluid. The impact of radiation and fluid suction/injection is fully discussed. The coupled nonlinear ordinary differential equations that describe the temperature and velocity components are obtained in a closed-form solution.

Furthermore, the physical aspects of engineering interests, such as the Nusselt number and shear stress, have been calculated. The effects of major fluid parameters are depicted in line graphs. Our results for the limiting scenario (R=0 and S=0) are compared to previous work, and they are remarkably similar, authenticating the accuracy and correctness of the present work.

Some intriguing findings from this work are summarized below:

(i) Increased levels of thermal radiation are seen to increase the temperature and velocity of the fluid.

(ii) Fluid injection is observed to increase the fluid's temperature and velocity, respectively.

(iii) The fluid flow is reduced in the presence of a magnetic influence due to the force that acts to oppose the flow.

(iv) The presence of the fluid injection parameter is envisaged to enhance the shearing stress and Nusselt number, respectively.

(v) It was concluded that higher values of the thermal radiation parameter decrease the skin friction and the heat transfer amount, respectively, at the plate (y = 0), while a contrasting behaviour happens at the plate (y = 1).

(vi) The action of Br is concluded to improve both the skin friction and the heat transfer rate at the wall (y=0), while a reverse phenomenon occurs at the upper plate (y=1).

(vii) In the future, the effect of the Darcy porous medium would be studied on this mathematical model.

Nomenclature

y dimensional horizontal position between the plates (m)

g gravitational force (m/s^2)

T_1 dimensional temperature of the surroundings (K)

T_2 dimensional temperature higher than T_0 (K)

B_0 constant magnetic field applied (T)

Y dimensionless horizontal position between the plates (-)

U dimensionless vertical velocity of the flow (-)

M magnetic parameter (-)

L channel width (m)

Nu Nusselt number (-)

k thermal conductivity (W/nK)

Br = Viscous Dissipation parameter (-)

R = Thermal Radiation parameter (-)

S = Suction/injection parameter (-)

q_r^* = The radiative heat flux [Wm^{-2}]

K^* = Roseland mean absorption parameter [m^{-1}]

Greek letters

γ' dimensional temperature jump (m)

γ dimensionless temperature jump (-)

λ' dimensional slip length (m)

λ dimensionless slip length (-)

τ skin friction coefficient

θ dimensionless temperature (-)

ν fluid kinematic viscosity (m^2 / s)

ρ fluid density (kg / m^3)

σ fluid electrical conductivity $(A^2 S^3 / kgm^3)$

σ^* = The Stefan-Boltzmann constant [$W\ m^{-2}\ K^{-4}$]

References

Ahmad, S. K., Muhammad, H. A., Hamza, M. M., Ojemeri, G., and Usman, U. (2024). Magnetized Oscillatory Jeffrey Fluid Flow in A Porous Channel with Unequal Wall Temperature, *Dutse Journal of Pure and Applied Sciences (DUJOPAS)*, 10(2c), 261-272.

Ajibade A. O., Umar A. M., and Kabir T. M. (2021). An analytical study on effects of viscous dissipation and suction/injection on a steady MHD natural convection Couette flow of heat-generating/absorbing fluid, *Advances in Mechanical Engineering*, 13(5), 1–12.

Bachok, N., Tajuddin, S. N. N., Anuar, N. S., and Rosali H. (2023). Numerical computation of stagnation point flow and heat transfer over a nonlinear stretching/shrinking sheet in hybrid nanofluid with suction/injection effects, *Journal of Advanced Research in Fluid Mechanics and Thermal Sciences*, 107(1).

Bejawada S. G. and Nandeppanavar M. M. (2023). Effect of thermal radiation on magneto-hydrodynamics heat transfer micropolar fluid flow over a vertical moving porous plate, Experimental and computational multiphase flow, 5(2), 149-158.

Chaudhary R. C. and Jain A. (2007). Combined heat and mass transfer effects on MHD free convection flow past an oscillating plate embedded in porous medium, *Romania Journal of Physics*, 52, pp. 505–524.

Gebhart, B. (1962): Effects of viscous dissipation in natural convection. *J. Fluid. Mech.* 14, 225–232. https://doi.org/10.1017/S0022112062001196.

Gireesha, B. J., Sowmya, G. Khan, M. I., and Öztop, H. F. (2020). Flow of hybrid nanofluid across a permeable longitudinal moving fin along with thermal radiation and natural convection, Computer Method and Programs in Biomedicine, 185, pp. 105166–105166.

Goud B. S., Srilatha P., Mahendar D., Srinivasulu T., and Reddy Y. D. (2023). Thermal radiation effect on thermostatically stratified MHD fluid flow through an accelerated vertical porous plate with viscous dissipation, Partial Differential Equation in Applied Mathematics, 7, 100488.

Hamza M. M., Sani D., Ojemeri G., and Shuaibu A. (2025). A Numerical Investigation of Porosity and Suction/ Injection Effects on Magnetized Slip Flow Affected by Temperature-dependent Property, Heat transfer Wiley, 55(4), 1-12. https://doi.org/10.1002/htj.70006

Hamza, M. M., Bello, I., Mustapha, A., Usman, U., and Ojemeri, G. (2023). Determining the role of thermal radiation on hydro-magnetic flow in a vertical porous super-hydrophobic microchannel, *Dutse Journal of Pure and Applied Sciences*, 9(2b), pp. 297-308.

Hasan, N. Z., Mohammed, Y., Nasreen, S.N. (2020). Effects of thermal radiation, heat generation, and induced Magnetic field on Hydromagnetic free convective flow of couple stress fluid in an isoflux-isothermal vertical channel, *Journal of Applied Mathematics*, vol.2020, Article ID 4539531.

Isa B. U., Yale I. D., Sarki, M. N., and Hamza M. M. (2024). Effects of Viscous Dissipative Fluid on Couette Flow in A Vertical Channel Due to Newtonian Heating, *International Journal of Science for Global Sustainability*, 10(1), pp. 76-82. DOI: https://doi.org/10.57233/ijsgs.v10i1.595

Jamaludin A., Naganthran K., Nazar R., Pop I. (2020). Thermal radiation and MHD effect in the mixed convection flow of Fe3 O4 water Ferrofluid towards a nonlinear moving surface, Processes, 8, (95). https:doi.org.10.3390/pr8010095

Jha, B. K., Altine M. M., and Hussaini, A. M. (2022). Role of suction/injection on free convective flow in a vertical channel in the presence of a point/line heat source/sink, *ASME Journal of Heat Transfer*, 144, 062602.

Jha, B. K., Azeez, L. A., and Oni, M. O. (2019). Unsteady Hydromagnetic-Free convection flow with suction/injection, *Journal of Taibah University*, 13, 136–145.

Jha, B. K., Isah, B. Y., and Uwanta, I. J. (2018). Combined Effect of Suction/Injection on MHD Free-Convection Flow in a Vertical Channel with Thermal Radiation, *Ain Shams Engineering Journal*, 9(4), 1069–1088.

Kamis, N. I., Bashir, Md. F., Shafie, S., Khairuddin T. K. A., and Jiann L. Y. (2021). Suction effect on an unsteady Casson hybrid nanofluid film past a stretching sheet with heat transfer analysis, *IOP Conference Series: Materials Science and Engineering*, 1078.

Kamis, N. I., Jiann, L. Y., Shafie, S., Khairuddin T. K. A., and Faisal Md. B. (2022). Magnetohydrodynamics boundary layer flow of hybrid nanofluid in a thin film over an unsteady stretching permeable sheet, *Journal of Nanofluid*, 11(1), 74-83.

Kamis, N. I., Rawi, N. A., Jiann, L. Y., Shafie, S., and Ilias, M. R. (2023). Thermal characteristics of an unsteady hybrid nano-Casson fluid passing through a stretching thin-film with mass transition, *Journal of Advanced Research in Fluid Mechanics and Thermal Sciences*, 104(2).

Karouei, S. H. H., Shani, M. B., Sekaloo, M., Eimeni, S. H. H., Pasha, P., and Ganji, D. D. (2024). Computational modelling of magnetized hybrid nanofluid flow and heat transfer between parallel surfaces with suction/injection, *International Journal of Thermofluids*, 22, 100613.

Khalid, A., Khan, I., and Khan, A. (2015). Unsteady MHD free convection flow of Casson fluid past an oscillating vertical plate embedded in a porous medium, *Engineering Sciences and Technology*, 18, 309–317.

Mishra, A. K., Paul T., and Singh, A. K. (2022). Mixed convention flow in a porous medium bounded by two vertical walls, *Forsch Ingenieurwes*, 67, 198–205.

Narender G., Govardhanb K., and Sarmaa G. S. (2021). Viscous dissipation and thermal radiation effects on the flow of Maxwell nanofluid over a stretching surface, *Int. J. Nonlinear Anal. Appl.* 12(2), 2008-6822. http://dx.doi.org/10.22075/ijnaa.2020.18958.2045.

Ojemeri G. and Onwubuya I. O. (2023). Significance of viscous dissipation and porosity effects in a heated superhydrophobic microchannel, *Journal of Engineering and Technology (JET)*, 14(2), 1-19.

Ojemeri G., Shuaibu A., Onwubuya I. O., Omokhuale E., and Rufa'i T. A. (2025). Casson fluid flow in a magnetized oscillatory system affected by variable thermal conductivity with asymmetric heating, *Journal of Mathematical Sciences and Computational Mathematics* (JMSCM), 6(1), 145-162. DOI: doi.org/10.15864/jmscm.6110

Omokhuale E. and Ojemeri G. (2024). Couette flow in the presence of viscous dissipative fluid along an upstanding channel affected by Newtonian heating: homotopy perturbation approach, *Journal of Basic Physical Research*, 12(1), pp. 1-12. ISSN: 2141-8403.

Omokhuale E., Abubakar J. and Ojemeri G. (2024b). Mixed Convection Flow of a Magnetized Nanofluid with Viscous Dissipation in an Oscillatory System, *International Journal of Science for Global Sustainability*, 10(2), pp. 229-241. DOI: https://doi.org/10.57233/ijsgs.v10i2.672.

Omokhuale E., Ojemeri G., Muhammad A., and Usman H. (2024a). Significance of Viscous Dissipation on Hydromagnetic Oscillatory Flow Affected by Nanoparticles Through a Boundary Layer Regime, International Journal of Development Mathematics 1(1), pp. 098-113.

Onwubuya I. O. and Ojemeri G. (2023). Evaluation of mixed convection-radiation flow of a viscous fluid restricted to a vertical porous channel: A comparative study, *Caliphate Journal of Science and Technology (CajoST)*, 5(3), pp. 67-75.

Parthiban S. and Prasad V. R. (2023). Thermal radiation and Hall current effects in a MHD non-Darcy flow in a differentially heated square enclosure, Lattice-Boltzmann simulation, Journal of porous media, Begell house, 26(5), pp 37-56.

Raptis A. (1998). Flow of microplolar fluid past a continuously moving plate by the presence of radiation, International Journal of Heat and Mass Transfer, 41, pp. 2865-2866.

Raptis A. (1999). Radiation and viscoelastic flow, International Journal of Heat and Mass Transfer, 26, pp. 889-895.

Uka U. A., Emeziem I. C., Ayinde S. A., Adenika C., and K. O. Agbo (2023). Enhancement of heat transfer with viscous dissipation impact on fluid flow past a moving wedge in a permeable domain, *Eskişehir Technical University Journal of Science and Technology A- Applied Sciences and Engineering*, 24(3), pp. 164-176, DOI:10.18038/estubtda.. 1197024

Uwanta, I. J. and Hamza, M. M. (2014). Effect of Suction/Injection on Unsteady Hydro-Magnetic Convective Flow of Reactive Viscous Fluid Between Vertical Porous Plates with Thermal Diffusion, *International Scholarly Research*, Article ID 980270, 1–14.

Yale I. D., Uchiri A. M. T., Hamza M. M., and Ojemeri G. (2023). Effect of viscous dissipation fluid in a slit microchannel with heated superhydrophobic surface, *Dutse Journal of Pure and Applied Sciences*, 9(3b), pp. 290-302.